THE

BUILDER'S GUIDE;

OR

A PRACTICAL TREATISE ON THE SEVERAL ORDERS

OF

GRECIAN AND ROMAN ARCHITECTURE,

TOGETHER WITH THE

GOTHIC STYLE OF BUILDING;

CONSTITUTING A COMPLETE EXPOSITION OF THE

MOST MODERN AND APPROVED METHODS

ADOPTED BY SKILFUL ARCHITECTS IN THE VARIOUS DEPARTMENTS OF

CARPENTRY, JOINERY, MASONRY AND SCULPTURE,

EMBRACING ALL THEIR NECESSARY DETAILS,

AND BY A PLAIN AND COMPREHENSIVE ARRANGEMENT,

PARTICULARLY ADAPTED TO THE WANTS OF THE LESS EXPERIENCED.

ILLUSTRATED AND EMBELLISHED WITH

SEVENTY FOLIO PLATES.

DRAWN ON A LARGE SCALE.

BY CHESTER HILLS,
PRACTICAL ARCHITECT.

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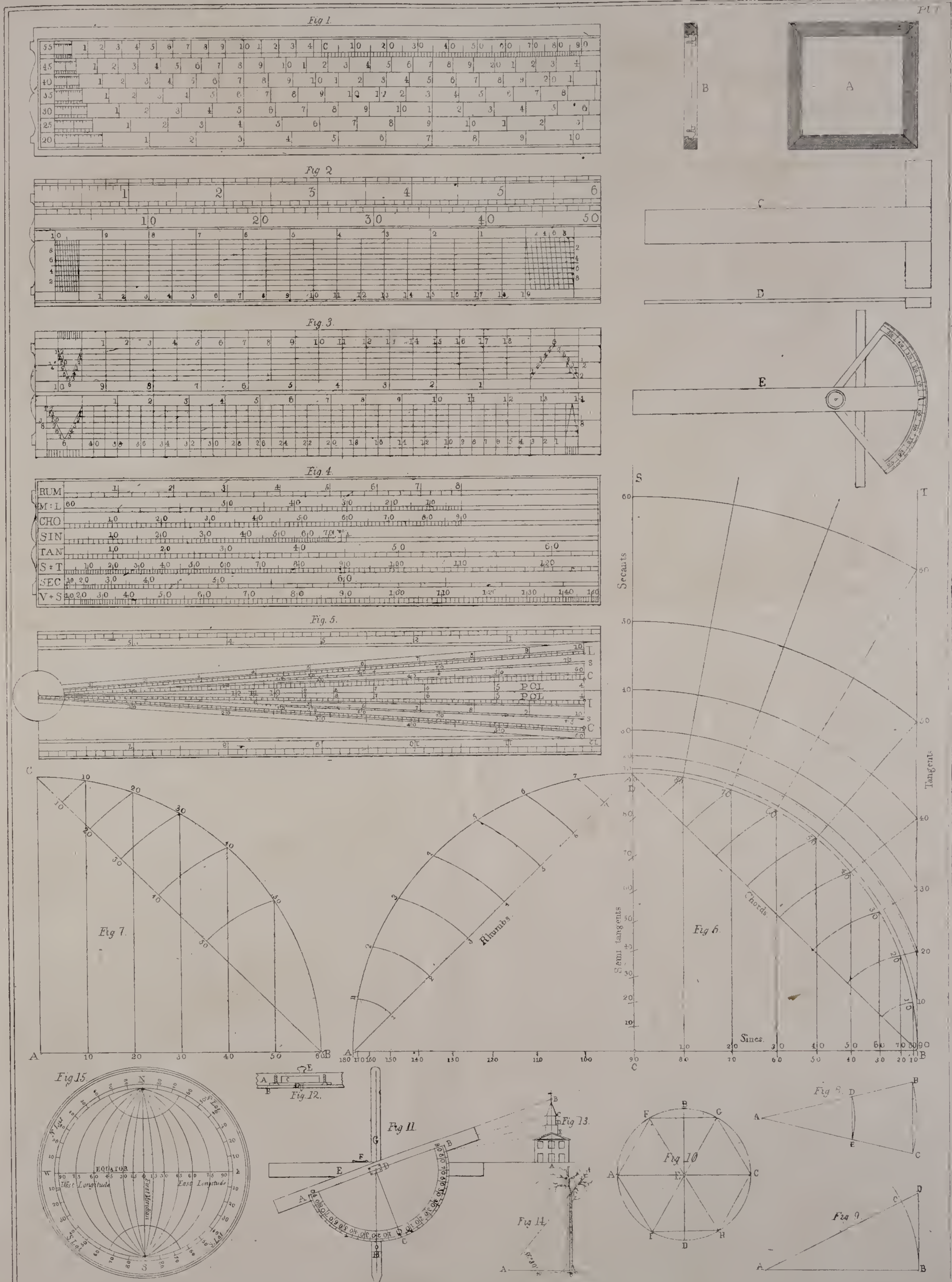

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THE BUILDER'S GUIDE.

INTRODUCTORY REMARKS.

It is necessary, to the understanding of this work, that the Practical Builder should have a knowledge of the properties, relations, and positions of lines as explained in some common treatise for representing geometrical truths.

The course adopted by the Ancients is generally regarded as the most satisfactory; it not only accustoms the Architect to reason correctly, which is indispensable, but also directs him in a course distinct from analysis, which in important mathematical research renders propositions clear to his mind, without the assistance of rule and compass. It is only by being competent to demonstrate from principles previously established, and continue a chain of reasoning in such a manner as will render the conclusion from one truth, part of the data requisite for the proof of a succeeding proposition, that the Architect can become distinguished.

This science being fully explained in the writings of Archimedes and Euclid, and having served as a model for all successive works, I have thought proper to adopt in this treatise so much of those "elements" as will be required by the Architect in building.

If the builder attempts to apply the rules of Geometry to his art, without the knowledge of theory, his efforts will prove abortive; or should he at all succeed, yet his work would be void of proportion and incomplete. It is only by a competent knowledge of this science, that the Architect can accomplish his work in a simple and elegant style; or the Artist so construct his lines as to be able to complete his design.

The completion of a design is usually left to the skill of the workman, who is supposed to be thoroughly acquainted with the execution of the task he undertakes. But if he is ignorant of the "geometrical construction" of the object to be executed, he is not only incompetent for the task he has commenced, but at every additional advance, displays to the world his inability and ignorance: such a person, so long as he remains uninformed, will be unprepared for any undertaking, and his labours in no manner useful, unless under the guidance and direction of another.

The definitions, theorems and problems which are here subjoined, are intended to instruct the uninformed, and prepare him to proceed with the remaining part of this work, wherein their application will be found absolutely necessary. The terms are as clearly stated as the nature of the present work will admit; and the theorems and problems are placed in succession, so that nothing is introduced "as taken for granted," but every thing has been previously proved and explained.

The selection, though limited, is deemed sufficient to render the Architect able to proceed through the remaining part of the treatise, with facility and advantage.

DEFINITION OF TERMS IN GEOMETRY.

1. *A Point.*—Abstractedly considered, a point is said to have position without magnitude, and is therefore represented to the eye by the smallest *visible mark, or dot.*

2. A *Line* is length without breadth or thickness; it is represented by the motion of a visible point.

3. A *right or strait line* is the shortest that can be drawn between two given points—as A B (fig. 1, pl. 2.)

Every line which is not strait, or composed of strait lines, is a *curve line*, thus A B (fig. 2.)

4. A *superficies or surface* is extention of length and breadth without thickness.

5. A *Plane Superficies* is a flat surface which coincides in every point, with a right line.

6. A *Plane Figure, or Diagram,* is the lineal representation of any object on a plane surface. If the lines forming the figure be straight, the figure is said to be *rectilineal.*

7. *Parallel Lines* are in every part equally distant from each other, and cannot meet how far soever, either way, they be produced, as A B and C D (fig. 3. and 4.)

8. An *Angle* is the space enclosed between two lines meeting in a point. See A B C (fig. 5.); the letter A denotes the vertex or angular point, this is a rectilineal angle, which is formed by two straight lines meeting in the point (A.)

9. *Converging Lines* are right lines so inclined to each other as to meet if produced to a certain point. Thus, AB and CD (fig. 6,) converge to each other, and if produced will meet in the angle O.

10. *Right and Oblique Angles.*—If one right line meets another, so as to make the angles on each side equal, each angle is called a *Right Angle*, and the line which meets the base, or lower line, is called a *perpendicular*. Thus in (fig. 7) the line CD is drawn perpendicular to AB and makes the angles on both sides of CD equal; each of these angles is called a *right angle*. In (fig. 8) the line C D does not make the angles on each side of it equal to each other; therefore CD is said to be drawn obliquely to the lines AB; while in (fig. 7.) CD is at right angles with the line AB.

11. An *Acute Angle* is less than a right angle.

12. An *Obtuse Angle* is greater than a right angle. In (fig. 8.) the line CD makes the angles on each side of it unequal; therefore, one must be greater than the other: the greater is an *obtuse*, and the lesser, an *acute* angle: for it is obvious that whatever be the position of the line CD, relative to AB, the excess of one angle more than a right angle the other must want in order to be equal to the same.

Examples.—(Fig. 9) is an *Acute* Angle; (fig. 10.) a *Right* Angle, and (fig. 11) an *Obtuse* Angle.

13. A *Plane Triangle* is a space inclosed by three right lines, as ABC, (fig. 12.)

14. A *Right-Angled Triangle* is that which has one right angle; as ABC (fig. 12.) The longest side is called the *Hypothenuse*, and the other two, *Legs*, or base and perpendicular. Thus, AC the Hypothenuse, AB the base, and BC the perpendicular.

15. An *Acute-Angled Triangle* is a triangle which has all its angles acute; as (fig. 13 and 14.)

16. An *Obtuse-Angled Triangle* is a triangle having one obtuse angle; as (fig. 15.)

17. An *Equilateral Triangle* is a triangle having all its sides equal; as (fig. 13.)

18. An *Isosceles Triangle* is a triangle having two equal sides; as (fig. 14.)

19. A *Salene Triangle* is a triangle having no two of its sides equal; as (fig. 15.)

20. A *Parallelogram* is a figure of which the opposite sides are parallels. Thus the (fig. 16, 17, 18 and 19) are parallelograms.

21. When the *parallelogram* has a right angle, it is called a *rectangle*. Thus the (figures 16 and 17) are rectangles.

22. If the sides of the *rectangle* be equal, it is called a square. See (fig. 16.)

23. If the two adjacent sides be unequal, the rectangle is termed an *oblong*; as (fig. 17.)

24. If only two opposite angles of a parallelogram be equal, the figure is called a *rhombus*; as (figures 18 and 19.)

25. If two adjacent sides of a rhombus be equal, the figure is called a *rhom-bo-id*; as (fig. 18.)

26. Every figure, inclosed by four right lines, is called a *quadrangle*, or *quadrilateral*. Thus (figures 16, 17, 18, 19, 20 and 21) are quadrangles or quadrilaterals.

27. When all the sides of a quadrilateral are unequal, it is called a *trapezium*; as (fig. 20.)

28. If two sides of a trapezium be parallel, it is called a *tra-pe-zo-id*; as (fig. 21.)

29. Figures having equal sides and equal angles, or equilateral and equiangular figures, formed by more than four right lines, are called regular *polygons.*

30. A regular polygon of five sides is called a *pentagon*; as (fig. 22.)

31. A regular polygon of six sides is called a *hexagon*; as (fig. 23.)

32. A regular polygon of seven sides is called a *heptagon*; as (fig. 24.)

33. A regular polygon of eight sides is called an *octagon*; as (fig. 25.) Of nine sides, an *eneagon* or *nonagon*. Of ten sides, a *decagon*. Of eleven sides, an *undecagon*. Of twelve sides, a *duodecagon*. Of fifteen sides a *quindecagon*; but polygons having more than twelve sides are commonly expressed as such, with the number of sides given.

34. A circle is a plane figure formed by one uniform curved line which is called its circumference; as (fig. 26.)

35. The centre of a circle is the point in the middle of it, as A, in (fig. 26) and the line A B drawn from the centre to the circumference, is the radius of the circle; all lines thus drawn are equal.

36. The diameter of a circle is a right line drawn through the centre and terminated by the circumference; as A B (fig. 27.)

37. A chord of a circle is a right line drawn from one point of a circle to another, and dividing it into unequal or equal parts or segments. In the latter case, the chord is also the diameter. Thus C D (fig. 28) is a chord as well as A B (fig. 27.)

38. A *Semi-circle* is one half of a circle, as divided into two equal parts by the diameter.

39. A *Segment* of a circle is that portion which is cut off by a chord. Thus, in (figures 28 and 29,) C D and F H, are segments; and (fig. 30) though a semi-circle, is still a segment, terminated by the diameter.

40. A *Sector* is the portion of a circle formed by two radii and the intercepted part of the circumference; as A B C, (fig. 31.)

41. A *Quadrant* is the fourth part, or quarter of a circle; or in other words, a sector contained by two radii, forming a right angle at the centre, and the intercepted part of the circumference; as A B C (fig. 32.)

42. An *Arc* is any portion of the circumference of a circle.

43. The *Sine* of an arc is a line drawn from one end of the arc perpendicular to a diameter drawn through the other end of the same arc. Thus K B (fig. 33) is the sine of the arc A B, K B being a line drawn from one end B of that arc, perpendicular to D A which is the diameter passing through the other end A of the arc.

44. The *Co-sine* of an arc is the sine of the complement of that arc, or of what that arc wants to make it a quadrant: thus A H being a quadrant, the arc B H is the complement of the arc A B; B E is the sine of the arc B H, or the co-sine of the arc A B.

45. The *Versed* sine of an arc, is that part of the diameter contained between the sine and the arc; thus K A is the versed sine of the arc A B, and D C K is the versed sine of the arc D H B.

46. The *Tangent* of an arc is a right line drawn perpendicular to the diameter passing through one end of the arc, and terminated by a line drawn from the centre through the other end of the arc; thus A G is the tangent of the arc A B.

47. The *Co-tangent* of an arc is the tangent of the complement of that arc to a quadraut; thus H F is the tangent of the arc H B or the co-tangent of the arc A B.

48. The *Secant* of an arc is a right line drawn from the centre through one end of the arc to meet the tangent drawn from the other end; thus C G is the secant of the arc A B.

49. The *Co-secant* of an arc is the secant of the complement of that arc to a quadrant; thus C F is the secant of the arc B H, or co-secant of the arc A B.

50. What an arc wants of being a semicircle is called the supplement of the arc; thus, the arc D H B is the supplement of the arc A B. The sine, tangent or secant of an arc is the same as the sine, tangent, or secant of its supplement; thus the sine of eighty degrees is equal to the sine of one hundred and the sine of seventy is equal to the sine of one hundred and ten, &c.

51. *Equivalent figures* are such as have equal surfaces, without regard to their form.

52. *Identical figures* are such as would entirely coincide, if the one be applied to the other.

53. In *Equiangular figures*, the sides which contain the equal angles, and which adjoin equal angles are homologous.

54. Two figures are similar, when the angles of the one are equal to the angles of the other each to each, and the homologous sides are proportionals.

55. In two circles, similar sectors, similar arcs, or segments, are those which have equal angles at the centre. Thus if the sector A B C (fig. 34) be similar to the sector D E F in (fig. 35) then the angle A B C will be equal to the angle D E F; or if the arc A C be similar to the arc D F, then the angle at B will be equal to the angle at E. Also if the segment G M H (fig. 36) be similar to the segment K N L in (fig. 37) then the angle I, will be equal to the angle R.

56. The altitude of a figure is a right line drawn from the top, or vertical angle perpendicular to the base or opposite side, or to the base produced or continued. Thus, C D is the altitude of the triangle A B C in (fig. 38) or C D is the altitude of A B C (fig. 39.)

The altitude of a parallelogram is the perpendicular which measures the distance of two opposite sides, taken as bases. Thus, E F (fig. 40) is the altitude of the parallelogram D B.

57. The area of a figure is the quantity of surface containing a certain number of units of any given scale; as of inches, feet, yards, &c.

DEFINITIONS OF THE ELLIPSE.

58. That portion of the primary line terminated at each extremity by the vertices of the curve, is called the major, or transverse axis.

59. A straight line, drawn perpendicularly to the axis major, from any point in it to meet the curve, is called an ordinate.

60. The middle of the axis major is called the centre of the figure. Thus, A A (fig. 41) is the axis major, P M an ordinate to it, and the point C in the middle of A A is the centre of the ellipse A M A.

61. A straight line drawn through the centre perpendicularly to the axis major and terminated by the curve, is called the axis minor, or conjugate axis.

62. A third proportional to the axis major and minor, is called the parameter, or the latus rectum of the axis.

63. That point in the axis, cut by an ordinate, which is equal to half the parameter is called the focus. In (fig. 42) B B, drawn through C, in the semi-axis-minor; and if L L be a third proportional to A A, B B, then L L is the parameter, and the point F, where it cuts A A, is the focus. Or thus; if two pins are fixed at the points A and B, as in (fig. 43,) a string being put about them, and the ends fastened together at C; the point C being moved round, keeping the string stretched it will describe a curve called an ellipsis, and the two points A and B, about which, the string is made to revolve, is the foci.

64. Any line drawn through the centre, and terminated at each extremity by the curve, is called a diameter.

65. A diameter which is parallel to a tangent at one extremity of another diameter, is called a conjugate diameter to that other diameter.

66. A straight line parallel to a tangent, at the extremity of any diameter, terminated at one extremity by that diameter, and the curve at the other, is called an ordinate to that diameter.

67. The portion of a diameter between the centre and an ordinate, is called the abscissa of that ordinate, or of that diameter. In (fig. 44) the straight line A A, drawn through the centre, C, is a diameter, and if S T be a tangent at A, and the diameter B B be drawn parallel to S T, the diameter is called the conjugate diameter of A A and P M, parallel to S T or B B, is an ordinate to the diameter A A; and the distance C P, on the diameter A A, is called the abscissa.

EXPLANATION OF TERMS AND SIGNS USED IN THIS WORK CONTINUED.

68. An Axiom is a self-evident proposition.

A Theorem is a truth, which becomes evident by means of a train of reasoning called a demonstration.

A Problem is a question proposed, which requires a solution.

A Lemma is a subsidiary truth, employed for the demonstration of a theorem, or the solution of a problem.

The common name proposition, is applied indifferently to theorems, problems, and lemmas.

A Corollary is an obvious consequence deduced from one or several propositions.

A scholium is a remark on one or several preceding propositions which tends to point out their connexion, their use, their restriction, or their extension.

An Hypothesis is a supposition, made either in the enunciation of a proposition, or in the course of a demonstration.

The sign + is pronounced, *plus*: it indicates addition, and denotes that whatever number or quantity follows the sign, must be added to those that go before it; thus 6+5 signifies that 5 is to be added to 6, or A+B implies that the quantities represented by A and B, are to be added together.

Again, A+B+C+D implies that B is to be added to A; C to the sum of A, and B and D, to the sum of A, B, C.

The Sign — is pronounced minus; it indicates subtraction, and denotes that the number following it must be subtracted from those going before it; thus 7—3 signifies that 3 must be subtracted from 7 or m-n implies that the quantity represented by n, is to be subtracted from that represented by m. Suppose for instance that m is 7, and n 3; then 7—3 will be 4. Therefore m-n denotes the remainder arising by subtracting n from m.

The sign × indicates multiplication; and shows that the numbers placed before and after it are to be multiplied; thus 5×8 signifies that 5 is to be multiplied by 8, which makes 40; and 4×6×2, signifies the continued product of 4 by 6 and by 2 which makes 48; or A×B signifies that A is to be multiplied by B.

The sign ÷ indicates division, and signifies that the numbers that stand before it are to be divided by the number following it, as 75÷16 shows that 75 is to be divided by 16; division may also be denoted by placing two points between the numbers thus 75:16 represents 75 divided by 16, or by placing the numbers thus $\frac{75}{16}$ which signigfies 75 divided by 16, ( ) or —. either of these marks are used for connecting numbers together: Thus $\overline{4+5}\times8$, or (4+5) +8, signifies that the sum of 4 and 5 are to be multiplied by 8.

The sign = indicates equality, and shows that the numbers or quantities placed before it are equal to those following: thus 8×10=80, or 8 multiplied by 10 is equal to 80, and 6+2×7=56, or A×B=C D.

The sign : :: : signifies proportion, and is marked thus 6:12::10:20, that is, as 6 is to 12, so is 10 to 20, or A:B::C:D, that is, as A is to B, so is C to D.

Roots are usually represented by the following characters or exponents :—

$\sqrt{}3$, or $3^{\frac{1}{2}}$, denotes the square root of the number 3.

$\sqrt[3]{}5$, or $5^{\frac{1}{3}}$, denotes the cube root of the number 5.

7^2, denotes that the number 7 is to be squared.

8^3, denotes that the number 8 is to be cubed.

The square root of the line A B is designated by A B 2; its cube by A B3.

A number placed before a line, or a quantity, serves as a multiplier to that line or quantity; thus 3 A B, signifies that the line A B is taken three times; $\frac{1}{2}$A signifies the half of the angle A.

° Signifies Degrees; thus 35° represents 35 degrees.

′ Signifies Minutes thus; thus 26′ or 26 minutes.

″ Signifies Seconds; thus 54″ or 54 seconds.

‴ Signifies Thirds or sixtieth parts of seconds; thus 43‴ or 43 thirds.

Again, 35° 26′ 54″ 43‴ implies 35 degrees, 26 minutes, 54 seconds, and 43 thirds.

S Signifies Sine.—Sec. signifies Secant.—Tan. signifies Tangent.

Co-sine, Co-tangent or Co-secant of an arc signifies the sine, tangent or secant of the complement of that arc respectively.

∠ Signifies Angle with an s at top angles $\angle^s$ or with a d angled $\angle^d$.

△ Signifies Triangle. □ Signifies a Square.

AXIOMS.

69. 1. Things that are equal to the same thing, or to equal things, are equal to one another. 2. If equal things be added to equals, the sums are equal. 3. If equal things be taken from equals, the remainders will be equal. 4. If equal things be added to unequals, the sums are unequal. 5. The halves of equal things are equal, and double the parts of equal things are equal. 6. Magnitudes that mutually agree, or fill equal spaces are equal to one another. 7. The whole is greater than a part, and equal to the sum of all its parts. 8. Only one right line can be drawn from one point to another. 9. Two right lines can not be drawn through the same point parallel to another right line, without coinciding with each other. 10. All right angles are equal to each other. 11. Equal circles have equal semi-diameters.

POSTULATES OR DEMANDS.

70. A Postulate signifies something which may be assumed as granted.—
1. That a right line may be drawn from any one point to any other point.
2. That a right line may be produced or continued at pleasure in a right line.
3. That a circle may be described from any centre with any radius.

THEOREMS.

THEOREM I.

71. *All right angles are equal to each other.* Let the straight line C D (fig. 45, and 46, pl. 2) be perpendicular to A B, and G H to E F ; the angles A C D and E G H will be equal to each other.

Demonstration.—Take the four distances C A, C B, G E, G F, all equal ; the distance A B will be equal to the distance E F, and the line E F being placed on A B, so that the point E falls on A, the point F, will fall on B. Those two lines will thus coincide entirely ; for otherwise there would be two straight lines extending from A to B, which (Def. 69 8) is impossible : and hence G the middle point of E F, will fall on C, the middle point of A B. The side G E being thus applied to C A, the side G H must fall on C D. For suppose, if possible, that it falls on a line C K, different from C D : then since by hypothesis (10) the angle E G H=H G F, A C K would in that case be equal to K C B. But the angle A C K is greater than A C D ; and K C B is smaller than B C D, but by hypothesis A C D=B C D ; hence A C K is greater than K C B. Therfore the line G H cannot fall on a line C K different from C D ; therefore it falls on C D, and the angle E G H on A C D ; therefore all right angles are equal to each other (69. 6.)

THEOREM II.

72. Every straight line, which meets another, makes with it two adjacent angles the sum of which is equal to two right angles. Let A B and C D (fig. 47.) be the straight lines meeting each other at C, then will the angle A C D + the angle D C B be equal to two right angles.

At the point C, erect C E perpendicular to A B. The angle A C D, is the sum of the angles A C E, E C D ; therefore A C D + B C D is the sum of the three angles A C E, E C D, B C D ; but the first of those three angles, is a right angle and the other two together make up the right angle B C E hence the sum of the two angles A C D and B C D is equal to two right angles.

73. Cor. 1. If one of the angles A C D, B C D in (fig. 48) is right the other must be right also.

74. Cor. 2. If the line D E is perpendicular to A B, reciprocally A B will be perpendicular to D E.

For since D E is perpendicular to A B, the angle A C D must be equal to its adjacent one D C B and both of them must be right. But since A C D is a right angle, its adjacent one A C E must also be right ; hence the angle A C E=A C D ; therefore A B is perpendicular to D E.

75. Cor. 3. (fig. 49.) The sum of all the successive angles B A C, C A D, D A E, E A F, formed on the same side of a straight line B F, is equal to two right angles ; because their sum is equal to that of the two adjacent angles, B A C, C A F.

THEOREM III.

76. Whenever two straight lines intersect each other, the opposite or vertical angles which they form are equal.

Let A B and D E (fig. 50) be the given straight lines intersecting each other at C ; then is the angle E C B=A C D, and the angle A C E=D C B.

For since D E is a straight line the sum of the angles A C D, A C E, is equal to two right angles ; and since A B is a straight line, the sum of the angles A C E B C E, is also equal to two right angles ; hence the sum A C D+A C E is equal to the sum A C E+B C E. Take away from both, the same angle A C E ; there remains the angle A C D, is equal to its opposite or vertical angle B C E. It may be shown in the same manner, that the angle A C E is equal to its opposite angle B C D.

77. Scholium. The four angles formed about a point by two straight lines which intersect each other are together equal to four right angles ; for the sum of the two angles A C E, B C E, is equal to two right angles ; and the other two A C D, B C D, have the same value ; therefore the sum of the four, is four right angles.

In general if any number of straight lines, C A, C B, C D, &c. as in (fig. 51.) meet in a point C, the sum of all the successive angles A C B, B C D, D C E, E C F, F C A, will be equal to four right angles ; for if four right angles were formed about the point C by means of two lines perpendicular to each other the same space would be occupied either by the four right angles, or by the successive angles A C B, B C D, D C E, E C F, F C A.

THEOREM IV.

78. Two triangles are equal when an angle and the two sides which contain it, in the one are respectively equal to an angle and the two sides which contain it, in the other.

Let the angle A (figs. 52 and 53) be equal to D, the side A C equal to the side D F, the side A B equal to D E ; then will the triangle A B C be equal to D E F. For these triangles may be applied to each other, so that they shall perfectly coincide. If the side D E be placed on its equal A B the point D will fall on A, and the point E on B ; and since the angle D is equal to the angle A, when the side D E is placed on A B, the side D F will take the direction A C. Besides, D F is equal to A C ; therefore the point F will fall on C, and the third side E F will exactly cover the third side B C ; therefore (69. 6.) the triangle D E F is equal to the triangle A B C.

79. Cor. When in two triangles, these three things are equal, namely, the angle A=D, the side A B=D E, and the side A C=D F, the other three are equal also, namely, the angle B=E the angle C=F and the side B C=E F.

THEOREM V.

80. In an isosceles triangle the angles opposite to the equal sides are equal.

Let the side A B (fig. 54) be equal to A C, the angle C will be equal to B.

Join A the vertex, and D the middle point of the base B C. The triangles A D B, A D C, have all the sides of the one respectively equal to those of the other, A D being common, A B=A C (hyp.) and B D=D C by construction ; therefore by the last proposition, the angle B is equal to the angle C.

81. Cor. An equilatoral triangle is likewise equiangular, that is to say has all its angles equal.

82. Scholium. The equality of the triangles A B D, A C D, proves also that the angle B A D is equal to D A C, and B D A to A D C ; hence the latter two are right angles ; hence the line drawn from the vertex, of an isosceles triangle to the middle point of its base is perpendicular to that base and divides the angle at the vertex into two equal parts.

In a triangle which is not isosceles, any side may be assumed indifferently as the base ; and the vertex is, in that case, the vertex of the opposite angle. In an isosceles triangle, however, that side is specially assumed as the base which is not equal to either of the other two.

THEOREM VI.

83. From a given point without a straight line, only one perpendicular can be drawn to that line.

Let A (fig. 55) be the point, and D E the given line.

Let us suppose we can draw two, A B and A C. Produce one of them A B, till B F is equal to A B and joins F C.

The triangle C B F is equal to A B C ; for the angles C B F and C B A are right, the side C B is common and the side B F=A B ; therefore (78) those triangles are equal and the angle B C F=B C A. The angle B C A is right by hypothesis ; therefore B C F must be right also. But if the adjacent angles B C A, B C F, are together equal to two right angles, the line A C F must be straight ; from whence it follows, that between the same two points A and F, two straight lines can be drawn : which is impossible ; hence it is equally impossible that two perpendiculars can be drawn from the same point to the same straight line.

84. Scholium. At a given point C in the line A B, it is equally impossible to erect two perpendiculars to that line for (see the diagram of Art. 72) if C D and C E were those two perpendiculars the angle B C D would be right as well as B C E, and the part would thus be equal to the whole.

THEOREM VII.

85. Two right angled triangles are equal when the hypothenuse and a side of the one are respectively equal to the hypothenuse and a side of the other.

Suppose the hypothenuse A C=D F, (figs. 56 and 57) and the side A B=D E; the right angled triangle A B C will be equal to the right angled triangle D E F.

Their equality would be manifest, if the third sides BC and E F were equal. If possible suppose that those sides are not equal, and that B C is greater. Take B G=E F ; and join A G. The triangle A B G is equal to D E F ; for the right angles B and E are equal, the side A B=D E, and B G=E F ; hence these triangles are equal (78) and consequently A G=D F. Now (Hyp.) we have D F=A C ; and therefore A G=A C. But the oblique line A C cannot be equal to A G, which lies nearer the perpendicular A B ; therefore it is impossible that B C can differ from E F ; therefore the triangles A B C and D E F are equal.

THEOREM VIII.

86. If a straight line falling upon two other straight lines makes the alternate angles equal to one another, these two straight lines are parallel.

Let the straight line E F, (fig. 58) which falls upon the two straight lines A B, C D make the alternate angles A E F, E F D equal to one another ; A B is parallel to C D.

For if it be not parallel, A B and C D being produced shall meet either towards B, D, or towards A, C ; let them be produced and meet towards B, D, in the point G ; therefore G E F is a triangle, and its exterior angle A E F is greater than the interior and opposite angle E F G ; but it is also equal to it which is impossible : Therefore, A B and CD being produced do not meet towards B, D. In like manner it may be demonstrated that they do not meet towards A, C ; but those straight lines which meet neither way, though produced ever so far are parallel (7) to one another, A B therefore is parallel to C D.

THEOREM IX.

87. If a straight line falling upon two other straight lines makes the exterior angle equal to the interior and opposite upon the same side of the line ; or makes the interior angles upon the same side together equal to two right angles ; the two straight lines are parallel to one another. Let the straight line E F (fig. 59) which falls upon the two straight lines A B, C D make the exterior angle E G B equal to G H D, the interior and opposite angle upon the same side ; or let it make the interior angles on the same side B G H, G H D together equal to two right angles ; A B is parallel to C D.

Because the angle E G B is equal to the angle G H D, and also to the angle A G H, the angle A G H is equal to the angle G H D ; and they are the alternate angles ; therefore A B is parallel (86) to C D. Again because the angles B G H, G H D are equal (hy. hyp.) to right angles, and A G H, B G H are also equal (71. 1.) to two right angles, the angles A G H, B G H are equal to the angles B G H, G H D : Take away the common angle B G H ; therefore the remaining angle A G H is equal to the remaining angle G H D ; and they are alternate angles ; therefore A B is parallel to C D.

THEOREM X.

88. To draw a straight line through a given point parallel to a given straight line.

Let A (fig. 60) be the given point and B C the given straight line it is required to draw a straight line through the point A parallel to the straight line B C.

In B C take any point D, and join A D; and at the point A in the straight line A D make the angle D A E equal to the angle A D C: and produce the straight line E A to F.

Because the straight line A D which meets the two straight lines B C, E F, makes the alternate angles E A D, A D C equal to one another, E F is parallel (87) to B C, therefore the straight line E A F is drawn through the given point A parallel to the given straight line B C, which was to be done.

THEOREM XI.

89. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

Let A B C (fig. 61) be a triangle and let one of its sides B C be produced to D; the exterior angle A C D is equal to the two interior and opposite angles C A B, A B C; and the three interior angles of the triangle, viz. A B C, B C A, C A B, are together equal to two right angles.

Through the point C draw C E parallel (87) to the straight line A B; and because A B is parallel to C E and A C meets them, the alternate angles B A C, A C E are equal. Again, because A B is parallel to C E, and B D falls upon them the exterior angle E C D is equal to the interior and opposite angle A B C, but the angle A C E was shown to be equal to the angle B A C; therefore the whole exterior angle A C D is equal to the two interior and opposite angles C A B, A B C, to these angles add the angle A C B, and the angles A C D, A C B are equal to the three angles C B A, B A C, A C B, but the angles A C D, A C B, are equal (71) to two right angles; therefore also the angles C B A, B A C, A C B are equal to two right angles.

90. Cor. 1. All the interior angles of any rectilineal figure are equal to twice as many right angles as the figure has sides, wanting four right angles.

For any rectilineal figure A B C D E (fig. 62) can be divided into as many triangles as the figure has sides, by drawing straight lines from a point F within the figure to each of its angles. And by the proceeding proposition, all the angles of these triangles, are equal to twice as many right angles as there are triangles, that is as there are sides of the figure; and the same angles are equal to the angles of the figure, together with the angles at the point F, which is the common vertex of the triangles; that is, together with four right angles. Therefore twice as many right angles as the figure has sides, are equal to all the angles of the figure, together with four right angles that is the angles of the figure are equal to twice as many right angles as the figure has sides, wanting four.

91. Cor. 2. All the exterior angles of any rectilineal figure are together equal to four right angles.

Because every interior angle A B C (fig. 63,) with its adjacent exterior A B D is equal (71) to two right angles; therefore all the interior, together with all the exterior angles of the figure, are equal to twice as many right angles as there are sides of the figure; that is by the foregoing corollery, they are equal to all the interior angles of the figure, together with four right angles; therefore all the exterior angles are equal to four right angles.

THEOREM XII.

92. The opposite sides and angles of a parallelogram are equal to one another and the diameter bisects it, that is divides it into two equal parts.

Note.—A parallelogram is a four sided figure; (20) of which the opposite sides are parallel; and the diameter is the straight line joining two of its opposite angles.

Let A C D B (fig. 64) be a parallelogram of which B C is a diameter; the opposite sides and angles of the figure, are equal to one another; and the diameter B C bissects it.

Because A B is parallel to C D, and B C meets them, the alternate angles A B C, B C D are equal to one another; and because A C is parallel to B D, and B C meets them, the ultimate angles A C B, C B D are equal to one another, wherefore the two triangles A B C, C B D have two angles A B C, B C A in one, equal to two angles B C D, C B D in the other, each to each and the side B C which is adjacent to these equal angles, common to the two triangles; therefore their other sides are equal, each to each, and the third angle of the one to the third angle of the other, viz: the side A B to the side C D, and A C to B D, and the angle B A C equal to the angle B D C. And because the angle A B C is equal to the angle B C D, and the angle C B D to the angle A C B, the whole angle A B D is equal to the whole angle A C D: and the angle B A C has been shown to be equal to the angle B D C; therefore the opposite sides and angles of a paralellogram are equal to one another, also its diameter bisects it; for A B being equal to C D, and B C common, the two A B, B C are equal to the two D C, C B, each to each; now the angle A B C is equal to the angle B C D; therefore the triangle A B C is equal (78) to the triangle B C D, and the diameter B C divides the parallelogram A C D B into two equal parts.

THEOREM XIII.

93. Parallelograms upon the same base and between the same parallels, are equal to one another.

Let the parallelograms A B C D, E B C F in (figs. 65, 66, and 67) be upon the same base B C, and between the same parallels A F, B C; the parallelogram A B C D is equal to the parallelogram E B C F.

If the sides A D, D F of the parallelograms A B C D, D B C E opposite to the base B C be terminated in the same point D; it is plain that each of the parallelograms is double (92) of the triangle B D C; and they are therefore equal to one another.

But if the sides A D, E F, opposite to the base B C of the parallelograms A B C D, E B C F, be not terminated in the same point; then because A B C D is a parallelogram, A D is equal (92) to B C; for the same reason E F is equal to B C; wherefore A D is equal (1 Ax) to E F; and D E is common; therefore the whole, or the remainder, A E is equal (2 or 3 Ax) to the whole, or the remainder D F; now A B is also equal to D C, therefore the two E A, A B are equal to the two F D, D C, each to each; but the exterior angle F D C is equal to the interior E A B, wherefore the base E B is equal to the base F C, and the triangle E A B (78) to the triangle F D C. Take the triangle F D C from the trapezium A B C F, and from the same trapezium take the triangle E A B; the remainder will then be equal (3 Ax,) that is the parallelogram A B C D is equal to the parallelogram E B C F.

THEOREM XIV.

94. Equal triangles upon the same base, and upon the same side of it, are between the same parallels.

Let the equal triangles A B C, D B C (fig. 68) be upon the same base B C, and upon the same side of it; they are between the same parallels.

Join A D: A D is parallel to B C; for, if it is not, through the point A draw (88) A E parallel to B C, and join E C. The triangle A B C is equal to the triangle E B C, because it is upon the same base B C and between the same parallels B C, A E. But the triangle A B C is equal to the triangle B D C; therefore also the triangle B D C is equal to the triangle E B C, the greater to the less, which is impossible; Therefore A E is not parallel to B C. In the same manner it may be demonstrated that no other line but A D is parallel to B C: A D is therefore parallel to it.

THEOREM XV.

95. If a parallelogram and a triangle be upon the same base, and between the same parallel; the parallelogram is double of the triangle.

Let the parallelogram A B C D (Fig. 69,) and the triangle E B C be upon the same base B C and between the same parallels BC, A E; the parallelogram A B C D is double of the triangle E B C.

Join A C; then the triangle A B C is equal to the triangle E B C, because they are upon the same base B C and between the same parallels B C, A E.

But the parollelogram A B C D is double (92) of the triangle A B C, because the diameter A C divides it into two equal parts; wherefore A B C D is also double of the triangle E B C.

THEOREM XVI.

96. The complements of the parallelograms which are about the diameter of any parallelogram, are equal to one another.

Let A B C D (fig. 70) be a parallelogram of which the diameter is A C; let E H, F G be the parallelograms about A C, that is through which A C passes and let B K, K D be the other parallelograms, which make up the whole figure A B C D, and are therfore called the complements. The complement B K is equal to the complement K D.

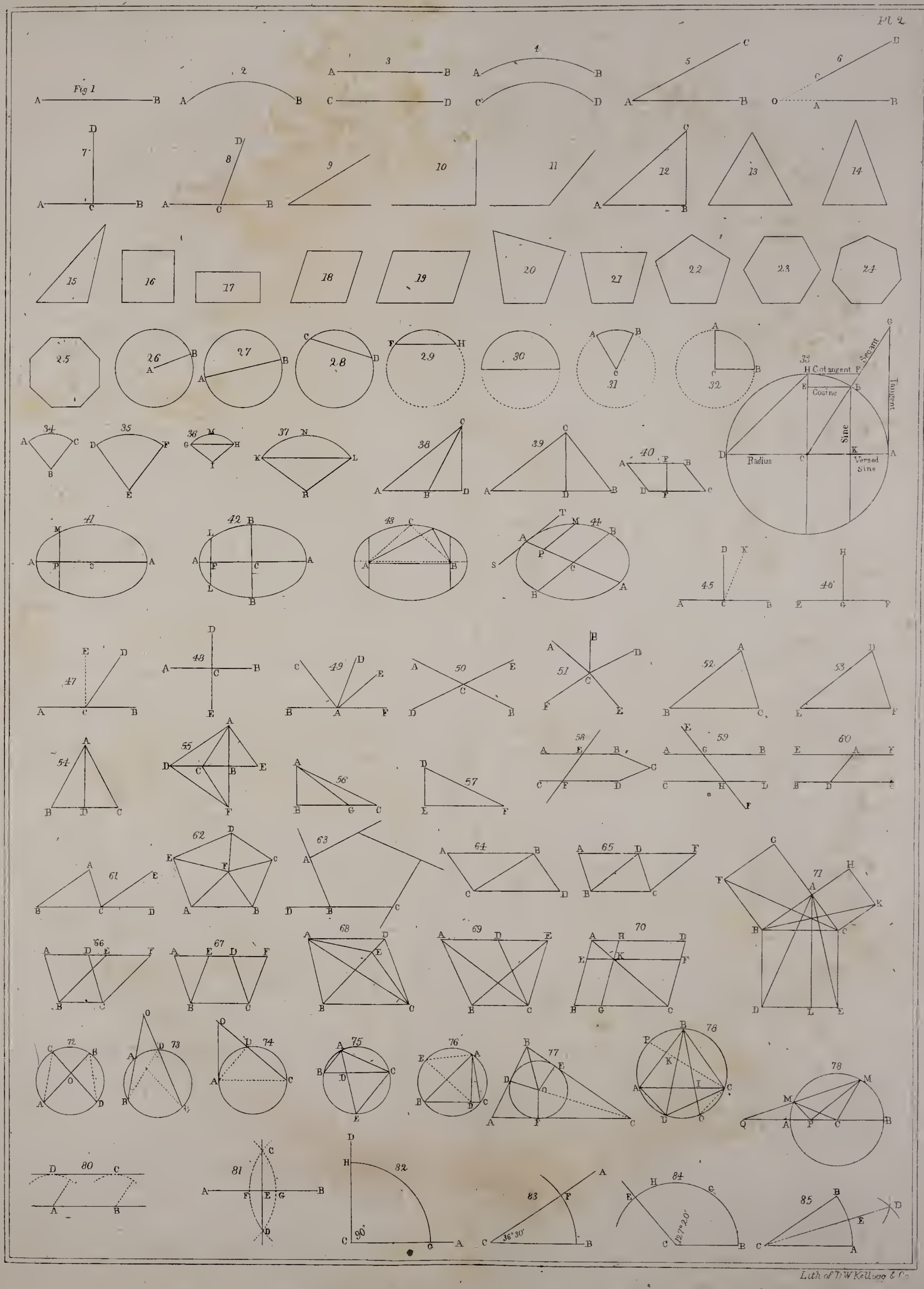
Because A B C D is a parallelogram and A C its diameter, the triangle A B C is equal (90) to the triangle A D C: And because E K H A is a parallellogram, and A K its diameter, the triangle A E K is equal to the triangle A H K: For the same reason, the triangle K G C is equal to the triangle K F C. Then because the triangle A E K is equal to the triangle A H K, and the triangle K G C to the triangle K F C; the triangle A E K together with the triangle K G C is equal to the triangle A H K together with the triangle K F C. But the whole triangle A B C is equal to the whole A D C; therefore the remaining complement B K is equal to the remaining complement K D.

THEOREM XVII.

97. In any right-angled triangle the square which is described upon the side subtending the right angle is equal to the squares described upon the sides which contain the right angle.

Let A B C (fig. 71) be a right angled triangle having the right angle B A C; the square described upon the side B C is equal to the squares described upon B A, A C.

On B C describe the square B D E C, and on B A, A C the squares G B, H C; and through A draw (86) A L parallel to B D or C E and join A D, F C; then because each of the angles B A C, B A G is a right angle (22) the two straight lines A C, A G upon the opposite sides of A B, make with it at the point A the adjacent angles equal to two right angles; therefore C A is the same stright line with A G; for the same reason A B, and A H are in the same straight line. Now because the angle DBC is equal to the angle FBA, each of them being a right angle, adding to each the angle A B C, the whole angle DBA will be equal to the whole FB C; and because the two sides AB, BD, are equal to the two FB, BC, each to each, and the angle D B A equal to the angle F B C, therefore the base A D is equal (78) to the base F C; and the triangle A B D to the triangle F B C. but the parallelogram B L is double (95) of the triangle A B D, because they are upon the same base B D, and between the same parallels B D, A L, and the square G B is double of the triangle B F C, because these also are upon the same base F B and between the same parallels F B, G C. Now the doubles of equals are equal to one another, therefore the parallelogram B L is equal to the square G B: And in the same manner, by joining A E, B K, it is demonstrated that the parallelogram C L is equal to the square H C. Therefore the whole square B D E C is equal to the two squares G B, H C, and the square B D E C is described upon the straight line B C, and the squares G B, H C, upon B A, A C: Wherefore the square upon the side B C, is equal to the squares upon the sides B A, A C.



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98. Cor. Hence in any right angled triangle, if we have the hypothenuse and one of the legs, we may easily find the other leg, by taking the square of the given leg, from the square of the hypothenuse the square root of the remainder will be the sought leg. Thus if the hypothenuse was 50 and one leg was 40 the other leg would be 30, for the square of 40 is 1600, and the square of 50 is 2500, by subtracting 1600 from 2500, it leaves 900, and the square root of 900 is 30. If both legs be given the hypothenuse may be found by extracting the square root of the sum of the squares of the legs, for we know that 6, 8 and 10 will form a right angled triangle, therefore the square of 6 is 36 and the square of 8 is 64, thus adding 36 and 64 together gives 100 whose square root is 10, which is the sought hypothenuse.

THEOREM XVIII.

99. The segments of two chords, which intersect each other, in a circle are reciprocally proportional.

Let the chords A B, and C D, (fig. 72) intersect at O, then will A O : D O : : O C : O B. Join A C and B D. In the triangles A C O, B O D the angles at O are equal, being vertical; the angle A is equal to the angle D, because both are inscribed in the same segment; for the same reason the angle C=B; the triangles are therefore similar and the homologous sides give the proportion A O : D O::C O:O B.

100. Cor. Therefore A O.O B=D O.C O; hence the rectangle under the two segments of the one chord is equal to the rectangle under the two segments of the other.

THEOREM XIX.

101. If from the same point without a circle, two secants be drawn terminating in the concave arc, the whole secants will be reciprocally proportional to their external segments.

Let the secants O B, O C (fig. 73) be drawn from the point O: then will O B: O C::O D : O A. For joining A C, B D, the triangles O A C, O B D have the angle O common; likewise the angle B=C; these triangles are therefore similar; and their homologous side give the proportion O B : O C::O D : O A.

102. Cor. The rectangle O A.O B is hence equal to the rectangle O C.O D.

103. Scholium. This proposition, it may be observed, bears a great analogy to the preceeding, and differs from it only as the two chords A B, C D, instead of intersecting each other within, cut each other without, the circle. The following proposition may also be regarded as a particular case of the proposition just demonstrated.

THEOREM XX.

104. If from the same point without a circle, a tangent and a secant be drawn, the tangent will be a mean proportional between the secant and its external segment.

From the point O (fig. 74,) let the tangent O A, and the secant O C be drawn; then will, O C:O A::O A:OD, or $O A^2$=O C.O D.

For joining A D and A C, the triangles O A D, O A C have the angle O common; also the angle O A D, formed by a tangent and a chord, has for its measure half of the arc A D; and the angle C has the same measure; hence the angle O A D=C therefore the two triangles are similar, and we have the proportion, O C:O A::A O:O D. which gives $O A^2$=O C.O D. (100).

THEOREM XXI.

105. If either angle of a triangle is bisected by a line terminated on the opposite side, the rectangle of the sides including the bisected angles is equal to the square of the bisecting line together with the rectangle contained by the segment of the third side.

Let A D (fig. 75) bisect the angle A; then will A:B A C=$A D^2$+B D.D C.

Describe a circle through the three points A, B, C; produce A D till it meets the circumference, and joins C E.

The triangle B A D is similar to the triangle E A C; for by hypothesis the angle B A D=E A C: also the angle B=E since they both are measured by half of the arc A C; hence these triangles are similar and the homologous sides give the proportion, B A : A E::A D : A C; hence B A . A C=A E.A D; but A E=A D+D E and multiplying each of these equals by A D, we have A E. A D=$A D^2$+A D. D E; now A D. D E=B D. D C hence finally B A.A C=A D +B D. D C.

THEOREM XXII.

106. In every triangle the rectangle contained by two sides, is equal to the rectangle contained by the diameter of the circumscribed circle, and the perpendicular let fall upon the third side.

In the triangle A B C (fig. 76) let A D be drawn perpendicular to B C; and let E C be the diameter of the circumscribed circle; then will A B A C=A D C E.

For joining A E, the triangles A B D, A E C are right-angled the one at D, the other at A; also the angle B=E; these triangles are therefore similar, and they give the proportion, A B: C E :: A D : A C and hence A B . A C=C E.A D.

107. Cor. If these equal quantities be multiplied by the same quantity B C there will result A B, A C, B C=C E, A D, B C; now A D, B C is double of the surface of the triangle; therefore the product of the three sides of a triangle is equal to its surface multiplied by twice the diameter of the circumscribed circle.

The product of three lines is sometimes called a *solled*, for a reason that shall be seen afterwards. Its value is easily conceived, by imagining that the lines are reduced into numbers, and multiplying these numbers together.

108. Scholium. It may also be demonstrated, that the surface of a triangle is equal to its perimeter multiplied by half the radius of the inscribed circle.

For the triangles A O B, B O C, A O C as in (fig. 77;) which have a common vertex at O, have for their common altitude the radius of the inscribed circle; hence the sum of these triangles will be equal to the sum of the bases A B, B C, A C, multiplied by half the radius O D; hence the surface of the triangle A B C is equal to the perimeter multiplied by half the radius of the inscribed circle.

THEOREM XXIII.

109. In every quadrilateral inscribed in a circle, the rectangle of the two diagonals is equal to the sum of the rectangles of the opposite sides.

In the quadrilateral A B C D (fig. 78); we shall have A C.B D=A B.C D+A D.B C.

Take the arc C O=A D, and draw B O meeting the diagonal A C in I.

The angle A B D=C B I, since the one has for its measure half of the arc A D, and the other half of C O, equal to A D; the angle A D B=B C I, because they are both inscribed in the same segment A O B; hence the triangle A B D is similar to the triangle I B C, and we have the proportion A D:C I::B D:B C. hence A D.B C=C I.B D. Again the triangle A B I is similar to the triangle B D C; for the arc A D being equal to C O, if O D be added to each of them, we shall have the arc A O=D C; hence the angle A B I is equal to D B C; also the angle B A I to B D C, because they are inscribed in the same segment; hence the triangles A B I, D B C, are similar, and the homologous sides give the proportion A B:B D::A I:C D; hence A B.C D=A I.B D.

Adding the two results obtained, and observing that A I.B D+C I.B D=(A I+C I.) B D=A C.B D, we shall have A D.B C+A B.C D=A C.B D.

110. Scholium. Another theorem concerning the inscribed quadrilaterial may be demonstrated in the same manner.

The similarity of the triangles A B D and B I C give the proportion B D:B C::A B:B I; hence B I.B D=B C.A B. If C O be joined the triangle I C O, similar to A B I, will be similar to B D C, and will give the proportion B D:C O::D C: O I; hence O I.B D=C O.D C, or because C O=A D, O I.B D=A D.D C.

Adding the two results, and observing that B I.B D+O I.B D is the same as B O, B D, we shall have B O.B D=A B.B C+A D.D C.

If B P had been taken equal to A D and C K P been drawn, a similar train of reasoning would have given us, C P.CA=A B.A D+B C.C D.

But the arc B P being equal to C O, if B C be added to each of them it will follow that C B P=B C O; the chord C P is therefore equal to the chord B O and consequently B O.B D and C P.C A are to each other as B D is to C A; hence, B D:C A::A B.B C+A D.D C:A D.A B+B C.C D.

Therefore the two diagonals of an inscribed quadrilateral are to each other, as the sums of the rectangles under the sides which meet at their extremities.

These two theorems may serve to find the diagonals when the sides are given.

THEOREM XXIV.

111. If a point be taken on the radius of a circle and this radius be then produced and a second point be taken on it without the circumference of the circle these points being so situated that the radius of the circle shall be a mean proportional between their distances from the centre then if lines be drawn from these points to any point of the circumference, the ratio (of these lines) will be constant.

Let P (fig. 79) be the point within the circumference, and Q the point without; then if C P : C A :: C A : C Q, the ratio of Q M and M P will be the same, for all positions of the point M.

For by hypothesis, C P : C A:: C A : C Q: or substituting C M for C A, C P: C M::C M: C Q; hence the triangles C P M, C Q M, have each an equal angle C contained by proportional sides; hence they are similar (102); and hence the third side M P is to the third side M Q as C P is to C M or C A.

But by division, the proportion C P : C A :: C A : C Q gives C P : C A :: C A — C P : C Q — C A or C P : C A :: A P : A Q; therefore M P : M Q :: A P : A Q.

PRACTICAL GEOMETRY.

PROBLEM I.

112. At a given distance, parallel to a given straight line A B, (fig. 80. pl. 2.) to draw a straight line, C D.

In the given straight line A B, take any two points as A D or B C in your compasses, with one foot in A describe the arc at D, also in B describe another at C draw a line through C and D and it is done; for the line C D will be parallel to the line A B.

PROBLEM II.

113. To bisect or divide a given straight line A B (fig. 81.) by a perpendicular.

Take any distance in your compasses greater than half the line A B, then with one foot in B describe the arc C F D; with the same distance, and one foot in A, describe the arc C G D; cutting the former arc in C and D; draw the line C D, and it will bisect A B in the point E perpendicular.

PROBLEM III.

114. To make an angle that shall contain any proposed number of degrees, from a given point, in a given line.

Case 1. When the given angle is right or contains 90° let C A (fig. 82) be the given line, and C the given point.

On C erect a perpendicular C D and it is done, for the angle D C A is an angle of 90°, or thus on the point C, as a centre with the chord* of 60° in your compasses, describe an arc G H and set off thereon from G to H the distance of the chord of 90° and from C through H draw C *H D*, which will form the required angle D C A of 90°.

115. Case 2. When the angle is acute as (fig. 83;) as for example 36° 30′ let C B be the given line and C the point at which the angle is to be made.

With the chord of 60° in your compasses and one foot in C as a centre describe the arc F B, on which set off from B to F, the given angle 36° 30′ taken from the line of chords; through F and the centre C draw the right line A C, and it is done, for the angle A C B will be an angle of 36° 30′ as was required.

116. Case 3. When the given angle is obtuse, as for example 127° 20′ let C B (fig. 84) be the given line and C the angular point.

Take the chord of 60° in your compasses, and with one foot in C as a centre, describe an arc B G H E, upon which set off the chord of 60° (which you already have in your compasses) from B to G, and from G to H; then set off from G to E, the excess of the given angle above 60° which is 67° 20′ taken from the line of chords; or you can set off from H to E the excess of the given angle above 120° which is 67° 20′ draw the line C E and it is done, for the angle E C B will be an angle of 127° 20′ as required.

PROBLEM IV.

117. To bisect a given arc of a circle as A B (fig. 85) whose centre is C.

Take in your compasses any extent greater than the half of A B, and with one foot in A describe an arc; with the same extent and one foot in B, describe another arc, cutting the former in D; join C D and it is done, for this line will bisect the arc A B in the point E, and divides it into two equal parts.

PROBLEM V.

118. From a given point B (*Geometry, Plate* III. *fig.* 1.) at the extremity of a given straight line, A B, to draw a perpendicular.

Take any point, E above the line A B, and, with the radius B E, describe the arc *d* B C cutting A B in *d*: draw the straight line *d* E C, and join B C, which will be the perpendicular required.

PROBLEM VI.

119. From a given point C (fig. 2) to let fall a perpendicular to a given straight line A B.

From the point C, with any radius greater than the distance of A B, describe an arc cutting A B at *e* and *f*; from the points *e* and *f*, as centres with any equal radius greater than the half of A, describe arcs cutting each other in D, and draw C D, which will be the perpendicular required.

PROBLEM VII.

120. To describe the segment of a circle, which shall have a given length or chord, A B (fig. 3) and a given breadth or versed sine, C D.

Bisect the straight line A B by a perpendicular C E; from the point D, where the perpendicular cuts the chord A B, make D C equal to the breadth, or versed sine; join A C; and make the angle C A E equal to the angle A C E; from E as a centre, with the radius E A or E C, describe the arc A C B which will be the segment required.

PROBLEM VIII.

121. Through three given points A, B, C, (fig. 4) to describe the circumference of a circle.

Join A B, B C and bisect each of the lines A B and B C, by a perpendicular, and let the perpendiculars meet each other in I: from the centre I, with the distance I A, I B, or I C, describe the circle A B C, which is that required.

PROBLEM IX.

122. Upon a given straight line A B (fig. 5) to describe an equilateral triangle.

From the centres A and B, with the radius A B, describe arcs cutting each other at C, join A C and B C, then A B C will be the equilateral triangle required.

PROBLEM X.

123. Upon a given straight line A B (fig. 6) to describe a square.

From the point B draw B C perpendicular to A B, make B C equal to A B: from the points A and C as centres, with a radius equal to A B or B describe arcs cutting each other in D and join A D and D C; then A B C D is the square required.

* For the description of the line of chords see (fig. 6, pl. 1.) which will be explained hereafter.

PROBLEM XI.

124. Upon a given straight line A B, (figs. 7 and 8) to describe a regular polygon of any number of sides.

Produce the side A B to P, and on A P, from the centre B, describe a semicircle A C P; divide the semi-circumference A C P into as many equal parts as the number of sides intended: through the second division, from P draw the line B C; bisect A B and B C by perpendiculars cutting each other in S; from S, with the radius A S, B S or C S describe a circle A B C D E, then carry the side A B or B C round the remaining part of the arc, which will be found to contain the remaining sides of the number required.

Fig. 7 is an example of a pentagon; fig. 8 is an example of a hexagon; but in this figure, we need not proceed by the general method; we have only to make a radius of the given side A B and take the points A and B as centres; and from the arcs A G and B G and strike a circle with the radius G A or G B, which will contain the side A B six times.

PROBLEM XII.

125. In a given square, A B C D (fig. 9) to inscribe a regular octagon, so that four alternate sides of the octagon, may coincide with four sides of the square.

Draw the diagonals A C and B D, cutting each other in S: on the sides of the square make A L, A F, B E, B H: C G, C K; and D I, D M, each equal to half the diagonal; join M E, F G, H I, K L; then will F G H I K L M E F be the octagon required.

PROBLEM XIII.

126. In a given triangle A B C (fig. 10) to enscribe a circle.

Bisect any two angles, A and B by the straight lines A E and B E and the point E, the intersection of these two lines will be the centre of the inscribed circle; draw E D perpendicular to A B cutting A B in D; from E with the radius E D, describe the circle D F G, which will be enscribed in the triangle A B C, as required.

PROBLEM XIV.

127. A circle, D E F, (fig. 11,) and a line A B, touching it, being given, to find the point of contact.

From the centre C draw the perpendicular C D cutting A B in D, which is the point of contact required.

PROBLEM XV.

128. Two straight lines, A B, B C (fig, 12) forming any angle being given to describe a circle to touch each of these lines at a given point A, in one of them.

Make B C equal to B A, and draw A D perpendicular to A B, and C D perpendicular to B C: from the point of intersection D, with the radius D A or D C describe the circle A C E which is that required.

PROBLEM XVI.

129. In a given circle A B C D (fig. 13) to enscribe a square.

Draw the diameters A C and B D at right angles, and join A B, B C, C D, D A, then A B C D will be the square required.

PROBLEM XVII.

130. To describe a segment, A B C of a circle by means of an angle.

Let A B (fig. 14) be the length or chord and D C the versed sine, join C A and C B produce C A to F and C B to E making C E and C F of any length, not less than the chord A B prepare two straight edges, C E and C F and fasten them together at the angle C so that their outer edges may form the angle A C B and to keep them to the extent, fix another slip across them at D to keep them tight: put in pins at A and B, and move the angle thus formed round these pins, hold a pencil to the angular point at C, it will describe the segment required.

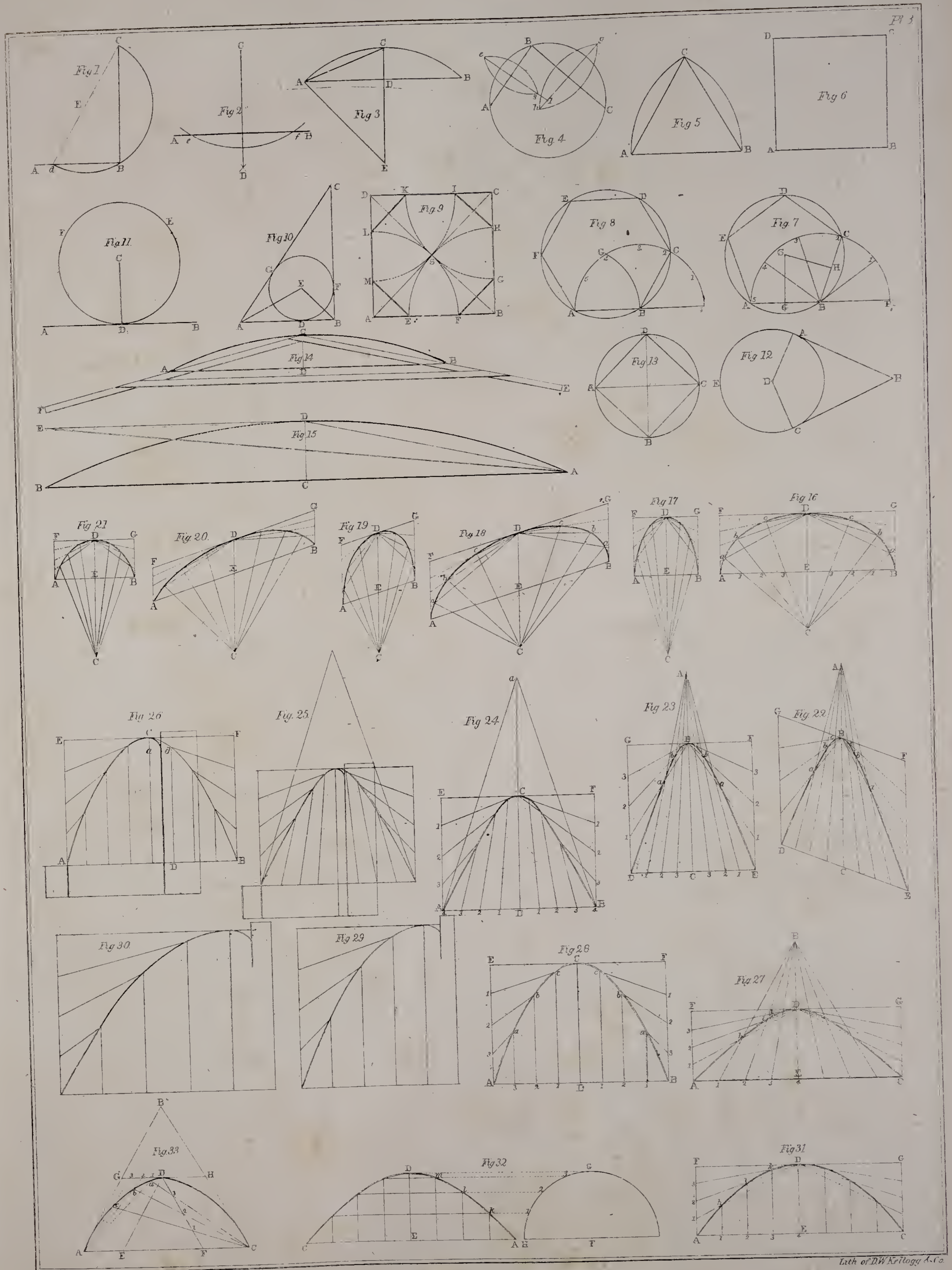
ANOTHER METHOD.

131. Let A B (fig. 15) be the length of the chord, and C D the versed sine, (or the perpendicular height in the middle) join A D, and draw D E parallel to A B, making D E of any length not less than A D, form a triangular piece of wood, A D E: bring the angular point D of the triangle, to the point A, (put in pins at the points A D B) and move the triangle, so that the side D A may slide upon A, and the side D E upon D; then if during the motion a pencil be held at the angular point D, with its point tracing over the plane, the arc A D will be described by the point of the pencil, the arc A D being described, the arc D B will be described, in a similar manner: and consequently, the whole segment of the circle, as required to be done.

PROBLEM XVIII.

132. To describe an ellipsis, or any segment of an ellipses, having a diameter and a double ordinate, by means of points being found in the curve, without finding the parameter.

Let A B (fig 16,) be a diameter (or double ordinate) let C D be its conjugate, and let E D be the height of the segment. Through D draw F G parallel to A B; also through the points A and B draw A F, and B C parallel to D E cutting F G in F and G. Divide A E and E B into a like number of equal parts, as four; likewise B G, and A F, into the same number of equal parts. From the point D, through the points 1, 2, 3, in A F, and B G, draw 1 D, 2 D, 3D. From



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the point C, through the points 1, 2, 3, in A B draw Ca, Cb, Cc, cutting the lines 1 D 2 D 3 D, in a, b, c, they will be in the periphery of the ellipsis; a curve being traced through these points, will form the ellipsis required.

But if the curve is very large, as in practical works the best way is to put in nails or pins at the points, a, b, c, &c. bend a slip round them, and draw a curve by it, it will appear quite regular.

(Figures 17, 18, 19, 20, and 21) are drawn in a similar manner.

PROBLEM XIX.

133. To describe a hyperbola by finding points in the curve, having the diameter or axis A B (in figs. 22 and 23) its abscissa B C and double ordinate D E. Through B draw G F parallell to D E; from D and E draw D G and E F parallel to B C cutting G F in F and G. Divide C D and C E, each into any number of equal parts, as four through the points of division 1, 2, 3, draw lines to A.

Likewise divide D G and E F, into the same number of equal parts, viz. four from the divisions on D G and E F draw lines to B and a curve being drawn, through the intersections at B a b c E, will be the hyperbola required.

(Figures 24, 25, 26, 27, 28, 29, and 30.) Need no explanation, they being drawn in a similar manner.

PROBLEM XX.

134. To describe a parabola upon a given ordinate, A E, and a given abscissa E D (fig. 31.) Make E C equal to E A, and complete the rectangle A F G C; so that the opposite side may pass through D. Proceed as in problem 15 excepting that, instead of drawing the lines 1, 2, 3, &c. to A as in (figs. 22 and 23) draw them perpendicular to A C.

PROBLEM XXI.

135. To describe the figure of the sines (fig. 32) describe the quadrant F H G equal to the heighth of the figure, and divide the arc H G into any number of equal parts, the more of these the more perfect will be the operation; and extend the chords to double the number of parts upon the line A C which is a continuation of F H, and make the points of division.

Draw the lines 1 k, 2 l, 3 m, &c. perpendicular to A C; and from the points 1, 2, 3, &c. of division in the quadrant draw lines 1 k 2 l 3 m, &c. parallel to A C, and through the points A k l m, &c. draw a curve which will be the figure of the sines, as required.

PROBLEM XXII.

136. To describe a conic section, to touch two right lines A B (fig. 33) and B C, in the points A and C, and to pass through a given point D.

Join the points A, and C; through D, draw D E and D F, parallel to B A and B C, through D, draw G H parallel to A C, cutting B A and B C in G and H, and divide D G, and D H, D E, and D F each into the same number of equal parts. From C through the points 1, 2, 3, in D F draw the lines C a, C b, C c &c. from A, through the points 1, 2, 3, in D G, draw 1 A, 2 A, 3 A, cutting the former in a, b, c, which are in the curve.

In the same manner may points be found between D and C.

PROBLEM XXIII. PLATE 4.

137. To describe a triangle, whose three sides shall be equal to three given lines provided that any two of them are greater than the third.

Let A, B, C, (*Geometry, plate* 4, fig. 1) be the three given straight lines.

Take one of the given lines as A, and make the base of the triangle D E. Upon E, with the length of B in your compasses, describe an arc at F. Upon D with the length of C, describe another arc intersecting the former at F, and join D F, and F E, then D F E, is the triangle required.

In this manner a triangle may be made equal to another given triangle; for this is only making the sides of the triangle equal to those of the given triangle.

PROBLEM XXIV.

138. To form a trapezium from a given triangle, let G, I, H, (fig. 2) be the given triangle; and described in the same manner as (fig. 1.) The two sides G K, and I K being given, take the length of G K in your compasses and upon G describe an arc at K; upon I with the length of I K describe another arc intersecting the former at K, and join G K and I K: then is G K I H the trapezium required.

The line drawn G I, divides the trapezium into two obtuse angled triangles, and is called the base line, and to let fall the perpendiculars K, H to M and N on the base line G I; then it becomes four right angled triangles.

PROBLEM XXV.

139. To make a rectangle equal to a given triangle A B C (fig. 3.) It is required to make a rectangle equal to the gieen triangle. Draw D C perpendicular to A B, and divide D C into two equal parts at g, through g draw E F, parallel to A B from B draw B F, perpendicular to AB, through A draw AE parallel to BF. Then the rectangle A B E F will be equal to the triangle A B C, as required to be done.

PROBLEM XXVI.

140. To make a square equal to a given rectangle. Let A B C D (fig. 4) be the given rectangle produce the side A B of the rectangle to h, and make B h, equal to B C draw B G perpendicular A B; and on i h as a diameter, describe the semicircle A G h, and on the straight line B G describe the square B G F E; which is the thing required to be done.

We now see that a triangle may be reduced to a rectangle, and a rectangle may be reduced to a square; therefore a triangle may be reduced to a square.

(Figures 5 and 6) these two figures are described similar to the figures 1 and 2.

PROBLEM XXVII.

141. Given two circles to find a third equal to them both.

Let A B and A C (fig. 7) be the diameters of the given circles, perpendicular to each other at the point A; join C B on which, describe a circle and the thing is done.

PROBLEM XXVIII.

142. Given any two similar figures, to find another equal and similar to them both.

Let E and F (fig. 8) have their sides A B and A C placed perpendicular to each other join B C on which describe a figure similar to E or F, and the thing is done.

PROBLEM XXIX.

143. Given three straight lines to find a fourth proportional.

Let A C (fig. 9) be one of the given lines, make any angle with the line C E and from B, one extremity of another given line, draw B D the third any how to meet C E, and through A draw A E parallel to B D, and A E will be the fourth proportional required.

PROBLEM XXX.

144. Having given two lines, to find the third proportional.

Let C B and C D (fig 10.) be the given lines, and let them have any inclination at the point C; join B D, and produce C D and C B to A and E making C A equal to C D; through A draw A E parallel to B D, and C E will be the third proportional required.

PROBLEM XXXI.

145. To find a line equal to a given arc of a circle.

Let A B C (fig 11) be the given arc, join A C which bisect in D, by the perpendicular B D; join A B, and produce A C till A E be equal twice A B; divide C E into three equal parts, and make E F equal to one of these parts; then will A F be nearly equal to the arc A B C.

PROBLEM XXXII.

146. To divide a given line in the same proportion as another line is divided.

Let d e (fig 12.) be the line proposed for division. On d e describe an equilateral triangle, d C e; produce the sides C d and C e till each of them be equal to A B; join A B and from C to the points of division, f, g, h, draw the lines Cf, Cg, and Ch, which will cut d e in the points i, h, l, in the same proportion as A B is cut. Or thus—

147. Let B D (fig. 13) be the given line whose divisions are required.

Upon B D raise B A perpendicular at the point B, which make equal to B D; produce A B till A C be equal to the given divided line C E, draw C E perpendicular to A C, and produce A D to E, to the points of division f, g, h, i; from A draw the lines Af, Ag, Ah, and Ai, which will divide the line B D, in the given proportion.

PROBLEM XXXIII.

148. To divide a quadrant of a circle into any number of equal parts.

Bisect the diameter A B (fig 14) perpendicularly in C, produce C E till E F be equal to three fourths of A C or B C, join F A which produce to meet D G, drawn through D parallel to A B: divide DG into the proposed number of equal parts in the points h, i, k, l, and join Fh, Fi, Fk, and Fl, which will divide the quadrant A D, into the proposed number of equal parts nearly.

PROBLEM XXXIV.

149 Having the abscissa E D (figs. 15 and 16) and a double ordinate A B to describe a parabola. Produce E D to B, making D B equal to D E and join A B and C B. Divide A B into any number of equal parts, numbering them from A to B, and divide B C into the same number of equal parts, numbering them from B to C, join 11, 22, 33, &c. and the parts of the straight lines, comprehended between the entersections, will form the parabola, being all tangents at different points.

PROBLEM XXXV.

150. To describe a parabola by a continued motion.

Let G H (fig. 17) be the straight edge of a ruler, and K L Q the internal right-angle of a square, and let the edge which is parallel to K L coincide with the straight edge G H. Suppose now one end of a string fastened at F, and the other end to the end G of the square. Let F I be perpendicular to G H, meeting G H in I. Suppose the ruler to remain fixed, and the square to be moved, keeping the upper edge against the straight edge G H: then if both parts, F M, M Q, be kept straight by a pencil at M the point M will describe the half of the parabolic curve.

PROBLEM XXXVI.

151. Given the axis major and two foci of a conic section, to describe the curve.

Let A B (figs. 21 and 27) be the axes major and the points Ff, the foci. Pro-

duce B A, if necessary, to F, and make A I equal to A F; from the remote focus f, describe an arc Q I. Through Q draw f M: then find the point M, by sloping with a compass or dividers, so that F M may be equal to M Q; then M is a point in the curve.

By employing F the same manner as has now been done, in respect to f, we shall obtain the curve r Bs.

N. B. In the ellipse (fig 27) it will be most convenient to describe one-half by means of the focus f, and then the other by the other focus F.

In the parabola (fig. 20) the arc Q I will become a straight line, this being understood the point M, and every other point will be found as in (figs. 21 and 27.)

PROBLEM XXXVII.

152. To describe an elipse having the two axes given.

On A B (fig. 22) describe the rectangle G H I K, whose sides G H and G K are epual to the given axes. Divide C D into any number of parts, q, r, s, t, and D G into the same number a, b, c, d,; from B through the points of division q, r, s, t, draw lines B M &c. and from A to the points of division, a, b, c, d, draw Aa, Ab, &c. meeting the former in M, then will M be a point in the curve: and thus may all the points be found, through which draw the curve itself. If the curve is large it would be well to put in pins or nails at the points and bend a slip of wood around to draw the curve by.

PROBLEM XXXVIII.

153. The transverse and conjugate axes A B and D D in (fig. 23,) of an ellipsis being given to find the two foci, from thence to describe an ellipsis.

Let F and f be the foci make F M+f M equal to A B, supposing F M+f M to be a cord or string. Then move the point M, round taking care to keep the string always tight and the point M will in its motion, trace out the curve A D B D, which is the ellipse required.

PROBLEM XXXIX.

154. To describe an ellipse round a given rectangle.

In (fig. 24) let I T, be half the longest side of the rectangle and make I G equal to I T: join T G, cutting A B in H then G I is the semi-transverse and G H the difference of the semi-axes. Therefore, the curve may be described. Or thus—

In (fig. 25) g, i, is the semi-transverse, and $g. h$, the difference of the semi-axes. The point g, is supposed to move in the groove exhibited in one arm of the trammel, while h, moves in the other and the point i, traces out the curve.

PROBLEM XL.

155. To describe an ellipse by means of circles.

From the centre C (fig. 26) with a distance equal to the semi-conjugate, describe the quadrent D $h. g$, and from the same centre, with the semi-transverse as a distance, describe the arc i B, bisect D h g in h; join C h which produce to i through h draw h, k parallel to C B and from i draw i k perpendicular to h k then will k be a point in the curve, produce D C to L, and bisect the distance Dk, perpendicularly, which bisecting line produce to meet D C in L, then will L be the center of the circle, describing one part of the curve.

Again; through l draw l m parallel to h k meeting the arc D n m, in m; join m B, which produce to meet the curve in n, join n l cutting A B in g; g is the center of the circle whence the vertical part of the curve is described.

HYPERBOLA PROBLEMS XLI.

156. Given the transerve axis of an hyperbola and an ordinate, to find the conjugate axis and assymtotes, (which are two straight lines such as, if produced ndefinitely with the curve will never meet each other) and thence to describe the curve itself

Let Aa (fig. 28) be the transverse axis, and let P M be an ordinate. Make P D equal to A P. Then on a D describe the semi-circle a N D, produce P M to N. Draw A R pependicular to C D, and make A R equal to C A. Join N R and produce N R and D A, if necessary, to meet each other in S; and draw M S, cutting A R in Q, produce Q A to T and make A T equal to A Q. Then Q T will be the conjugate axis or A Q, A T, will each be the semi-conjugate axis.— Through the points C, T, draw J H; and through the points C Q, draw I K: then J H and I K are the assymtotes by which the curve may be described.

157. The three curves of the conic sections,—Problem.

The vertical section of a right cone being given and the position of the axes of a conic section, to describe that section.

Let A V B (fig. 29) be the section of a cone through its axes; let ig be the line of the axes, and let it cut the sections A V B at h, and the opposite side B V, produced, at g. On g h describe the semicircle h q s g. Draw Vp parallel to A B, cutting the axis in p. Bisect h g in r and draw p q, r s, perpedicularly to h g. Make p w equal pV; then with the transverse axis h g, at the ordinate p w, describe the ellipse h w t g, cutting r s at t; then r t is the semi-conjugate axes.

GEOMETRY OF SOLIDS.

DEFINITIONS OF SOLIDS.

158 A Right Cylinder is that which is formed by the revolutions of a rectangle about one of its sides; the line round which the rectangle revolves is called the axis (plural axis); and the circles generated by the two opposite sides of the rectangles perpendicular to the axes, are termed the ends or bases. The surface of the cylinder, generated by the line parallel to the axis, is termed the curved surface, which is either straight or convex, according as a straight edge is applied, parallel to the axis, or in any other direction.

159. A Right Cone is that which is formed by supposing a right-angle triangle to revolve about one of its legs or perpendicular sides; the fixed leg, or line is called the axis; the surface generated by the other leg is called the base; and the surface formed by the hypothenuse, or sides opposite the right angle is denominated the curved surface. which is either straight or convex, according as a straight edge is applied upon the surface from the vertex, or in any other direction

160. A Sphere or Globe is that which is formed by supposing a semi-circle revolves upon its diameter; the diameter upon which the semi-circle revolves is called the axis, and the surface formed by the arc of the semi-circle is called the curved surface which is convex, in whatever way it may be tried hy a straight edge.

161. An Ellipsoid is formed or generated by supposing a semi-ellipse to revolve upon one of its axis the axis thus fixed is called the axis of the ellipsoid, aud the surface generated by the curve is termed the curved surface.

PROBLEM XLII.

162. To describe a conic section, from the cone, through a line given in position in the section passing through the axis.

Let A B C (Figures 1, 2, and 3 plate 5) be the section of a right cone, and let D E be the line of section. Through the apex or top ot the cone, C, draw C F parallel to the base A B of the section, and produce E D to meet A B in D, as in figures 2 and 3 or A B produced in G, as in fig. 1, as also to meet C F in F. On A B describe a semi circle which will be equal to half the base of the cone, in the semi-circle take any number of points, a, b, c, &c. Draw D d, in figures 2 and 3 and Gd in fig 1, perpendicular to A B and Gd in fig. 1. perpendicular to G F; as also Dd, figures 2 and 3, perpendicular to D F: From the points a, b, c, &c. draw lines a e, b f, c g, &c., cutting Gd (fig. 1) and D d (figs. 2 and 3) in the points e, f, g, &c. in figure 1, make in Ge, Gf, Gg; &c. equal to Ge, Gf, Gg, &c. and in Dd, (figs 2 and 3) make De, Df, Dg, &c. equal to De, Df, Dg, &c.— Through the points e, f, g, &c. draw lines to F. Through the points a, b, c, &c, draw lines perpendicular to A B. From the points of section in A B, draw lines to the vertex C of the cone cutting the sectional line, D E, in l, m, n, &c. Through the points of section l, m, n, &c. draw lh, mi. nk, &c. perpendicular to D E. Through the points D, h, i, k, in fig. 1 or d, h, i, k, &c. in figs. 2 and 3, draw a curve which will be the conic section required.

OBSERVATIONS.

163. In the first of these figures, the line of section cuts both sides of the section of the cone in this case the curve D h i k and e E is an Ellipse. In fig. 2 the line of section D E is parallel to the side A C of the section of the cone; in this case the curve d h i &c. E is a Parabola. In fig. 3. the line of section, D E is not parallel to any side of the cone; it must therefore, when produced with the sides of the section through the axis, meet each of these two sides in different points, in this case, the section d, h, i, &c. E is either an Ellipse or *Hyperbola* but the case is determined to be an *hyperbola* by the line of section meeting the opposite side B C at A C, where it cuts above the vertex, at the point B.

Here we may observe, that the line of section, D E is the same as that which has before been called the abscissa the part E B produced, contained between the two sides of the section is called the axis major; and the line D d, perpendicular to D E, an ordinate.

Hence the same section may be found by the method already shown in the problem; viz. by drawing any straight line d e b, fig. 4; make d e equal to D E, fig. 3, and e b equal to E B, fig. 3. Through d draw the straight line D D at right angles to d b: make d D equal to D d, fig. 3. then with the axis major b e the abscissa e d, and the ordinate d·D, on each side of the abscissa describe the curve of the hyperbola which will be of the same species as that shown in fig. 3.

PROBLEM XLIII.

164. To describe a cylindric section through a line given in position upon the section passing through its axis (fig. 4.)

This is no more than a particular case of the last problem.

For a cylindric may be considered as a cone, having its apex at an infinite distance from its base; or practically, at a vast distance from its base, in this case all the lines for a short distance, would differ insensibly from parallel lines; and this is the construction shown at fig. 5, which is therefore evident. But as the section of a cylinder so frequently occurs I shall here give a more practical description of it. Thus:—

Let A B H I (fig. 5) be a section of a right cylinder, passing through its axis, A B being the side which passes through the base, and let D E be the line of section. On A B describe a semi-circle; and in the arc take any number of points a, b, c, &c. from which draw lines perpendicular to the diameter A B, cutting it in Q, R, S, &c.: perpendicular to A B or parallel to A I or B H, draw the lines Q q, R r, S s, &c. cutting the line of sections D E, in the points q, r, s, &c. from the points of section q, r, s, &c. draw the lines qi, rk, sl, &c. perpendicular to the line of section, D E, make the ordinates qi, rk. sl, &c. each respectively equal to the ordinates Qa, Rb, Sc, &c.; and through the points D, i, k, l, &c. to E draw a curve, which will evidently be the section of the cylinder, as required.

The same may be done in this manner; viz:—Bisect the line of section D E in the point t. Draw t m perpendicular to D E.

Make t m equal to the radius of the circle which forms the end of the cylinder; then with the axis major DE, and the semi-axis minor tm describe a semi-ellipse, which will be the section of the cylinder required.

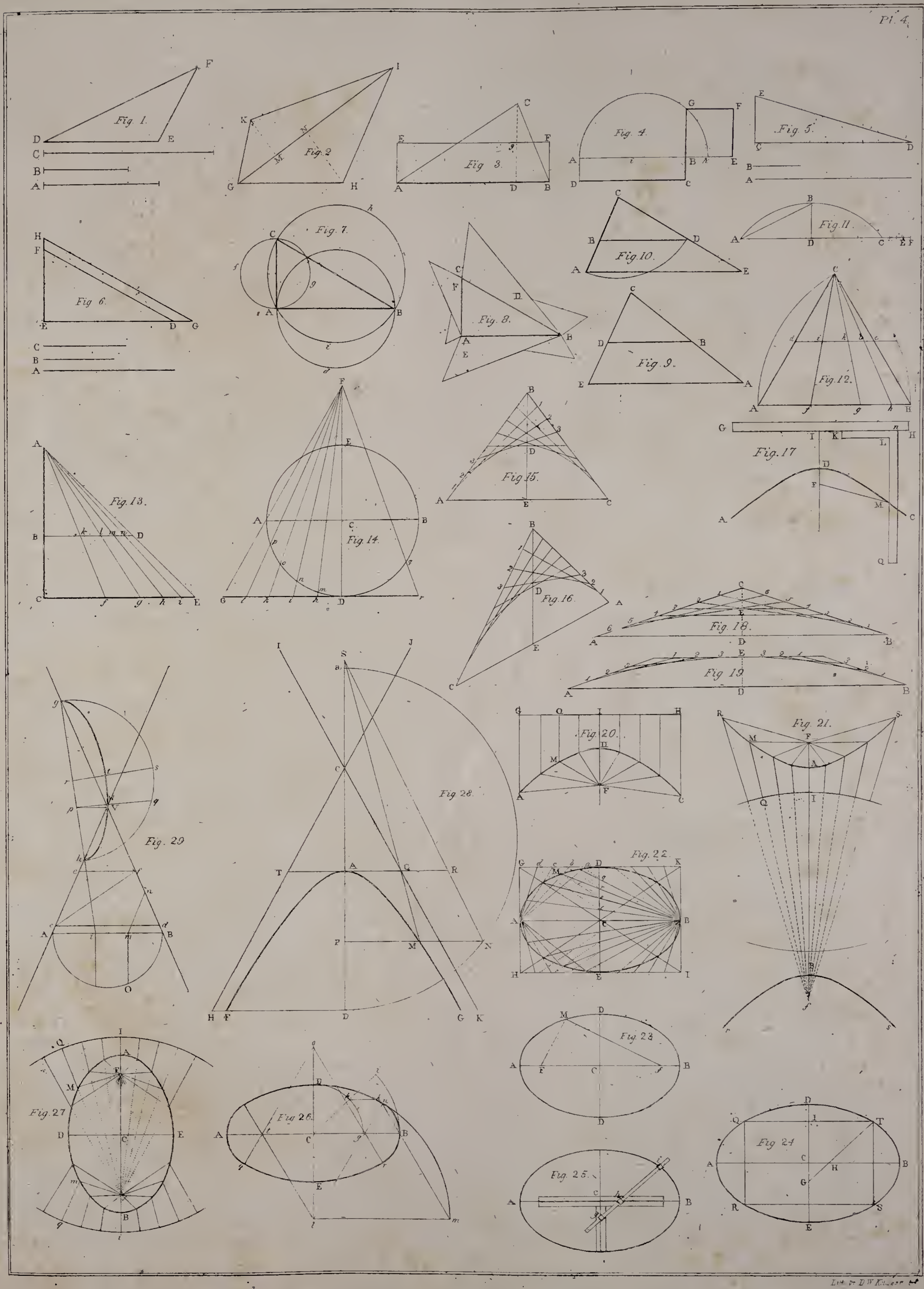


Fig. 1.
Fig. 2
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23
Fig. 24
Fig. 25.
Fig. 26.
Fig. 27
Fig. 28.
Fig. 29

A DEFINITION.

165. A cuneoid is a solid ending in a straight line, in which, if any point be taken, a perpendicular from that point may be made to coincide with the surface, the end of the cuneoid may be of any form whatever.

The cuneoid which occurs in architecture, has a semi-circular or a semi-elliptical end, parallel to the straight line to which the perpendicular is applied.

PROBLEM XLIV.

166. To find the section of a cuneoid with a semi circular base the given data being a section through the axis, perpendicular to the vertex or sharp end, and the line of section upon that end.

Let A B C (fig. 6) be the section through the axis perpendicular to the sharp edge, and let D E be the line of section.

This construction is similar to that of finding the section of a cylinder, excepting that instead of drawing parallel lines from the base A B, they are in this figure drawn from the points of section in A B to the point C which is the vertex of the cuneoid: the ordinates Q*a*, R*b*, Sc, &c. being transfered respectively to *qi*, *rk*, *sl*, &c.; and the curve D, *i*, *k*, *l*, & to E being drawn through the points D, *i*, *k*, *l*, &c. by hand.

CARPENTRY.

Carpentry is the art of applying timber in the construction of buildings. The cutting of the timbers, and adapting them to their various situations so that one of the sides of every timber may be arranged according to some given surface as indicated in the designs of the architect, requires profound skill in geometrical construction.

For this purpose it is necessary, not only to be expert in the common problems generally given in a course of practical geometry, but to have a thorough knowledge of the sections of solids and their coverings. Of these subjects, the first has already been explained in the series of problems given in the geometrical part of this work and I am now about to treat on the other; that is the method of covering them.

As no line can be formed on the edge of a single piece of timber so as to arrange with a given surface, nor on the intersection of two surfaces, (by workmen called a groin) without a complete understanding of both, the reader is required not to pass them, until the operations are perfectly familiar to his mind.

For the more effectually rivetting the principles upon the mind of the student, it is requested that he should model them as he proceeds and apply the sections and coverings found on the paper to the real sections and surfaces by bending them around the solids.

The surfaces which timbers are required to form are those of cylinders, cylindroids, cones, cuneoids, spheres, ellipsoids, &c. either entire or as terminated by cylinders, cylindroids, cones, and cuneoids.

The formation of Arches, Groins, Niches, Angle brackets, Lunettes, Roofs &c. depend entirely upon their sections or upon their coverings or upon both.

This branch of carpentry from its being subjected to Geometrical rules and described in schemes or diagrams upon a floor, sufficiently large for all the parts of the operation has been called *descriptive carpentry*.

In order to prepare the reader's mind for this subject, it will be necessary to point out the figures of the sections, as taken in certain positions.

All the sections of a cylinder, parallel to its base are circles.

All the sections of a cylinder parallel to its axis are parallelograms. And if the axis of the cylinder be perpendicular to its base all these parallelograms will be rectangles. If a cylinder be entirely cut through the curved surface and if the section is not a circle it is an ellipse.

All the sections of a cone parallel to its base are circles; all the sections of a cone passing through its vertex are triangles; all the sections of a cone which pass entirely through the curved surface and which are not circles are ellipses: all the sections of a cone which are parallel to one of its sides, are denominated parabolas, and all the sections of a cone which are parallel to any line within the solid passing through the vertex are denominated hyperbolas.

All the sections of a sphere or globe made plane are circles.

The solid formed by a semi-ellipse revolving upon one of its axis is termed an ellipsoid.

All the sections of an ellipsoid are similar figures: those sections perpendicular to the fixed axis are circles and those parallel thereto are similar to the generating figure.

OF THE COVERINGS OF SOLIDS.

PROBLEM I.

Let A B C D (fig. 7 pl. 6) be the generating section of the frustum.

On B C describe the semi-circle B E C, and produce the sides B A and C D, of the generating section A B C D to meet each other in F.

From the centre F with the radius F A describe the arc A H: and from the same centre F with the radius F B, describe the arc B G; divide the arc B E C of the semicircle into any number of equal parts; the greater the number the more correct will be the result of the operation; repeat the chords of one of those equal arcs, upon the arc B G, as often as the arc B E C contains equal parts; then through G, the extre-mity of the last part, draw G F, cutting the arc A H at H then will A B G H be the covering required, of the frustum of a right cone.

To find the covering of the frustum of a right cone when cut by two concentric cylindric surfaces, perpendicular to the generating section.

Let A B C D (fig. 7 also) be the given section, and A D, B C, the line on which the cylindric surface stands, (find the arc B G as before described) and mark the points 1, 2, 3, &c. of division both in the arc B G and in the semi-circumference: from the points 1, 2, 3, &c. draw lines to F; also from the points 1, 2, 3, &c. in the semi-circumference draw lines perpendicular to B C so that each line thus drawn may meet or cut it. From the points of division in B C draw more lines to F, cutting the arc B C in *a*. *b*, *c*, *&c.* from the points *a*, *b*, *c*, *&c.* draw lines parallel to B C to cut the side B A from the centre F through each point of section in B A, describe an arc, cutting the lines drawn from each of the points 1, 2, 3, &c. in B G at *a*, *b*, *c*, *&c.* then will B *e* G be the curve, which will cover the line B C on the plan, or B C will be the seat of the line B *e* G.

In the same manner A H the original of the line A D, will be found; and consequently, B *e* G H A will form the covering over the given seat A B C D as required to be done.

PROBLEM II.

To find the covering of a right cylinder.

Let A B C D (fig. 8 pl. 5) be the seat or generating section.

Produce the sides D A and C B to H and G, and on B C describe a semi-circle and make the straight line B G equal to the semi-circumference; draw G H parallel to A B, and A H parallel to B G, then will A B G H be the covering required.

PROBLEM III.

To find the covering of a right cylinder contained between two parallel planes perpendicuular to the generating section (fig. 9. pl. 5.) Through the point B draw I K, perpendicular to A B; and produce D C to K, on B K describe a semi-circle and make B I equal to the length of the arc of the semi-circle by dividing it into equal parts and extending them on the line B I. Through the points of section 1, 2, 3, &c. in the line B I, draw lines 1*a*, 2*b*, 3*c*, &c. parallel to B A, and through the points 1, 2, 3, &c. in the arc of the semi-circle, draw the other lines 1*a*, 2*b*, 3*c*, &c. parallel to B A cutting A D in *a*, *b*, *c*, &c. draw *aa*, *bb*, *cc*, &c. parallel to B K: then through the points *a*, *b*, *c*, &c, draw the curve A H and A H will be the edge of the covering over A D.

In the same manner the other opposite edge B G will be found, and the whole covering will therefore be A B G H.

PROBLEM IV.

A B C D (fig. 10 pl. 5) being the seat of the covering of a semi-cylindric surface, contained between the surface of two other concentric cylinders of which the axis is perpendicular to the given seat; it is required to find the covering.

Through B draw I K perpendicular to A B; and produce D C to K. On B K describe a semi circle and divide its circumference into equal parts, at the points 1, 2, 3, &c; the more of these the truer will be the opperation; and repeat the chord on the straight line B I, as often as the arc contains equal parts, and mark the points 1, 2, 3, &c. of division. Through the points 1, 2, 3, &c. in the arc of the semi-circle, draw the lines 1 *a*, 2*d*, 3*c*, &c. parallel to B A; and, through the points 1, 2, 3, &c. in B I, draw lines 1*a*, 2*b*, 3*c*, &c. parallel B A,—Draw *aa*, *bb*, *cc*, &c. parallel to K I, and through all the points *a*, *b*, *c*, &c. draw the curve line A H, which is one of the edges of the covering.

In the same manner the other edge B G will be found; and consequently, the whole covering A B G H.

PROBLEM V.

To find the covering of that portion of a semi-cylinder contained between two concentric surfaces of two other cylinders the axis of these cyilnders being perpendicular to ABCD (fig. 11, *pl*, 5.) Join BC and in this case BC will be perpendicular to AB. Produce C B to G; and on B C describe a semi-circle. Divide the arc of the semi-circle into any number of equal parts, and extend the chords upon the straight line B G marking the points of section both in the semi-circle and in the straight line B G. Through the points 1, 2, 3, &c. in the arc of the semi-circle, draw lines 1*a*, 2*b*, 3*c*, &c. parrallel to A B; and through the points 1, 2, 3, &c, in B G, draw the lines 1*a*, 2*b*, 3*c*, &c. parallel to A B; also draw *aa*, *bb*, *cc*, &c. parallel to B G, and, through the points *a*, *b*, *c*, &c. draw a curve, which will from one of the edges of the soffit: the opposite edge is formed in the same manner.

GEOMETRICAL CONSTRUCTION OF HIP ROOFS.

To find the bevels for cutting the various timbers in a hipid roof, and the length and backing of the hip rafters.

Let A B C D (fig. 12, *pl.* 5) be the plan of the building, or the outlines of the wall-plates, A E, B E, and C E, D E, the seats of the hips, draw E A, E B, and E C, E D; the base lines over which the hip rafters are to stand, let P *n*, *m*, be the pitch of the roof, and *o n* the perpendicular height, draw E F at right angles with E A and equal to the height of the perpendicular *o n*, then draw A F which is the length of the hip rafter, draw *h*, *k*, *h*, at any distance from the angle A and at right angles with A E make *k i* equal to *k r*, or from *k* to the nearest point of the top line of the hip rafters draw *h i* and *i h*, which is the backing of the hip rafter required. This method will give the backing of the hip rafter, whether the building be square or bevelling.

To find the bevels of a purlin upon a hip rafter giving the seat of a common rafter, and the seat of the hip rafter, and the angle which the common rafter makes with its seat.

Let A B C D, (figs. 13 and 14) be the outlines of the wall plates A F, D F, and B E, C E, the seats of the hips and E F the seat of the ridge-piece. Place the section-purlin in its real position with respect to the common rafter. Produce that side of the section of the purline, of which the bevel is required, upon

the hip toward the seat; from one extremity of the line thus produced, and, with the length of the said line as a radius, describe a circle. Draw three lines, parallel to the wall-plate, to meet the hiped line; viz. one from the centre of the circle, one from the point where the line meets the circle and the third a tangent to the circle. From the point in the seat of the hip-rafter where the middle line meets the said seat draw a line perpendicular to that middle line to meet the tangent; join the point where this perpendicular meets the tangent to the point where the line drawn from the centre meets the seat of the hip-rafter, and the angle formed by the line thus joining, and the line drawn from the centre of the circle will be the bevel of the purlin.

Example, (fig. 13 *pl.* 5) A F bring the seat of a hip-rafter, I F that of a common rafter and F I L the angle which the common rafter, makes with its seat, and *a b c d* the section of the purlin. Now suppose it were required to find the bevel of that side of the purlin represented by *a d*. Produce *a d* to any point, *f*; and from *a*, with the radius *a f*, describe a circle *f g h*, parallel to the adjacent wall-plate, A B draw three lines to cut the seat A F, of the hip; viz; from the centre *a* draw *a i* and from the point *f*, where *a f* meets the circle draw *f k*, the former cutting the seat in *i*, and the latter in *k*. Draw *k l*, perpendicular to *f k*, and draw the tangent *e l*, cutting *k l* in *l*; and join *i l* then *l i a* is the angle required.

In the same manner by producing *a b*, we shall find the angle formed upon the end of the side of which its section is *a b*.

In order that the differe nt inclined planes, which form the sides of a roof, should have an equal inclination to the horizon, the seats of the hip-rafters ought to bisect the angles of the wall-plates.

When a roof is wider at one end than at the other as in order as in (fig. 15) to prevent its winding let IK and OP be the seats of the two common rafters, passing through each extremity of the ridge-piece and let the rafters I L and K L be found as before; divide O P into two equal parts, in, E; draw E R perpendicular to O P. Make the angle E P R equal to the angle F K L; then E R will be the height of the roof upon the seat O P.

If this should be objected to, because it makes the ridge higher at one end than at the other, let E (fig. 17) be the end of the seat of the ridge next the narrow end of the roof.

Bisect all the four angles of the roof by the straight lines A F, B E, C E, D F; and through E, draw E G, parallel to A B cutting A F in G; and draw E H parallel to C D, cutting D F in H; and join G H; then G H will be parallel to A D. This is true, because, since all the angles are bisected, if we imagine perpendiculars drawn from E to the three sides, the three straight lines thus drawn will be equal; and because E G is parallel to A B, the perpendiculars drawn from the points E and G, to the straight line A B, are equal; from the same reasons, because E H is parallel to C D, the perpendiculars drawn from the points E and H to the straight line C D, are equal therefore the perpendicular drawn from the point G, to the straight line A B, is equal to the perpendicular drawn from H to the straight line C D.

And, since the angles B A D and C D A are bisected by the straight lines A. G and D H, the two perpendiculars, drawn from G to the sides A B and A D, are equal, as also the two perpendiculars from the point H to the sides D A and D C; but the perpendicular drawn from G, to the side A B, is equal to the perpendicular drawn from H to the side C D; therefore the perpendiculars, drawn from points G and H, to the straight line A D, are equal to each other; but when the perpendiculars drawn between two straight lines are equal, these two straight lines are parallel: therefore the straight line G H is parallel to A D.

Whence if all the angles of a roof be bisected, and if any point be taken in any one of the bisecting lines, and if a line be drawn through the point thus assumed, parallel to one of the adjacent sides, to meet the next bisecting line and so on from one to another, till only one line remains to be drawn, then if the point assumed be joined to the point where the parallel meets the last bisecting line the line thus joining will be parallel.

(Fig. 16) Is one half of the plan of the roof and the stretch-out of the rafters of (fig. 15)

OF DOMES.

(Fig. 1 *pl*, 6) is a design for an hemispherical dome, the ribs are constructed of thin boards and small pieces of plank. The principle of this form of roof consist in placing a number of hoops one above the other, and of such sizes as when properly placed, will form the contour of the dome. These hoops are here formed by pieces of plank as represented on the plan of the dome in (fig. 1 NO. 2) Near each one of these is a long mortice, the position of these is shown in the section *d d d &c.* in (fig 1 NO. 1) one of the ribs or rafters is shown in (fig. 1. NO. 3) with a mortice in the middle of it long enough to receive the thickness of two hoops. as also may be scen in (fig. 1 NO. 4). At each end of these ribs is a sliding mortice of half the length, as represented in the section *c c c &c.* in (fig. 1 NO. 1) when these are to be put together, the wall plate (which should be of two thicknesses of boards, and made to break joint) should be first laid, and then a piece of the rafter, as (fig. 1 NO 3) should be fixed upright in its proper place and secured by a tenon at the lower end, which must go through the plate. It should be observed, that the rafters are of two lengths which should break joint; of course one of the first pieces should he but half the length of No. 4. when one set of the rafters are fixed all round, the pieces which form the hoops or which I shall call the purlins, are fixed in them and secured by wooden keys which are driven, on each side of the rafter, through the mortice. By driving these keys, more or less, the hoop may be lenghtened or contracted, so as to bring it to the exact form or contour of the dome. After the first set of purlins are fixed and properly keyed, another set of rafters are placed, and then another set of purlins, until the dome is complete. The figure in the plate, for the sake of making its parts more clear has been drawn considerably out of proportion. the materials being much too large and a much greater number of purlins would be proper. This principle of covering may be extended to a great span, and when the rafters come too close together, at the top every other one may be left out as may be seen in the proceeding figures.

Figures 2 and 3, are so similar constructed to each other, a particular explanation is not required of both. Therefore I shall proceed with fig. 3.

(Fig. 3 No 1. *pl.* 6) is a design for an ellipsoidal dome, the plan being elliptic, and one of the vertical sections circular. The ribs are constructed without trusses. In order to divide them as equally as possible, a purlin is introduced to support the upper ends of the jack-ribs. As this dome is supposed to rise from an elliptic well hole the timbers are carrid below the base, from *a. b, c, d, e, f*, No. 1 is the elevation No. 2 the plan, showing the upper face of the wall-plates purlin and curb, (fig. 3, No 3, fig, 3. No. 5. fig. 3 No, 7,) are the entire ribs, to be placed upon A, C, E, in the plan, and (fig 3 No. 4 fig. 3, No. 6, fig. 3 No. 8) are the jack-ribs to be placed upon B, D, F, on the plan the upper ends of all the ribs terminate upon the curb, or upon the purlin, with a sally, or birds mouth, which is the usual method of fitting them (fig. 2 No 1) Is a design for an hemispherical dome, constructed in the same manner as the elliptic dome fig. 3.

In large roofs constructed of a domic form, without trussing the ribs may be made in two or more thicknesses, in such a manner that the common abutment of every two pieces, in the same ring may fall as distant as possible from the abutment of any other two pieces, in a different ring. The number of purlins must depend upon the diameter of the dome.

To find the form of the boards for an ellipsoidal dome, the plan being an ellipse, and the vertical section upon the axis-minor a semi-circle; so that the joints of the boards may be in planes passing through the axis-major of the plan.

Let A B C D, (fig. 4 No. 1. *pl.* 6) be the plan of the dome, A C the axis major and DB the axis-minor; E the centre. From E, with the distance ED, or EB describe the semi-circle B F D. Divide the arc into such a number of equal parts, that one of them may be equal to the breadth of a board, and let the points of division be at 1, 2, 3, 4, &c. draw the lines 1*a*, 2*b*, 3*c*, 4*d*, perpendicular to B D, cutting B D at the points *a*, *b*, *c*, *d*. Then upon A C, as an axis-major, and upon E*a*, E*b*, E*c*, E*d*, as so many axis-minor, describe the semi-ellipse A*a*C, A*b*C, A*c*C, A*d*C, which will represent the joints of the boards upon one side of the dome. Now since all the sections of this dome, through the line A C are identical figures, the vertical section upon the line A C, will be identical to the half plan A B C or A D C. Divide, therefore, B A into any number of equal parts, by the points of division, *e*, *f*, *g*, *h*, *i*, *k*, *l*; the more correct will be the operation. Draw the straight lines *em*, *fn*, *go*, *hp*, *iq*, *kr*, *ls*, perpendicular to A C, cutting A C at the points *m*, *n*, *o*, *p*. *q*, *r*, *s*, and the semi-ellipse A*d*C, in the points *t*, *u*, *v*, *w*, *x*, *y*, *z*, on the straight lines G H, (fig. 4 No. 2) set off the equal parts E*m*, *mn*, *no*, *&c.* from each side of the centre E each equal to one of the equal parts B*e*, *ef*, *fg*, *&c.* in the semi-elliptic curve, A B C, in the plan No 1.

Through the points *m*, *n*, *o*, *p*, *&c.*, No 2, draw *tt*. *uu*, *vv*, *&c.* perpendicular to G H. Make *mt*, *mt*, each equal to *mt*, in the plan No. 1, and *nu*, *nu*, No. 2, each equal to *nu*, in the plan No. 1; then through all the points *t*, *u*, *v*, *&c.* draw a curve on each side of the line A C, to reach from A to C, and each curve will be the edge of a board. If the work be large it would be well to tack in nails at the several points and bend a slip around to describe the curve.

(Fig. 4, No. 3) show the longitudinal elevation: viz: on the line A C of the plan.

(Fig. 4. No. 4) exhibits the transverse elevation, the contour being identical to that of the section on the line A B.

To find the covering for a sphere or globe which is supposed to be divided into four parts or quarters.

Let A C B D (fig. 5, pl. 6) be the circumference, and A B and C D the diameters which divide the circle, into four quadrants; divide the arc A C into a like number of equal parts, (six in this example) draw E G through the arc at L, and take the six divisions from the arc A C and set them off from E to F on the line E G, produce the same number of parts from F to G, draw F M and F N at right angles with E G; take three of the divisions from the arc A G, and set them off from F to I, and from F to H, on the line M N, on M as a centre describe the arc E H G, cutting the point in H, on N as a centre describe the arc E I G, cutting the point in I.

Then will E I G H be one quarter of the covering required.

METHODS OF BOARDING CIRCULAR ROOFS.

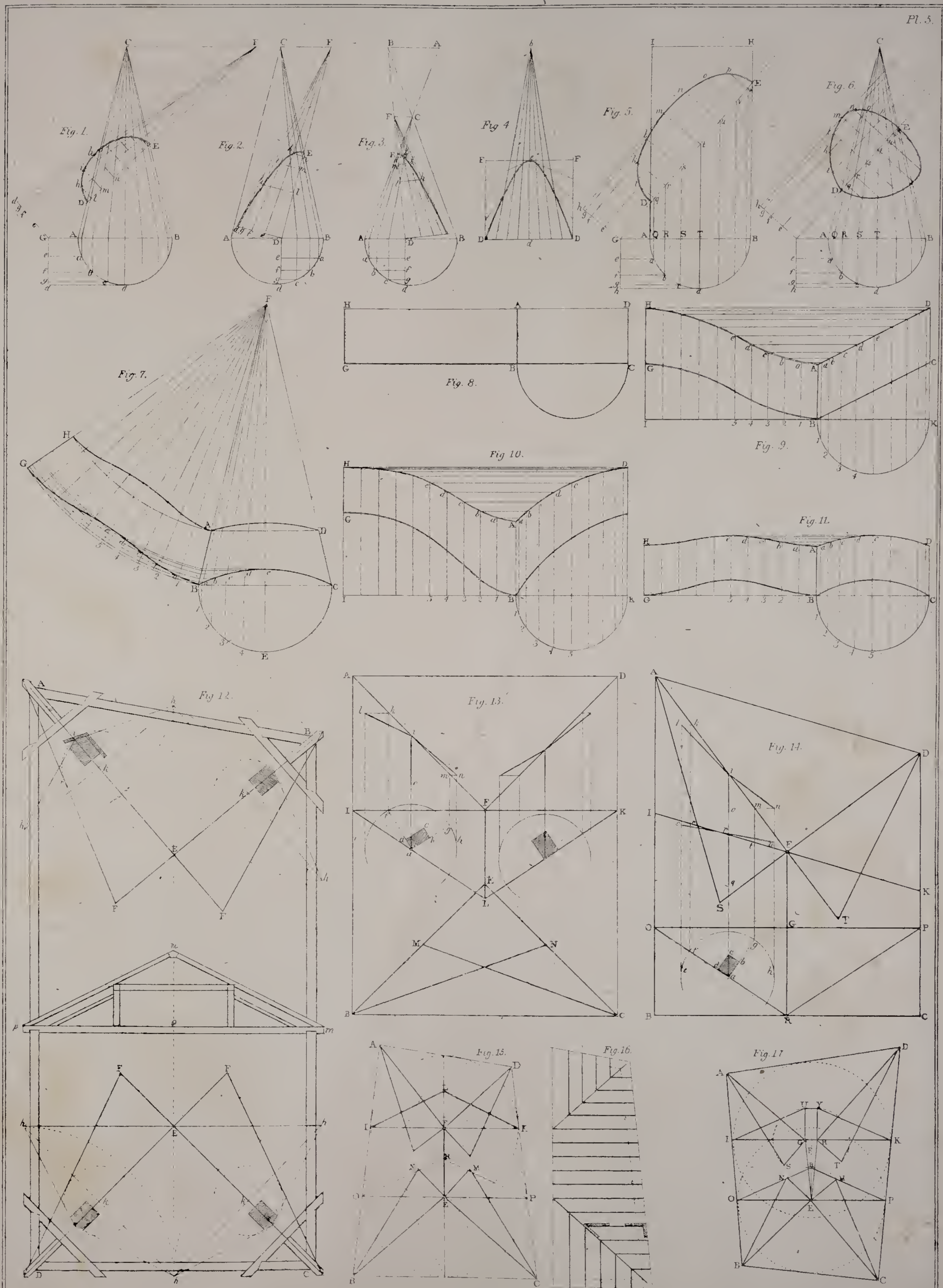
With regard to boarding of roofs for slates there are two methods.

In the first place; if a round solid be cut by two planes, each parallel to the base, the portion of the surface of the solid, between these planes, will nearly coincide with a conic surface, contained between sections perpendicular to the axis of the cone of the same diameter each as those made by cutting the round solid; therefore the whole of the round solid may be looked upon as so many conic frustums, laying one upon another; therefore to cover all the conic frustums is to cover the round solid.

The other method of covering a round solid is to suppose the base divided into equal parts and the solid to be cut hy planes passing through the points of division, and through the fixed axis; then the surface of the body will be divided into as many equal and similar parts: so that if any one of these portions of the solid be covered, the cover will of course fit any other portion thus divided; and as all the horizontal sections of each portion of the solid is the sector of a circle, the chords of all the sectors will be parallel to each other; therefore the curved surface will be nearly prismatic. This therefore affords another method of forming the boarding.

The first of these methods is called the horizontal method and the second the vertical method of covering a dome.

Let A B C (fig. 6 pl. 6) be a vertical section of a circular dome, through its axis and let it be required to cover the dome horizontally; bisect the base A C, in the point H, and draw H I perpendicular to A C cutting the semi-circumference in B, divide the arc B C into such a number of equal parts that each part may be less than the breadth of a board; that is to say, allowing the boards to be of a certain length, each part may be of the proper width, allowing for waste. Then if, between the points of division, we suppose the small arcs to be straight lines, as



Lith. of D.W. Kellogg & Co

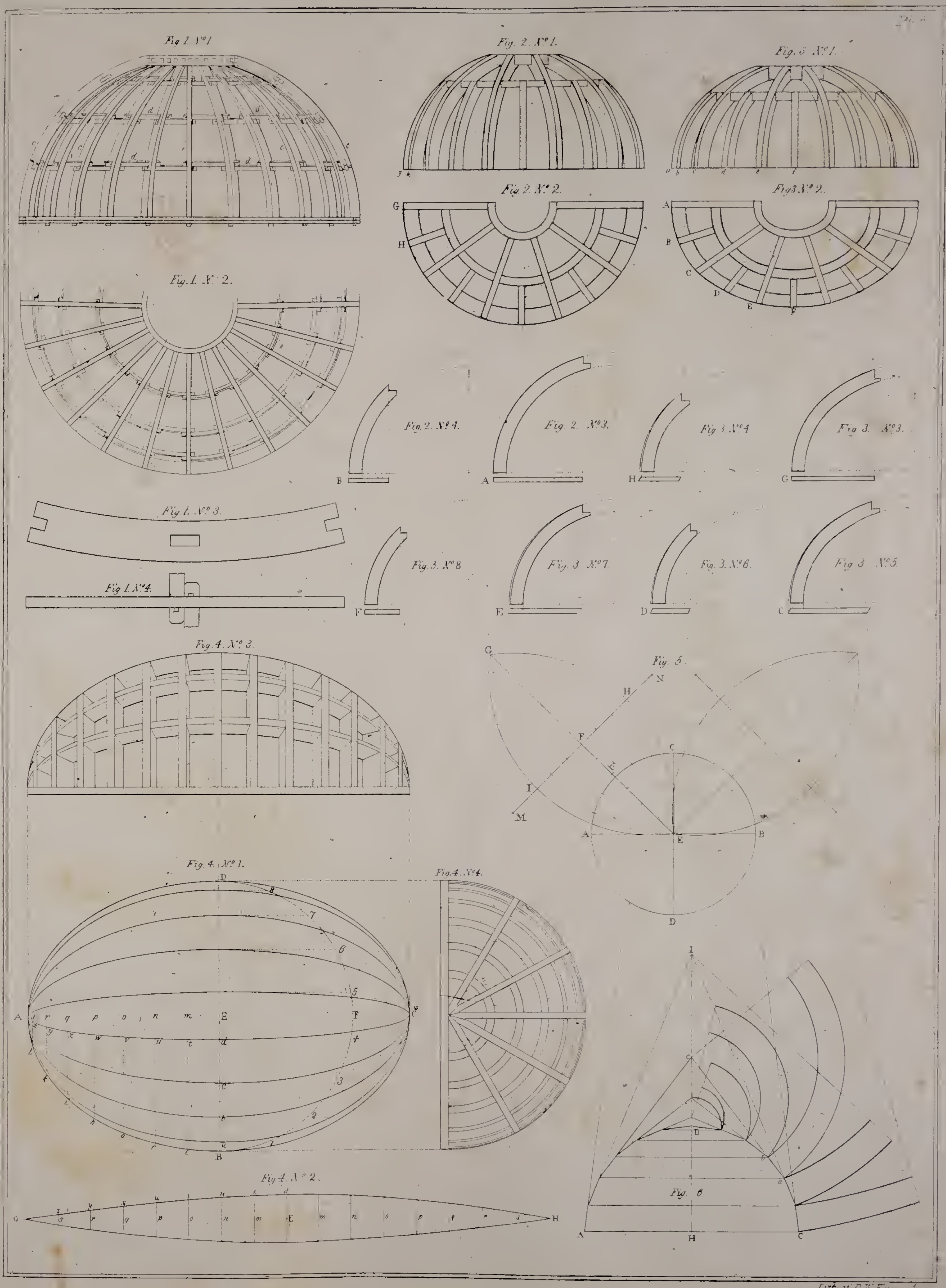


Fig. 1. No. 1
Fig. 2. No. 1.
Fig. 3. No. 1.
Fig. 2. No. 2.
Fig. 3 No. 2.
Fig. 1. No. 2.
Fig. 2. No. 4.
Fig. 2. No. 3.
Fig. 3. No. 4
Fig. 3. No. 3.
Fig. 1. No. 3.
Fig. 1. No. 4.
Fig. 3. No. 8
Fig. 3. No. 7.
Fig. 3. No. 6.
Fig. 3. No. 5.
Fig. 4. No. 3.
Fig. 5.
Fig. 4. No. 1.
Fig. 4. No. 4.
Fig. 4. No. 2.
Fig. 6.

Lith. of P. W. K...

they will differ very little from them, and if horizontal lines be drawn through the points of division to meet the opposite side of the circumference the trapezoids will be the sections of so many frustums of a cone and the straight line H I will be the common axis for every one of these frustums.

Now, therefore, to describe any board, which shall correspond to the surface of which one of these parts, *ab*, is the seetion, produce *ab* to meet H I in *c*; then will the radii *cb*, *ca*, describe two arcs ; then radiating the end to the centre the lines thus drawn will form the board required.

In the same manner any other board may be found ; as is evident from the principle described.

This kind of work should be described out on a floor or some other extensive plane ; by so doing you can draw all your moulds and cut the joints, both for the frame and the covering before it is erected. In case the dome should be so large that it cannot be described on a floor, take a thin board of suitable length for the first course of boarding, bend it around on the plain of the rafters and scribe it down to the base line of the dome, when it is well fitted, gauge it to a width and this will make you a pattern to mark out the remainder of the first course and for the second course take the upper edge of the pattern of the lower course to mark out the lower edge of the second course, when this is done gauge it to a width ; and for the third course, proceed in the like manner and so on until the roof is completed.

To find the forms of the boards for covering an annular vault.

Let A D (fig. 1, pl. 7) be the outer diameter of the annulus, C G the inner, E the centre, and A C the thickness of the ring.

On A C describe the semicircle A B C : then if A B C be supposed to be set perpendicular to the plane of the paper, it will represent half the section of the ring. From E with the radius E A, describe the semi-circle A F D ; and from the same centre E, with the radius E C, describe the semi circle C H G , then A F D is the outer circumference and C H G is the inner circumference ; and consequently A F D G H C, A is the section of the ring, perpendicular to the fixed axis ; and the section A B C of the solid itself is perpendicular to the section A F D G H C.

To find the form of any board ; divide the circumference A B C of the semicircle into such a number of equal parts as the boards or planks out of which they are to be cut will admit.

Let *ab* be the distance between two abjacent points ; through the centre E draw H I perpendicular to A D : and through the points *a* and *b*, draw the straight line *ac*, meeting H I in the point *c* ; from *c*, with the radius *ca*, describe an arc ; and from the same centre, *c*, with the radius *cb*, describe another arc, and inclose the space by a radiating line at each end ; and the figure bounded by the two arcs, and the radiating line will be the form of the board required.

In the same manner the form of every remaining board may be found. It is obvious that, as common boards are not more than from ten to twelve inches in breadth, the boards formed for the covering cannot be long, if so they will produce much waste.

To cover an ellipsoidal dome the major-axis of the generating ellipse being the fixed axis.

Let A B C (fig. 2. *pl.* 7) be the section through the fixed axis or generating ellipse which will also be the vertical section of such a solid. Produce the fixed axis A C to I, and divide the curve A B C into such a number of equal parts that each may be equal to the proper width of a board. Then as before, draw a straight line through two adjacent points *a* and *b*, to meet the line A I in *c* ; then with the radii *ca* and *cb*, describe arcs and terminate the board at its proper length

(Fig. 2 No. 2) is an horrizontal section of the dome, exhibiting the plan of the boarding.

(Fig 3 *pl*, 7) is a section of a circular roof. The principle of covering it with boards bent horizontally, is exactly the same as in the preceding examples.

It is now necessary only to explain one general principle which extends to the whole of these round solids. The planes which contain the conic frustums are all perpendicular to the fixed axis, which is represented by H C, in all the figures. Produce *ab* to meet the fixed axis H I in *c* ; then with the radius *ca*, describe an arc, which two arcs will form the edges of the board, the ends are formed by radiating lines.

Either of these figures which we inspect we shall find this rule to apply as the boards approach nearer to the rovolving axis ; they may be made either wider or longer but as the boards approach near to the fixed axis the waste will be greater, and consequently, the boards must be shorter, when the boards come very near to the bottom of the dome, the centre, for describing the edges of the boards will be to remote for the length of a rod to be used as a radius.

In this case we may have recourse to the following method.

Let A B C (fig 7, *pl.* 7) be the section of the dome as before, and let *e* be the point in the middle of the breadth of a board : draw *ed* parallel to A C, the base of the section, cutting the axis of the dome in *g*, and join A*e*, cutting the axis in *f*. (Then by *Problem* 7 *Geometry*) describe the segment of a circle through the three points *d*, *f*, *e*, and this will give the curve of the edge of the board as required.

(Fig. 7. No 2) exhibits the manner of using the instrument, (this instrument is also described in *problem* 17 *geometry*) Thus suppose we make D E equal to *de*, fig. 7 No 1 ; Bisect D E in G and draw G F perpendicular to D E and make G F equal to *gf*, in (fig. 7 No. 1) draw F H parallel to D E and make F H equal to F E, and join E H : then cut a piece of board into the form of the triangle H F E ; then let H F E be that triangle ; then move the vertex F from F to E keeping the leg F E upon the point E; and the leg F, and the angular point F of the piece so cut, will describe the curve, or perhaps as much of it as may be wanted.

It must be here observed that the line described is the middle of the board ; but if the breadth of the board is properly marked off at each end, on each side of the middle, we shall be able to describe the are with the same triangle, or if the concave edge of the board is hollowed out the convex edge will be found by gauging the board off to its breadth.

As all the conic sections approach nearer and nearer to circles, as they are taken nearer to the vertex. so a parabola whose abseissa is small, compared to its double ordinate will have its curvature nearly uniform, and will, consequently coincide very nearly with the segment of a circle and as this curve is easily described I shall employ it here instead of a cireular arc as in No. 3 and 4.

Draw the chord D E as before, and bisect it in G. Draw G F perpendicular to D E and make G F equal to *gf*, in No 1 ; so far the construction of the diagrams, No. 3 and 4 are the same, but in what follows they are different, therefore I shall speak of each sceperately—No. 3.

Divide each half, D G, G E, into the same number of equal parts ; and, through the points of division draw lines perpendicular to D E ; also from the points D and E at the extremities, draw perpendiculars ; and make each of these perpendiculars equal to G F then divide each into as many equal parts as D G or G E is divided into, and, through the points of division draw lines to F, intersecting the perpendiculars, and through the points of intesection, draw a curve on each side of the middle point F. and this will be the form of the edge of the board nearly.

In No 4 make F H equal to *gf*, No. 1, and join D H and H E. Then divide D H and H E each into the same number of equal parts, then, through the corresponding points of division draw straight lines and the intersection of all the lines will form the curve sufficiently near for the purpose. The lines thus drawn being tangents to the parabolic curve.

The preceeding method of covering round solids requires all the boards to be of different curvatures, and continually quicker as they approach nearer to the crown ; but by the following method of covering a dome, with the joints in vertical planes, when the form of one of the moulders is obtained this form will serve for moulding the whole solid. The waste of stuff in this case is not less than in the other.

The method which I am about to explain is not only useful in the formation of the boards of a dome but in the covering of niches.

In (Figs. 4, 5, and 6 pl. 7) No. 1 is the plan, No. 2 the elavation ; the counter of the latter being a vertical section passing through the axis. Fig. 4 is a dome which represents a round body of which the vertical section is an ogee or a curve of contrary flexure, to (Figs. 5 and 6) Fig. 5 represents a dome whose eounter is a semi-circle, fig. 6 represents a segmental dome ;

I shall proceed first with (figs. 5 and 6.) Through the centre of the plan G, draw the diameter A C ; and the diameter B D, at right angles to A C ; and produce B D to E. Let B D figs. 5 and 6 be the base of the semi-section of the dome on B D apply the semi-section C F D ; and as the dome represented by fig. 5, is semi-circular, the point F will coinside with the point A in the eircumference of the plan. In fig. 5 and 6 divide the curve F D, of the rib into any number of equal parts, and extend the curve D F upon the straight line D E, from D to E; that is, make the straight line D E, equal in length to the curve D F. Through the points of division, in the curve D A, draw lines perpendicular to D G, cutting it at the points *a*, *b*, *c*, then, extending the parts of the arc between the points of division upon the line D E, from D to 1, from 1 to 2, from 2 to 3 &c. make D *d* equal to half the breadth of a board, and join *d* G ; produce the lines 1*a*, 2*b*, 3*c*, &c. draw through the curve D F to meet the line *d* G in the points *d*, *c*, *f*, &c. Through the points 1, 2, 3, &c. in DE, draw perpendiculars 1*g*, 2*h*, 3*i*, &c. make 1*g*, 2*h*, 3*i*, &c. respeetively equal to *ad*, *be*, *cf*, &c, and through the points *d*, *g*, *h*, *i*, &c. E draw a curve, which will form one edge of the board, the other edge being similar, we have only to describe a curve equal and similar, so as to have all its ordinates respectively equal from the same straight line D E.

In fig. 4, the form of the mould for the boards is found in a similar manner except that the curve D E is one side of the elevation, No. 2 ; Lines are drawn from the points of division in D E perpendicular to the diameter A C which is parallel to the base of No, 2 and the points of division are transfered from the radius G C to the radius G D which is the base of the section. The remaining part of the process is the same as in figs. 5 and 6

In Fig. 5 the curved edge of the board is a symmetrical figure of the sines ; the curves of the mould, fig. 6 is a smaller portion of the figure of the same curve and in fig. 4 the mould is a curve of contrary flexure ; and if the curve D E be composed of two arcs of circles, the curve of the edges of the mould for the boards will still be compounded of the figure of the sines set on contrary sides ; and if the curve D E be compounded of two elliptic segments, the edge of the mould for the formation of the boards will still be of the same species of curve ; viz. the figure of the sines.

This figure occurs very frequently in the geometry of building.

COVERINGS OF POLYGONAL ROOFS.

The plans of these roofs are here supposed to be regular polygons, and all the sections prallel to the base, similar to the base and consequently similar to one another.

They are made of prismatic solids meeting each other in planes perpendicular to the plane of the base ; and these mitre-planes meet each other in one common axis, which passes through the centre of each polygon.

In (pl. 7. fig. 8.) the plan is denoted by the letters A B C D E F A. Then the centre of the polygon being the point I, draw the lines AI, BI, CI, &c. Bisect any of the sides as A B, in the point L, and draw LI, then LI is perpendicular to A B.

Produce the line I L to M and let I L N be the section applied upon I L. In the curve L N take any number of points 1, 2, 3, at equal distances, and transfer these distances, to the line L M, so that L M may be equal to the arc L N. Through the points 1, 2, 3, &c. in L M, draw lines 1*g*, 2*h*, 3*i*, &c. parallel to, A B ; and through the points 1, 2, 3, &c. in the arc L N, draw lines 1*d* 2*e* 3*f* &c. also parallel to A B, cutting L I at the points *a*, *b*, *c*, &c. and B I at the points *d*, *e*, *f*, &c. Make 1*g*, equal to *a d*, 2*h*, equal to *b c*, 3*i* equal to *c f*, &c. Through the points *g*, *h*, *i*, &c. draw a curve, which will be the edge of the joint over the mitre.

To find the angle-rib through the points *d*, *e*, *f*, &c. draw *dk*, *el*, *fm*, &c. per-

pendicular to B I. Make *dk*, *el*, *fm*, &c. respectively equal to *a*1, *b*2, *c*3, &c. through the points *k*, *l*, *m*, &c. draw a curve which will be the edge of the angle-rb as required.

Fig. 9 exhibits the method of framing the ribs for such kinds of roofs.

Fig. 10. shows the manner of describing the covering and ribs of a domical roof.

Fig. 11 shows the manner of describing the covering and ribs of a roof whose vertical section is a figure of contrary curvature.

Fig. 12 exhibits the manner of forming one of the ribs for an ogee roof, or that of a contrary curvature.

GROINS AND ARCHES.

Groins are the intersections of the surfaces of two arches crossing each other.

CONSTRUCTION OF GROINED ARCHES.

Groined Arches may be either formed of wood and lathed for plastering, or be constructed of brick or stone.

When constructed of brick or stone, they require to be supported upon wooden frames, boarded over so as to form the convex surface, which each vault is required to have, in order to sustain the cross arches during the time of turning them. This construction is called a centre and is removed when the work is finished. The framing consists of equi-distant ribs fixed in parallel planes perpendicular to the axis of each body; so that when the undersides of the boards are laid on the upper edges of the ribs, and fixed, the upper sides of the boards will form the surface required to build upon.

In the construction of the centering for groins one portion of the centre must be completely formed to the surface of its corresponding vault, without any regard to the cross arches, so that the upper sides of the boards will form a complete cylindric or cylindroidic surface. The ribs of cross vaults are then set at the same equal distances as that now described: and parts of the ribs are fixed on the top of the boarding at the same distances and boarded in; so as to intersect the other, and from the entire surface of the groin required.

Groins constructed of wood in place of brick or stone, and lathed under the ribs, and the lath covered with plaster, are called plaster groins.

Plaster groins are always constructed with diagonal ribs intersecting each other, then other ribs are fixed perpendicular to each axis, in vertical planes at equal distances, with short portions of ribs upon the diagonal ribs; so that when lathed over, the lathed angle may be equally solid to sustain the plaster.

When the axis and the surface of a semi-cylinder cuts those of another of greater diameter, the hollow surface of the lesser cylinder, as terminated by the greater is called a cylindro-cylindric arch, and vulgarly a Welsh groin.

Cylindro-cylindric Arches or Welsh groins, are constructed either of brick, stone or wood. If constructed of brick or stone, they require to have centres which are required to be formed in the same manner, as those for groins; and if constructed of wood, lath and plaster the ribs must be formed to the surfaces.

In the construction of groins, and of cylindro-cylindric arches, the ribs that are shorter than the whole width are termed jack ribs.

Cellars are frequently groined with brick or stone, and sometimes all the rooms of the basement stories of buildings in order to render their superstructure proof against fire, the surfaces of brick or stone on which the first arch stones or course of bricks, are placed, are called the springing of the arches. It is evident that the more weight put on the side walls which sustain arches, the more will they be able to sustain the pressure of the arches; therefore the higher a wall is, the greater the weight will be on each of the side walls; and for this reason groins are often constructed with wood in upper stories, instead of brick or stone as not being liable to thrust out the walls, or bulge them by the lateral pressure of the arches. The upper stories of buildings are never groined with stone or brick, unless when the walls are sufficiently thick to sustain the lateral pressure of the arches.

The ceilings of Gothic buildings were frequently constructed with groined arches of stone, which are obliged to be supported with buttresses, at the springing points of the arches.*

GROINS AND ARCHES.

Given the plan of a rectangular groined arch or vault, of which the openings are of different widths, but of the same heighth and a section of one of the arches, as also the seats of the groins to find the covering of both arches so as to meet their intersection.

Let AAA &c. (fig. 1, No. 1, pl. 8) be the plan of the piers and *ab*, *cd*, the seats of the groins.†

Let the section of the arch standing upon the lesser opening, B C, be a semi-circle; it is required to find the section upon the greater opening and the ends of boards, so as to meet the groin, or line of intersection, of the two surfaces.

On the diameter B C describe a semi-circle, and divide the quadrant into any number of equal parts *ef*, *fg*, *gh*, &c. and from the points *e*, *f*, *g*, &c. draw the line parallel to the axis F*k*, to meet the seat *ab* of the groined line, or line of intersection of the two surfaces. From the points *k*, *l*, *m*, &c. of intersection, draw the lines *k*Q. *l*R, *m*S, &c. parallel to the axis of the other vault, to meet the line VQ, perpendicular to that other axis in the points Q, R, S, &c. Then upon any line D E transfer the points Q, R, S, &c. to *q*. *r*, *s*. &c. and draw *qv*, *rw*, *sx*, &c. perpendicular to D E and transfer the ordinates F*e*, G*f*, H*g*, &c. of the semi-circle, to *qv*, *rw*, *sx*, &c. and through the points *v*, *w*, *x* &c. draw a curve; then *q*, *v* E will be half the section required.

To find the covering of the semi-cylinder. Upon any straight line YZ, No. 2, set off the distances *lm*, *mn*, *no*, &c. each equal to the chord *ef* or *fg*, &c. in No. 1; and draw *l*K, *m*L, *n*M, &c. in No. 2, perpendicular to YZ. Make *l*K, *m*L *n*M, &c. in No. 2, equal to L*k* M*l* N*m*, &c. of No. 1, and through the points, K, L, M, &c. No. 2, draw a curve. Then will the figure K*l*Z be half the covering of the cylinder.

To construct the covering, No. 3, for the great opening.

In the straight line *vq*, No. 3 make *vu*, *ut*, *ts*, &c. equal to the parts. E*z*, *zy*, *yx*, &c. of the elliptic curve No. 1. In No. 3 draw *v*B, *u*O, *t*N, *s*M, &c, and make *v*B, *u*O, *t*N, *s*M, &c. No. 3 equal V*b*, U*o*, T*n*, S*m*, &c. No. 1: and in No. 3 draw a curve through the points B, O, N, M &c; then *qv*BK*q* will be the covering required.

To find the groin of a cylindro-cylindric arch.

Let AAAA (fig. 2 pl. 8) be the plans of four piers which from the opening of different widths. On the lesser opening P M as a diameter, describe a semi-circle. Divide the quadrant next to P into any number of equal parts, and through the points of section draw the lines 1G, 2H, 3I, &c. perpendicular to P M, cutting P M in B, C, D, &c. and through the same points 1, 2, 3, &c. draw the lines 1*a*, 2*b*, 3*c*, &c. parallel to P M, cutting a line *qe* perpendicular to P M, in the points *a*, *b*. *c*, : produce the line which contains the points *a*, *b*, *c*, through the greater opening; and upon the part of the line thus produced, which is intercepted between the piers A, A, describe a semi-circle.

Produce the line M P to *k* and from *q* describe arcs *af bg*, *ch*, &c. cutting B*k* in the points *f*, *g*, *h*, &c. draw *fk*, *gl*, *hm*, &c. parallel to the base of the greater semi-circle to cut the arc of the same in the points *k*. *l*, *m*. &c. From the points *k*, *l*, *m*, &c. draw the lines *k*G, *l*H, *m*I, &c. parallel to P M; then through the points G, H, I, K, L, draw a curve, G, H, I, K, L, which will be the seat of the groin.

To find the diagonal or groin-ribs of a vault, of which the lesser openings are semi-circles, and the groins in vertical planes passing through the diagonals of the piers.

On *ah*, (fig. 3. pl. 8) the perpendicular distances between two adjacent piers of the lesser opening describe a semi-circle *abh*; and in the arc take 1, 2, 3, &c. any number of points and draw the lines 1*l*, 2*m*, 3*n*, &c. cutting the diagonal *ik* in *l*, *m*, *n*, &c. Draw as before *lq*, *mr*, *ns*, &c. perpendicular to *ik* and through the points *i*, *q*, *r*, *s*, &c. draw a curve; then *iuk* will be the edge of the rib to be placed in the groin.

The edge of the ribs for the other opening, will be found thus; From the points *l*, *m*, *n*, &c. draw the lines *l*I, *m*K, *n*L, &c. parallel to the axis of the opening of the larger body cutting H B at the points C, D, E, &c. Make CI, DK, EL, &c. each equal to *c*1, *d*2, *e*3, &c. then through the points BI KL, &c. draw a curve: and the line thus drawn will be in the surface of the greater opening. so that B N H will be one of the ribs of the body-range,

The method of placing the ribs is, exhibited at the lower end of the diagram, fig, 3; the ribs of each opening being placed perpendicular to the axis of each groin.

To draw the arches of groins whether right or rampant so that their arches will intersect or mitre together as in (figs. 4, 5, and 6.) Fig. 4 is the plan of a rectangular groined arch or vault of which the openings are of different widths but of the same heighth.

This form is called the Gothic or pointed arch.

The section of the arch being given standing upon the lesser opening *abh*, and *ad* being divided off into a like number of equal parts as in Fig. 3, you can proceed to find the ribs for the wider opening as in the former examples.

A rampant arch one of which (see fig. 5) the abutments or seats rises from an inclined plane.

This form of a groined arch is frequently met with in the ceilings of the under sides of galleries in Gothic churches.

It is but seldom the ceilings of galleries run horizontally for they are generally raised up from the base line on the outer wall for the sake of clearing the tops of the lower range of windows or that more light may be produced.

To find the diagonal ribs of a vault as in (fig. 5) suppose *jh* and *jk* to be the base or horizontal lines, and *a*, *h*, and *i*, *n* the seats of the groins, draw *ja* and *ji* at right angles with *jh* and *jk* and make *ji* equal in height to *ja* draw the diagonals *ah* and *ih*, (the arch in the lesser opening being described as in fig. 27) divide the arc *ab* into a like number of equal parts (as above described) draw lines from the points 1, 2, 3, &c. through *c*, *d*, *c*, parallel to *ai* until they intersect the diagonal line *ik*, then in the wider opening draw lines through *l*, *m*, *n*, &c. parallel to *ij*, and transfer the ordinates 1*c*, 2*d*, 3*e*, &c. in the lesser opening to *lq*, *mr*, *ns*. &c. in the wider opening, and through the points *q*,*r*,*s*, &c. draw the curve, then will *i*, *p*, *u* be half of the section required.

On the remaining part of the plate are a number of examples of the Gothic or pointed arch, as designed and described in the works of A. Pugin.

7. The semi-circular Arch was the principal one used in all buildings until about the middle of the 12th, century, though a solitary instance of the pointed Arch may now and then be proved to be of earlier construction.

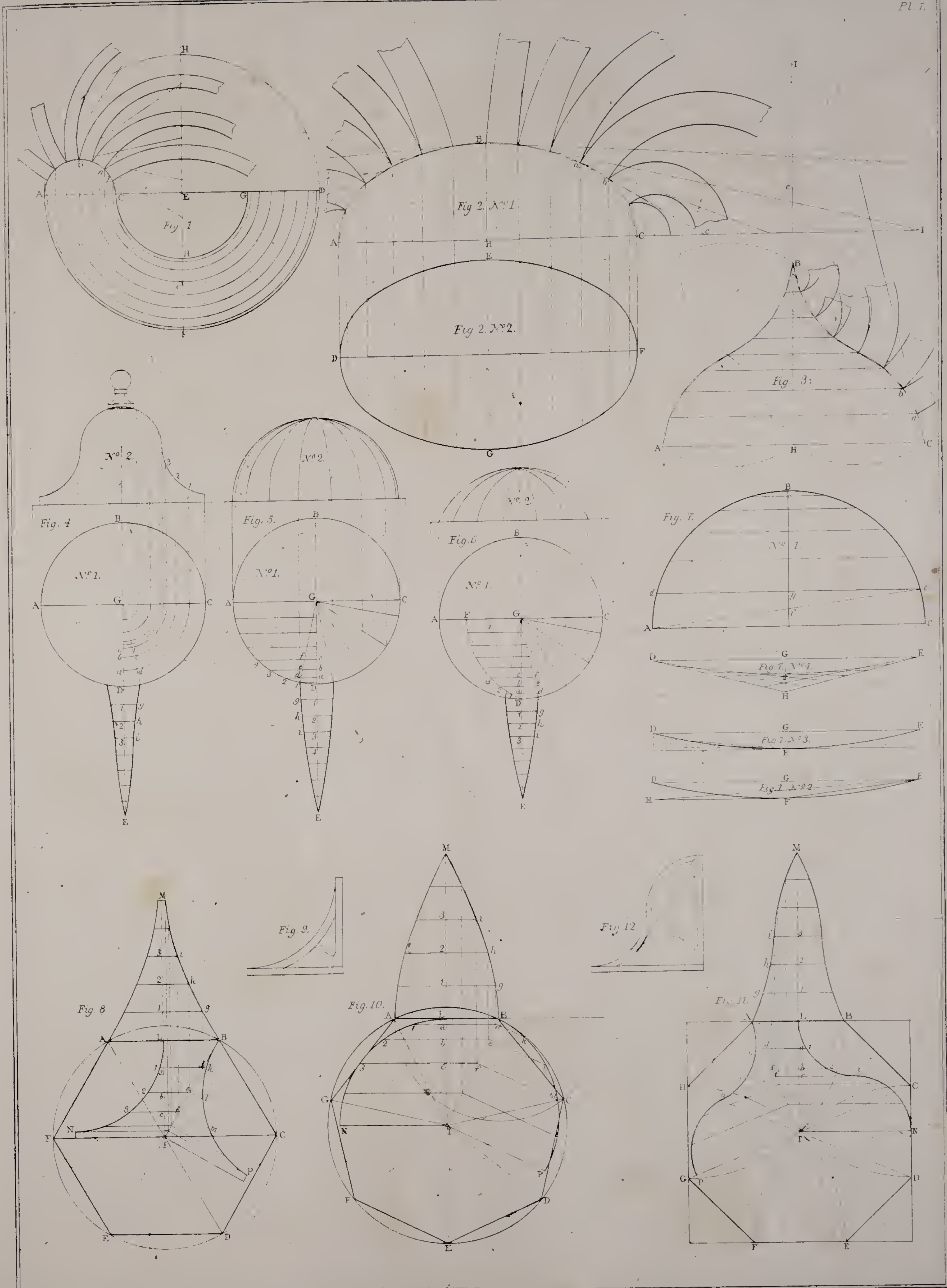
8. Arch described from one centre placed above the base line.—This form has been denominated the horse shoe, it is common in some buildings of eastern countries.

9. Semi-circular, but including a portion of the perpendicular jambs above the impost.—This form is seen in a side—arch of the rood tower of Malmsbury Abby Church where the transepts being narower than the nave and choir, two of the four arches were limited to a less breadth, though required to equal the others in height.

10.-11 and 12 Elliptical Arches described from three centres—Arches of this form are not only found in Norman buildings, mixed with the semi-circular but frequently over doors and windows in the early part of the fifteenth century, along with the pointed Arch and the other characteristics of the style of that period.

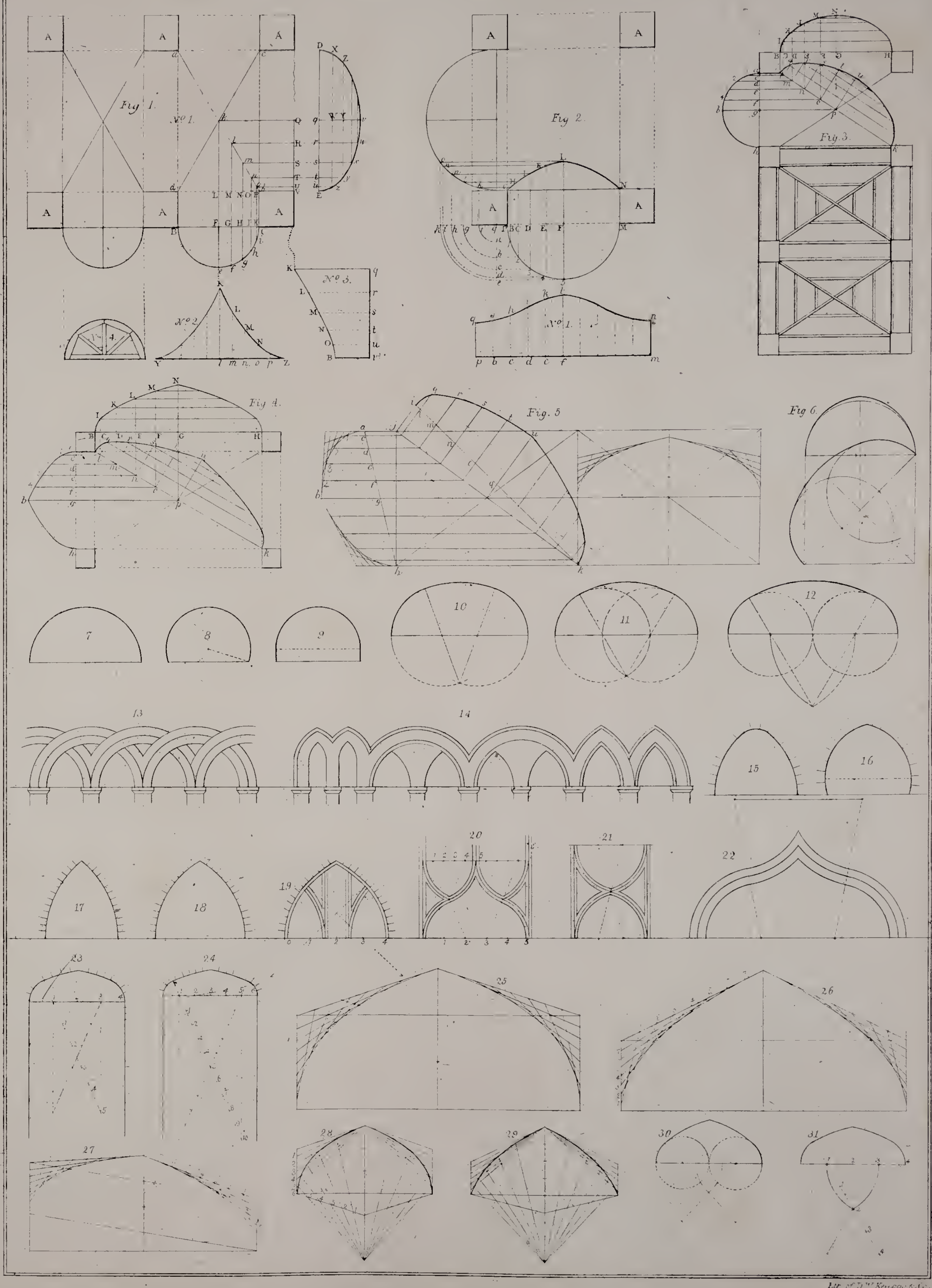
* A specimen of this kind of structure may be seen in A. Pugin's eminent works on Gothic Architecture taken from various ancient edifices in England (vol. 2, pl. 47.)

† The difference between the plan of any body and the seat of a point or line is distinguished thus. The plan is a figure upon which a solid is carried up so that all sections, parallel to the plan, are equal and similar to that plan, and the surfaces are perpendicular; but the seat of a line is not in contact with the line itself, but a perpendicular erected from any point in the seat will pass through the corresponding point of the line itself.



Lith. of D.W. Kellogg & Co.

Pl. 8.



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13. Semi-circular Arches intersecting each other.—Some instances occur of intersecting pointed Arches and others of Arches if they may be so called, described by strait lines forming a series of intersecting triangles raised on one base.

14. Semi-circular and Lancet Arches combined.—Such mixture is commonly found in buildings of the twelfth century, when the pointed Arch began to prevail.

15. Elliptical, resembling a pointed Arch only rounded at the top.

16. Moorish.—This form may be classed with the horse-shoe, No. 8.

It is described from two centres placed above the impost Arches, somewhat of this form are occasionally met with in buildings of the early pointed or Gothic style, they are only found placed over narrow apertures.

17. Lancet Arch described from two centres on the out side of the Arch—Those termed lancet have been happily applied to the tall narrow windows which enlighten the structers of the thirteenth century.

Salisbury Cathedral is the most complete specimen of that style. These lights have each a pointed arch at top, and the arch is frequently raised on straight lines above the mouldings of the impost where such mouldings occur; this is indeed the lancet from comparing the arch to the head of the lancet.

18. Equilateral where the point of the base and crown form an equilateral triangle.—This may be called the standard form of the pointed arch and is perhaps the most beautiful.

19. Four-centred pointed—some beautiful varieties of decoration were struck out from this form, but it must still be regarded as less perfect than the simple arch struck from two centres, as in Nos. 17 and 18, 20, 21, and 22. The combination of circles, and portions of circles, being so infinitely diversified in specimens of florid tracery especially in the larger windows of the fourteenth century it would be in vain to attempt to analyze all their principles. We may observe however that most of them were divided at first into a few large forms, and these again subdivided into as many openings as the space would allow, so that the openings were never broader than those of the perpendicular lights of the window and seldom less than one-half of the breadth of one of these. In proportioning the void and solid parts of windows we seldom find the mullion exceed one-third of the light in the larger divisions, nor smaller than one fifth.

23 and 24. Four-centred Arches whose centres must be upon the same diagonal lines, which are found by dividing the base-line of the arch into more or less parts, according to the fixed height of the arch,—These are some of the various forms of what has been called the Tudor arch; being chiefly found in buildings erected under the reigns of the princes of the house of Tudor, we find however that this flattened arch was used more than fifty years before the accession of Henry 7th, the first English sovereign of that name.

25 and 6. Methods of describing a pointed arch by the intersection of straight lines.—This arch may be classed with the four-centered, being of a less curve in the upper part than the lower. Many actual examples of arches appear to have been struck out, by the intersections of straight lines, in specimens of the latter period.

27. Rampant pointed, described also, by the intersection of straight lines.—(See what is said of fig. 5.)

28 and 29. Mode of describing a pointed arch by the crossing of straight lines. This arch also may be classed with the four-centered.

30 and 31. Four-centered pointed of the same class as Nos. 23 and 24, but differently described.

THE ROOFING.

The roof is that part of a building raised upon the wall, and extending overall the parts of the interior, in order to protect it from depredation, and from the severities and changes of the weather.

The Roof in Carpentry, consists of the timber-work which is found necessary for the support of the external covering.

The several timbers of a roof are, *principal rafters*, *tie-beams*, *king-posts*, *queen posts*, *struts*, *collar-beams*, *straining-sills*, *pole plates*, *purlins*, *ridge-peices*, *common rafters*, and *camber-beams*. The use of these will appear from the following description of them.

Principal-rafters, are the two pieces of timher, in a framed roof, that form the two equal sides under the covering.

A Tie-beam, is a piece of timber, connecting the end of the principal rafters, in order to prevent them from spreading, by the weight of the covering. The *tie-beam* is therefore used as a string, and is in a state of tension.

A *king-post* or *principal-post*, is a vertical piece of timber, extending from the meeting of the two principal rafters to the tie-beam, for the purpose of supporting the tie beam in the middle.

Queen-posts, are two pieces of timber, equidistant from the middle of the truss the one suspended from the head of one of the principal rafters, and the other with a level piece of timber between them.

Struts are those props which support the principal rafters in one or more points so as to divide them into equidistant parts.

A *collar-beam* is the piece of timber framed between two queen-posts.

A *straining-sill* is a horizontal peice of timber, disposed between the end of the queen-posts, to conteract the efforts of the struts, in pushing the principals nearer to each other.

A *pole-plate* is a beam over each opposite wall, supported upon ends of the *tie-beam* or upon the feet of the principal rafters.

Purlins are horizontal pieces of timber, supported by the principal rafters.

A *ridge piece* is a beam at the apex of a roof, supported by the king-post, only the heads of the principals.

Common rafters are inclined pieces of timber, parallel to the principal rafters, supported by the pole-plates.

Camber-beams are those timbers which are supported upon purlins over the collar-beams and support the boarding for a leaden platform.

Fig. 1, is a design for a roof for a very narrow span, which ought not to be

DESIGNS FOR ROOFS.—(PLATE 8.)

employed over a space exceeding fifteen or twenty feet Figs. 2, 3, 4, and 5 are examples which may be employed for a space of thirty or forty feet where the roof is shingled, or tinned.

Fig. 6 is a design for a roof for a narrow span, and calculated for an arched ceiling, having only one collar beam, without a tie at the bottom.

Figures. 7 and 8 exhibits two designs of trusses suitably constructed for the roofs of churches, and are calculated for a span of seventy or eighty feet.—fig. 9 is a truss suitable for a small church of forty or fifty feet span.

Figures 10, 11 and 12 shows the elevations, and the construction of the timber work of a roof and cupola, for a small church of forty or fifty feet span.—Fig. 10 represents the side of the truss and frame.

Fig. 12—represents the frame as seen in the length of the roof.—Fig. 11—shows the plan of the frame part of the deck of the belfrey which the upper part or story is framed into, these timbers should be well locked together and bolted; which I think will be sufficiently strong without the posts and timbers being extended down and framed into the collar beams, as they are generally for it is a bad practice to load down a roof with unnecessary timbers; the least timber that can be put into a roof to make it sufficiently strong the hetter.

The upper part of this cupola is framed out with planks or joist projecting out far enough to receive the entablature.

Fig 13.—is a design for a truss suitably constructed for a church or any other large building where it is designed to raise an arched ceiling and is calculated for, sixty or seventy feet span.

In this example it will be necessary to make the roof steeper than in the other examples heretofore described; and the timber also should be well seasoned, which will make the work better.—Fig. 14 shows the method of connecting the tie beam to that of the principal rafter.—Fig. 15–shows the longitudinal section of the tie beams and king post. The tie beams are locked into each other, so that the outsides of each are in the same plane. The king post is to be made in two parts, and a space cut away in each half so as just to admit the tie beams when locked together to pass through them.—Fig. 16.–shows three different methods for scarfing timber.

CONSTRUCTION OF STONE AND WOODEN BRIDGES.

Fig. 1—Shows the centre of Westminister Bridge which is partly supported by pieces strutting from the footings, and partly by piles driven into the bed of the river.*

Fig. 2—Is the centre of Blackfriars Bridge; which is entirely supported by pieces strutting from the footings and pier.—References.—AAA Timbers which support the centering.—BB.CC Upper and lower striking plates cased with copper.—DD Wedges between the striking plates for lowering the centre.—EEE Double trussing pieces to confine the braces.—FFF Apron pieces to strengthen the ribs of the centre.—GGG Bridgings laid on the back of the ribs.—HHH Blocks between bridgings to keep them at equal distances.—III Small braces to confine the ribs tight.—KKK Iron straps bolted to trussing pieces and apron pieces.—LLL Ends of the beams at the feet of the truss pieces.—MMM Principal braces.

Fig. 3–A longitudinal section of an arch of Waterloo bridge, showing the piles on which the piers are raised, the masses of bricks composing the standards, and the centre supporting the arch. The dotted line shows the direction of a curve in which the weight is so distributed that the different pressures to which the edifice is exposed have no tending to change the form of the arch.

Fig. 4—Shows a longitudinal section of the arch and centre of the Bridge that is biult over the little river crossing Main-street in the city of Hartford, (Conn.) The form of the centre of the bridge is different from those described: it is moreover well constructed. The bed of the river is a solid rock, and runs nearly on a horizontal plane between the two butments; during the summer season the river is generally shallow, which made it convenient for bedding the sills and timbers into which the struts are framed.—References.—A, shows a part of the longitudinal and transversection of the timbers running horizontally in the bed of the river, into which the struts are framed,—B Shows the form of the wedges that are placed under the feet of each strut for lowering the centre.—C Shows a part of the transversection of the frame.—D Is a section of the arch ribs.—E shows the manner in which the joints are connected.

* The first bridge that was built in England of any note, was what is called the "London Bridge." This bridge was originally begun in the year 1176 by a priest, called Peter, curate of St. Mary Colechurch, a celebrated architect of those times, it was thirty-three years in building; but this period will not appear surprising, when it is considered that it was built over a river in which the tide rises twice every day, from thirteen to eighteen feet. The bridge at first consisted of twenty arches, but in 1758 the middle pier was taken down and the two adjacent arches were converted into one, the span of which is seventy two feet; its breadth is forty-five feet, and for many ages there were houses along each side of it, but these were removed when the middle pier was taken down in 1758. The remaining arches are narrow, and the piers inconveniently large, being from fifteen to twenty-five feet in thickness. The passage over the bridge is commodious, but in other respects there is nothing remarkable about it.

The foundation Stone of Westminster Bridge was laid by the Earl of Pembroke (a nobleman distinguished by his taste in Architecture) on the 24th of January, 1739. It consists of thirteen large and two small semi-circular arches, of which the middle one is seventy six feet span, and the parapets forty four feet. The engineer was M. Labelye, a native of France; James King is believed to have aided in its design.

About ten years after this magnificent edifice was completed, another was begun about a mile lower down the river, known by the name of Blackfriars; the design was by Robert Mylne; it consists of nine arches of an elliptical form, of which the middle one is one hundred feet in span, and the breadth across the bridge is forty three feet six inches. The arches being elliptical, and of wider span than those of Westminster, the bridge of course has a lighter appearance. The general style of it bespeaks a mind emboldened by the success of his predecessors, to advance with cautious step in the practice of bridge building. It is a work of great merit and will stand a comparison with any other constructed in the same age. It was finished in $10\frac{3}{4}$ years.

But the glory of England in bridge building, may be seen in tho Strand or Waterloo Bridge, recently erected by Bennie, between Westminster and Blackfriars Bridges. Many other excellent Bridges have also been constructed in Great Britain, both of wood, iron, and stone.

The stone Bridge over the Little River, Hartford, Con. was built in the year 1832; the design of its centre was given and erected by James Chamberlain, a practical architect and builder of this city, and Elisha Rathbun being the master-mason. It consists of an arch of 90 feet span, and in breadth 100 feet.

The bridge over the Hockanum river in East-Hartford, which fell a few years since, was occasioned by driving over a large drove of fatted cattle. There has since been a substantial covered bridge erected after Town's patent.

The remaining part of the plate shows five designs which I have given for wooden bridges.

Fig. 5, is a side elevation of the frame, clearly showing all the timbers and the manner in which they are connected together. This design I think sufficiently strong for a bridge of ninety and even a hundred feet span If there be any cause for giving away, it will be from the spreading of the two outer butments: if these be built sufficiently strong to keep the lower struts to their proper place I think there will be no danger.

A bridge of this description should be filled in with puncheons between the principal timbers on each side of the truss or strings, standing vertically and not more than two feet apart from centre to centre, and covered over with boards on both sides, which should be matched or feather-edged, and strongly nailed with large nails or spikes, which not only keep the timber from being exposed to the storms, but makes the work more substantial, and it also prevents the timbers in a great measure from working or springing up and down which is of great injury to a bridge. This is the reason that Town's patent bridge has so good effect.

It is not by common travel that our bridges are injured and sometimes broken down, but by driving over large droves of cattle and horses. Drovers should be particularly careful in crossing a bridge, not to let their whole drove enter the bridge at the same time, for if their immediate weight does not break the bridge it will materially injure it.

Figures 6 and 7, are designs for bridges, which may be from fifty to sixty feet span or even wider, if they are built of good timbers and strongly covered.—Fig. 6 is supported by three iron bolts and two posts, the tenons may run through the sill, or tie beam and plate, and may be keyed. Fig. 7 is supported by three posts which are locked to the sill and plate, the posts are in two parts as represented in F.—G shows the manner in which they are locked to the sill and plate. The posts should be made of white-oak timber, as the ends which run below and above the sill and plate, will not be as liable to split. These being well locked and bolted together, will make a strong work, and will answer all the purposes of long iron bolts, and even better.

Figures 8 and 9 are of the same principle as figure 7, only calulated for a less span.

PLATE 11.

This plate exhibits a number of useful machines and their construction as made use of by builders and mechanics in general.

Figure 1, No. 1, and fig. 1, No. 2 shows the elevation of a machine that is called a crane; this machine is used to raise stone and other heavy bodies on to buildings, and it is also placed on wharves for the purpose of unloading small vessels and boats, it can be constructed so as to be taken apart and put together again as occasion may require.

Figure 1, No. 3, shows a plan of a frame drawn on a small scale.

Fig. 1, No. 4, shows a plan of the cap of the upper part of the frame; there must be a sufficient weight placed on the frame at A, to keep it in its place.

The cog wheel to which the wench or cylinder is attached and about which the rope winds is generally from 1 foot 6 inches to 2 feet in diameter, and the other wheel to which the crank is attached should be about one third of the size of the other.

Fig. 2, No. 1, fig. 2 No. 2, shows the elevation of a machine which is mostly used for hoisting peers for shop fronts, &c.

The frame part should be made of light timber, pine or spruce would be most proper, as it may be easily handled; the foot of the frame should be placed near the place where you intend to set your pier, and a little inclined as represented at No. 2; it may be secured by guy ropes, or by two light poles being made fast at the upper part of the frame. The cast iron cog wheels may be of the size of those of the crane, heretofore described.

Fig. 3, No. 1, fig. 3. No. 2, and fig. 3, No. 3, shows the plan and elevations of a wheel suitably constructed for raising goods &c. The wheel and machinery is generally erected in the upper part of wholesale stores for the purpose of hoisting and lowering goods from one loft to another.

The design that I have here given consists of three shafts; and to hoist heavy goods, the centre shaft should be used, which is attached to the great wheel, and to hoist light goods, such as boxes, bales, &c.; when they are to be raised many stories the two outer shafts will answer all purposes, and the work may be performed with much greater rapidity; you can either hoist upon one, and lower on the other, or hoist and lower both at the same time. The great wheel is generally made from 8 to 14 feet in diameter, and the shaft from 8 to 12 inches. ABC Shows a plan of a check or lever to stop the force of the wheel, which should be strongly secured at A, and a rope attached at the end B, leading over a pully at C, then passing through to the lower story.

Fig 4 shows a method of raising a stone column with *lewis* irons, D is the plan of a lewis which should be let into the end of the column very tight, having no room to play. A, A views of the capstan's. B and C a view and plan of the frame of a capstan. The frame which the large falls are attached to, may be secured by guy ropes or light poles as described in fig. 2, No. 2.

NOTE.—The weight of a stone column or any other form of a stone may be accurately ascertained, by finding the number of cubic feet and inches it contains by the rules given in (section 2d of mensuration) and multiplying the number of feet thus found by 125, which is the number of pounds contained in a cubic foot of stone.

A column that is 2 feet 6 inches at the lower diameter, and two feet at the upper, and 15 feet in length, contains about 60 cubic feet, and $60 \times 125 = 7500$ pounds; a column of this size, and even larger may be raised.

Fig. 5 shows a method of raising a truss by a gin pole.

This should be of a suitable length to raise the truss to its destined heighth, and should be made either of pine or spruce, so as to be easily raised or lowered: a stick that is from 10 to 12 inches in diameter at the bottom and from 6 to 8 at top, will be sufficiently large to raise a truss from 60 to 90 feet span.

This when placed upright should be secured by small falls for the guyes which will be more convenient, and safer than guy ropes, three in number will be all that is necessary, letting them run off at different angles, and properly secured at the bottom. A shows the manner in which they are secured at the top, and also the big block of the large fall, the fall part runs down and passes through a snatch block at B, then off on to the capstan at C.

In raising the trusses of a church they should be put together on the main floor and well secured; commence raising them at one end of the building, and when you have got one raised and placed to its proper place and well braced, slip the gin along where the second one is to stand, and this being raised and properly secured to the other, proceed in this manner through the whole building; a good set of hands working under a master workman, will generally be able to complete the whole in one day.

Fig. 6, shows the manner of rasing a purchase which is called by seamen a Spanish burden; A is a large single block which the runner is rove through, and one end of the runner is made fast to the double block at B, the other end hooks on to the weight at C, and also the single block of the fall, the fall part being rove through a snatch block at D, for the purpose of attaching a horse or a yoke of cattle, &c. This machine will be found useful for loading stone or large logs, on two set of wheels; this being placed near one end of a log and properly fixed for hoisting, fasten a horse to the fall and raise one end of the log up to a proper heighth, place one set of wheels under and lower the end, then the machine being placed at the other end, and raised in the same manner; this way of loading stone or logs is easily and quickly performed.

Figs. 7 and 8, show the elevations of a pile driver, a machine for driving piles into the ground, of which there are many kinds, some are worked by a number of men who raise a heavy weight to a small heighth, and then let it fall upon the pile, till by reiterated blows they drive it to the required depth. This machine is extremely simple. A long thick plank of wood is fixed up close to the pile, having a mortise through the upper end, in which a pully is fitted; a rope goes over this to suspend the rammer which is a large block of hard wood, properly hooped to prevent it from spliting. In rising and falling, it slides against the base of the plank, and is guided by irons which are fixed to the ram and bent around the edges of the plank in the manner of hooks. The plank when placed upright is secured by guy ropes in the manner of the mast of a ship; the end of the great rope which suspends the ram has ten or twelve small ropes spliced into it, for as many men to take hold of, to work it. They raise the ram up three or four feet by pulling the ropes all together, and then letting them go, the ram falls upon the pile head. When the pile becomes firm enough to cause the ram to rebound, they take care to pull the ropes instantly after the blow, that they may avail themselves of the leap it makes. This is the simplest form of the machine.

But for large works such as bridges, &c. the piles are driven by a different kind of machine: this has a very heavy iron ram as represented at E and F, in (figs. 7 and 8) with mechanical powers, by which it is raised to a very considerable heighth, and then let fall, instead of continually repeating small blows. This machine is constructed by two uprights erected on the frame, and supported by two braces, which is framed into the uprights and the cross feet or frame at the bottom, then let two pieces be framed horizontally, one on each side, running from the uprights into the braces at B, then let two maple planks be framed into these, and the bottom at A standing vertically, one on each side of the frame, for the crank and cylinder to run in, of which they should be from six to eight inches wide, and about two inches in thickness, the upper part of the frame may be constructed as represented in figs. 7 and 8, with a pully attached to the upper part of the frame, at I and J, for the rope to run over, of which one end of the rope is made fast to the cylinder at C, and the other end running over the pully at J, and so down to H, where it is made fast to the eye, or hoop of the tongs, H is a piece called a follower, consisting of a wooden block sliding between the uprights, and morticed to receive the iron tongs, which take hold of an eye on the top of a cast iron ram, or weight at E.

The rope is attached to the follower by an iron hoop, of which the form of it is seen in fig. 7, through which the centre pin of the tongs passes near H of the follower; the ram or weight E being held by the tongs, is drawn up by turning the crank at D till the tails of the tongs come to the inclined planes at the upper part of the frame, which opening the lower ends of the tongs, disengages them from the eye of the ram, and it falls upon the head of the pile. G is the plan, or form of the ram, showing the enter grooves, made in the edges by which it is guided as it rises and falls.

The fillets of iron are fixed withinside of the uprights, and they should be from 1 to $1\frac{1}{4}$ inches in thickness, and about 2 in width, and they should be let into the sides of the uprights, one inch and properly fastened.

DESIGNS FOR FRONTISPIECES AND PORTICOES. PLATE 12.

There are two designs for frontispieces and two for porticoes, of a suitable size for modern buildings.

In drawing these examples I have followed as nearly as possible their respective orders, or style; there is a scale of feet and inches applied to each example of which their general proportions may be measured, though it is not expected that the architect will govern his work by these scales, or at least in all parts.

Fig. 1, No. 1, shows the elevation of a frontispiece cased with a Grecian architrave (for further particulars respecting the form of the archatrave, see figs. 8 and 9, pl. 16.) The frame of the door, and side lights is recessed into the hall or house, forming a kind of porch which is quite fashionable at the present day.

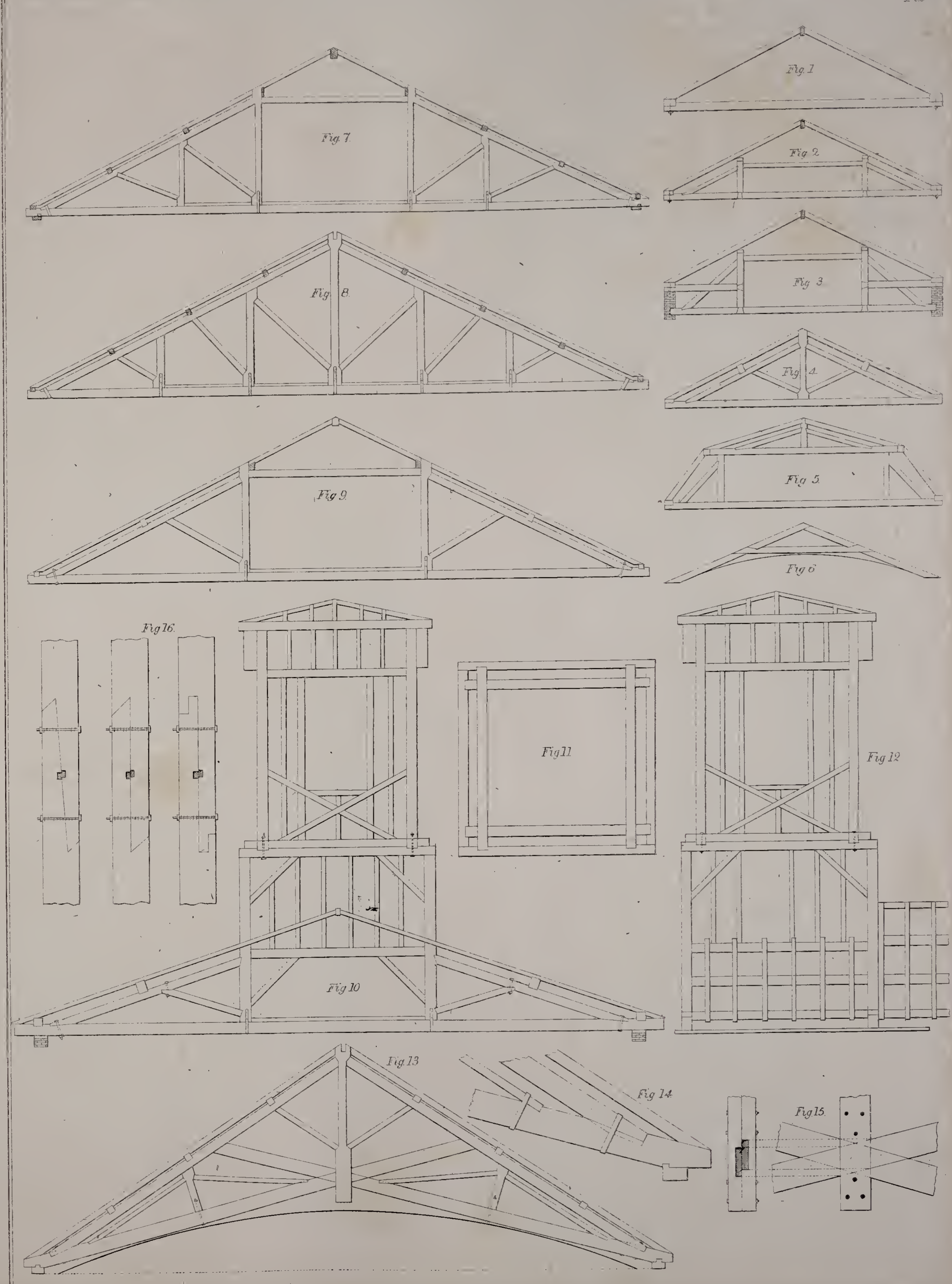
Fig. 1, No. 2, shows a section of the jambs and the manner of casing the same.

Fig. 1, No. 3, the plan. A is the form of the panels at BB.

Figs. 9 and 10, pl. 19, shows a design of a console* drawn to a larger size, they should be neatly wrought, and also the ornaments, if otherwise, being so close to the eye, the work will look better plain, but if the work be neatly done, it will have a pleasing effect.

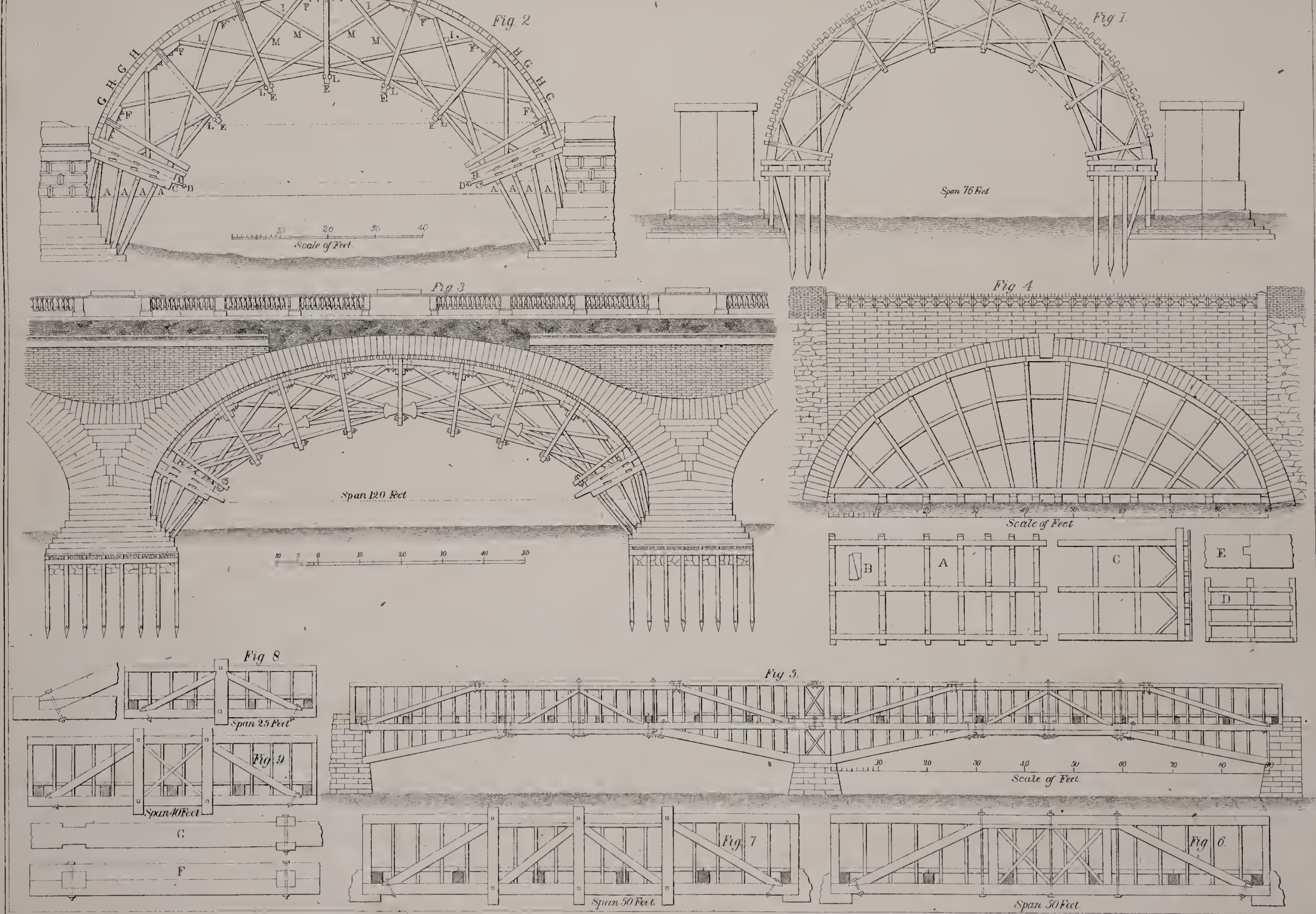
* Consoles are also called according to their form *ancones* or *trusses*, *mutules* and *modillions*.

Pl.9



Lith. of D.W. Kellogg & Co.

Pl. 10.



Lith. of D.W. Kellogg & Co.

(for further particulars respecting the form of this door and ornaments see fig. 1. pl. 18.)

Fig. 2. No. 1. Shows the elevation of another which is more plain the antae and entablature is taken from the order of the little Ionic Temple on the river Illyssus. To proportion this frontespiece let the height of the antae be divided into eight or eight and a half parts and give one of these parts for the diameter of the antae, which make a scale of sixty minutes, and then proportion the mouldings as given in plates 10, and 11, volume 1 of orders.

The frame of the door and sidelights is recessed as in the other example see (fig. 2, No. 2.) the plan; C represents the form of the diamond panels at DD

Fig. 3. The elevation of a portico of the doric order. Its general porpotions is nearly the same as the examples of the order as represented on Plates, 5 and 6, volume 1 of orders.

The columns are six diameters in height, and the entablature two making eight diameters for the whole height. This example is more suitable for a narrow door way than one of wider dimensions for it is not in character to have the columns two much spread in this order, and they cannot he well coupled unless the portico be of larger dimensions, (for a description of the door see fig. 2 pl. 18.)

Fig. 4. An example for a portico, of which its proportions and outlines are in imitation of that order represented on plate 3, volume 1 of orders.

In this example I have also made the columns six diameters in height and given about one diameter and fifty minutes for the height of the entablature. This design is calculated for a house of large dimensions; and the entablature should project four feet and ten inches from the face of the building being equal to one half of the length of the entablature in front. The soffit of the epistylium may be made from fifty four to fifty six minutes in breadth and make the antae equal in breadth to the soffit and give one fourth of the breadth to the antae for the projection; and I should recommend the antae capital, that of the Choragic Monument of Trysallus see Plate 9 of orders instead of the one given on plate 3. This example and also that of fig 3, may have the same proportions.

The plan of these two examples I have not given, but the door frames are calculated to be cased between the jambs or the walls of the building. F shows a seetion of the impost and rosettes (fig. 4.) G the section of the frets at H H which are drawn about one third the full size.

Plate 13 Exhibits two more examples for portieoes of a richer character.

Fig. 1 is an elevation of an Ionic portico; its proportions and outlines are in imitation of that order as represented on plate 14 volume 1. The columns I have made equal to eight and a half diameters in height, including the base and capital, the general rule is nine, the columns being so far spread it is necessary that they should be encased in size. The antae and soffit of the epistylium may be made equal in breadth to fifty four minutes and the soffit may be continued across between the antae the same as in front of the portico between the columns. C is the plan of the portieo and door way; The columns set on buttressess, of which the steps are carried up between them; the door and sidelights are recessed into the house, which make a better finish, and gives a more pleasant appearance; the door is of the Grecian style and is similar to the one of the Ionic portico of the Temple of Minerva as represented on plate 13 volume 1. the door may be decorated with ornaments, or left plain as choice directs, and one half of the number that is given here will be sufficient, unless the door he of a large size, (for further particulars respecting the form of this door and ornaments see fig. 1. plate 18.)

Fig 2. Is an elevation of a Corinthian portico; its proportions and outlines are in imitation of that order on Plate 23, volume 1 of orders as taken from the Choragic Monument of Lysicrates at Athens.

The columns I have made equal to nine and a half diameters in height including base and capital, being I think a suitable porpotion for a portico of these dimensions. (Ten diameters is the general porportion.) B is the plan of the portico and door-way which is similar to the other examples the door and side lights being recessed into the house and finished with an architrave round the side-lights. A shows the side view of the consoles setting under the soffit of the architrave. F shows the section of the impost and ornaments under the consoles in front: C shows a section of the front of the door, D the small ornaments, E the centre piece of the door.

DESIGNS FOR SHOP FRONTS PLATE 14.

This plate exhibits a number of designs for shop fronts and one design for a green house.

Figs. 1, 2, 3, 4 and 5, are designs with columns which are suitable fronts for city or country shops. Fig. 6 is a design for a building which is calculated for two shops or stores, one is represented shut, and the other open; in buildings of this description, the first story is generally built of free stone or granite; granite is the most substantial for the piers, as it is more hard than free stone.

Figs. 7 and 8, shows the side and end elevations of a green house supposed to stand joining some edifice as represented on plate 25. The antae and entablature is from the Choragic Monument of Trysallus as represented on (pl. 9 vol. 1 of orders) asigned to he used here; the windows on the side and ends may be made to admit glass of seventeen by eleven, those on the roof, or the sashes should be made of a different form; the bars of the sashes may be made of wood or cast iron with a suitable rabit for to receive the glass and they should not exceed six inches apart, the glass should be of a thick quality to prevent injury by hail.—Two thirds of the height of the roof will be all that is necessary to be covered with glass, and this may be divided into two parts or sashes, let the lower sash be made stationary to the frame or rafters, and the under side of the upper sash should lie on a plane with the upper side of the lower one so they may be lowered or raised by pulleys and the top rail of the upper sash should run into a grove under the butts of the shingles or whatever the upper part of the roof may be covered with.

DESIGNS FOR SHUTITNG WINDOWS (PLATE 15.)

Fig. 1 Shows a section of the sash-frame, shutters &c.

A, top of a stone sill. B, blinds.—C sill of the sash frame, being of a different section to the uprights.—D A section of the lower sash.—E Shutters showing their form when folded. F Soffit at the head of the window—G Section of the inside bead of the sash frame.—H Parting-bead, and serves to separate the upper from the lower sash, in order that they may work freely and independently of each other. I Pulley stile of the sash-frame. J Inside lining. K Weights to balance the sashes. L Back lining of the sash frame. M Outside lining. N Hanging stile. O Hook and hinge. P Brick work. QQ The rough furrings. R Back lining. S Grounds. T The plastering. U Pilaster or architrave. V shutters folded back into the boxing. W Knob. Fig. 2, Is a vertical seetion through the head of the sash-frame. A Stone cap. B Brick work. C Lintel to support the brick-work. D Rough furring. E Grounds. F Pilaster. G Showing a vertical section of the tablet. H Hanging stile. I Outside lining. J Head of the sash-frame. K The soffit tongued into the head of the sash frame. L Inside bead or stop of the sash frame. M The pulleys. N Parting-bead. O Hook and hinge. Fig. 3 Is a vertical section through the lower part of the window. A Stone sill. B Bottom rail of the blind. C Sill of the sash-frame. D Parting-bead. E Inside bead or stop. F Bead connected with the sill and back. G Shutter butt. H Section of the back under window. I The elbow. Figs. 4 and 5 are two examples showing a section of the stiles the mouldings and part of the pannel to shutters, drawn at full size for practice. Figs. 6, 7, and 8 are designs for mouldings. Fig. 6 is of a suitable size for out-side doors. Figs. 7 and 8, for inside doors, shutters &c.

Plate, 16. Fig. 1 exhibits an interior elevation of a window, with shutters, which fold into the boxing, parallel to the plane of the window or wall, as represented at fig. 5 below. This method of casing windows for shutters is convenent where the wall is not sufficiently thick to admit of boxing-room in the side jambs; or where the room cannot be spared in the interior.

Fig 2 Exhibits an interior elevation of a window with shutters, the jambs beingsplaying as represented on pl. 15; which clearly shows all its details placed in their proper position. Figs. 3 and 4 are two examples showing a section of the meeting rails and part of the vertical hars and stiles. A A shows two methods in which the meeting rails may be connected. B The groove to receive the glass. C and D Sections of the vertieal bars, and top rail. Fig. 5 Seetion of the sash frame and shutters, as represented in fig. 1. A The lower sash. B and C shutters folded into the boxing; the back shutters C is designed for hlind slats in room of being panneled; the slats should he made of hard wood and stand vertically in the frame of the shutter. D Pilaster. E, F and G section of the plinth, in which the pilaster is placed.

This plinth and pilaster may he brought forward so as to have the vertieal edge of the bead of the pilaster stand vertically at I as represented on the elevation in fig. 1. M Shows the plan and section at N in fig. 1 also.

Fig. 6. Shows a section of the pilasters the block against which the pilasters end at the upper angles of the window. A Is a section at B. And C is the section at D which the section of the moulding being transverse to that of A B: either of these examples will make a good finish; the blocks should be mitred together and well glewed and for modern size rooms $6\frac{1}{2}$ inches will be a suitable width for the blocks and $6\frac{1}{8}$ for the pilasters and let the blocks be about $\frac{3}{16}$ of an inch thicker than the pilasters. Fig. 7 exhibits another example for an interior window, which is cased with architraves and a surbace, the shutter is designed to slide into a boxing or between two partitions as represented in Fig. 10. The frame part of the shutter should be put together whole, and let the middle tier of muntins be double the width of the others and the stiles, and run a dado through the centre to receive tbe bead. Figs. 8 and 9 exhibits two examples for architraves, with their respective ornaments. These ornaments and all others which are liable to close inspection, should be well expressed, and neatly finished. Architraves should be proportioned to the size of the room and the apartments for which they are designed. Rooms which are from sixteen to eighteen feet square and ten feet between joints; seven to seven and a half inches will be a suitable proportion for these two examples.—For frontspieces and apartments which are of larger dimensions, they may be from nine to twelves inches in width, or wider according to the size and altitude. Fig. 10. Shows a section of a sash frame and shutter which is designed for a framed house. A Sill of the sash-frame. B Sill of the interior part of the surbase. C Shutter as standing in its vertical position which slides into a boxing, back to H between the two partitions. D is a rod of iron on which it runs, this rod may be from $\frac{1}{4}$ to $\frac{5}{16}$ in diameter, and one half of the thickness of it let into the sill or cap and properly fastened, and then let two window pulleys be let into the under side of the bottom rail of the shutter within about two inches of the stiles which will run much easier than on common sash rollers. E, E E, The rough furrings. F, F F. The vertical studs of the frame. G Outside covering.

DESIGNS FOR DOORS AND THEIR DECORATIONS. PLATE 17.

Figs. 1, 2, 3 and 4—Exhibits four designs for sliding doors.—Two of these examples are cased with architraves and pilasters, and Two with antae and entablatures. The doors to all of these examples are drawn of nearly the same width and heighth, it is necessary that the room should be higher between joints when finished with antae, instead of architraves or pilasters, it should not he less than twelve feet in height. The proportions of these two orders may be taken from figs. 1 and 2 pl. 19, which are drawn to a larger scale and expressed in minutes and parts; and the doors may be drawn from the scale which is given in feet and inches near Fig. 4. The other two examples which are finished with architraves and pilasters may be drawn from the scale near Fig 3. Fig. 5 shows the plan and manner of casing them. A and B shows two designs of iron rods for the doors to run on. A consists of a round rod of iron or brass about $\frac{3}{8}$ of an inch in diameter, one half of its size should be let into the floor and properly fastened with screws. This method is much cheaper than the other,

and will answer all purposes for light doors. Figs. 6, 7, 8 and 9 exhibits four designs suitably constructed for the same apartment, for which the sliding doors were intended, those should be arranged and proportioned according to the size and altitude of the room in which they are to be placed. Where a room is from 15 to 18 feet square, and from 9 to 10 feet in height, the common doors are generally 7 feet or 7 feet 2 inches in height, and 3 feet in breadth, and the folding or sliding doors which connect the two parlors, may be 8 feet in height and 3 feet 3 inches in breadth. If a room be of a larger size and from 10 to 12 feet between joints they may be from 7 feet 3 inches to 7 feet 6 in height, and from 3 feet to 3 feet 3 inches in breadth; and those that slide may be from 8 feet 3 inches to 8 feet 6 in height, and from 3 feet 3 to 3 feet 6 in breadth. The stiles top and frieze rails and muntins, may be from $4\frac{3}{4}$ to 5 inches in width; the bottom rails 9 inches, and the middle rails in Figs. 7 and 8 may be from 7 to $7\frac{1}{2}$ inches, and the stiles and rails &c. of the sliding doors should be made about $\frac{1}{2}$ inch wider than the others.

Plate 18—Exhibits a number of designs of doors, architraves, pilasters, base and surbase mouldings, impost mouldings, frets, &c.—Figs. 1 and 2 shows the manner in which the doors may be put together, of three of those designs which are given on pls. 12 and 13.—Fig. 1 is calculated and drawn for a door of 3 feet 4 inches in width, being nearly one half the full size. There is a scale twenty inches for the width of one half the door given, by which the parts may be measured. A, A, the turned ornaments, which should be for a door of this size, one placed at each angle on the inner frame; then let them be placed on the vertical part of the door so as the divisions will be equal to the breadth AA, which will be one half of the number that is described on the doors represented in Fig. 2 pl. 12, and Fig. 1 pl. 13. BB shows two designs for ornaments, which are drawn to a full size for practice. Fig. 2 shows a part of the frame of the door that is described in Fig. 3 on pl. 12, which is drawn one half the full size.

Figs. 3, 4, 5 and 6, shows three methods of casing doors with architraves and pilasters. Fig. 3 is cased with architraves standing on plinths at the bottom, which should be equal in height to the plinth of the base. A the partition plank. BB grounds. CC lath and plastering. DD Mopboards. E and G architraves. H rabbit casing. I furring. J part of the door. Fig. 4 is cased with pilasters which are worked whole or in one piece, and rabbited at AA and a bead turned on the edges. B is the butts which should be set out far enough to have the door swing clear of the casings, as represented by the dotted line. Fig. 5 and 6 shows the plan and vertical section of casing a door with pilasters. A in Fig. 5 is the plinth. B pilaster which stands on the cap of the plinth. C the base that finishes against the pilaster. DD plinth of base which is dadoed at E. F and GG the grounds and furrings. H lath and plastering. Fig 6 AA grounds. B and C facings which the pilasters set upon. Figs. 7, 8, 9, and 10. are designs for pilasters, architraves, and surbase mouldings for the finishing of windows. All these mouldings in Figs. 3, 4, 5, 6, 7, 8, 9 and 10, are drawn one third of the full size for practice for common size rooms which are from 9 to 10 feet in heighth.

A B C and D in Fig. 11 are given four designs for impost mouldings, which are drawn about three fourths of the full size for practice for common size doors.

Fig. 12 exhibits five designs for base mouldings, that are drawn one half the full size for practice for common size rooms.

On the lower part of this plate are designs given for frets.

Plate 19—Figs. 1 and 2 exhibits two designs of antae and entablatures which are more suitably constructed for frontispieces and interior finishing, than those of the orders.

Fig. 1 is imitated from the Choragic Monument of Trysallus, that stands at the foot of Acropolis or citadel of Athens.

Fig 2 is chiefly of my own design. The capital and mouldings are of the Grecian style. The heighths and projection of the mouldings and parts are expressed in minutes and eighths.

The remaining part of this plate is taken up with Grecian ornaments, and ornamented mouldings &c. which seems to require no particular description.

Fig. 3 represents the enriched moulding of the antae capital of the Temple of *Minerva Polias* at Athens as represented on (pl. 13, vol. 1.)

Fig. 4 *Egg and Tongue* with beads below, belonging to the same cap.

Fig. 5 represents the ornament between the bead and small projecting band of the same.

Fig. 6 is the ornament on the Cymatium of the door of the same temple, stretched out on a flat surface.

Figs. 9 and 10 represent a front and a side view of a console of the same door, most of the remaining part of these figures were taken from Grecian structures, which on account of their foilage are more graceful, than those of the Romans.

OF STAIRS AND STAIR-CASEING.

DEFINITIONS OF THE PARTS OF STAIRS.

A flight of steps so called, means an assemblage of steps, so formed and united, that by walking on them, we ascend or descend from one height to another.

The surfaces on which we set our feet are called *treads*; and these for their convenience of walking, are set at equal distances and parallel to each other.

In order to give a solid appearance to the whole, every adjacent pair of treads are connected by a third and vertical piece, called a *riser*. Each riser and tread when fixed together, is called a step. The wall which supports the ends of the steps is called the stair-case.

When the ends of the steps terminate upon a vertical prism or pillar, the prism or pillar is called a *newel*.

If the ends of the steps be cut through in the surface of the newel and the pillar or prism be supposed to be removed, the space left open by the removal of the solid is called the *well-hole*.

Stairs that have a well-hole, or hollow in the centre, are called *Geometrical stairs*.

The meeting of the sides which form the external angles of the steps is called the line of nosing: but sometimes the line of nosing is covered with a moulding and then this moulding is called the *nosing*.

When the steps are of equal breadth; that is when the distance from the line of nosing to the riser is every where equal, the steps are denominated *flyers*.

When the treads of the steps diminish in breadth toward the well-hole, the steps are called winders.

As the ends of the steps generally terminate upon a surface which is perpendicular both to the risers, and treads, the surface on which they thus terminate is generally that of a cylinder.

A number of contiguous flyers are called a *flight*.

When the tread of the step is so broad as to be equal to two or more of the other steps, and situated between floors, it is called a *resting place*.

If the tread of the resting-place form a right angle, that is if the two rises be perpendicular to each other, the resting-place is called a quarter space, or quarter-pace.

When the breadth of the tread of a step is contained between the same vertical plane, or makes two right angles round the axis of the well-hole, the tread is called a half space or half pace.

Half-space, and quarter-spaces are generally made on floors; and in this case are called *landing places*.

The carriage of a stair consists of several peices joined, or framed together; when they are so constructed is called the carriage of the stairs.

A flight of steps is genearally supported by two pieces of timber, placed under the steps and parrallel to the wall, being fastened at one or both ends, to pieces perpendicular thereto.

The pieces of timber or planks which are thus placed under the steps are called *rough strings*.

Dog-legged Stairs are those which have no well-hole, and consists of two flights without winders. The hand-rail, on both sides, is framed into vertical posts, in the same vertical plane, as well as a board which supports the ends of the steps. The boards are called *string-boards*, and the posts are called *newels*. The newels not only connect the strings, but they afford the principal support to the rail; and thus it may be affirmed that the newel posts and hand-rail, are all in one plane.

Open neweled Stairs are those which have a rectangular well-hole and are divided into two or three flights.

Bracketed Stairs are those where the string-board is notched so as to permit risers and treads to lie upon the notches, and pass over beyond the thickness of the string boards; the ends of the steps are concealed by means of ornamental peices called brackets.

Geometrical stairs, are generally bracketed; but the dog-legged and open-newelled stairs, only those of the best kind are bracketed.

A *Pitching Piece* is a piece of timber wedged into the wall, in a direction perpendicular to the surface of that wall, for supporting the rough strings at the top of the lower flight where there is no trimmer, or where the trimmer is too distant to be used for the support of the rough-strings.

Bearers are pieces of timber or planks fixed into and perpendicular to, the surface of the wall, for supporting the winders where they are introduced; the other end of the bearers is fastened to the string-board.

A *Notch Board* is a board into which the ends of the steps are let: it is fastened to the wall, or one of the walls of the staircase.

Curtail Step is the lowermost step of the stairs, and has one of its ends next to the well-hole formed into an ornament representing a spiral line.

These are the principal parts which belong to a stair or stairs; other parts connected with it belong to the hand-rail.

PROPORTIONS OF STAIRS &c.

The breadth of steps to common stairs is from nine to twelve inches. The breadth in elegant houses and public edifices, ought never to be less than ten, nor more than fifteen inches.

A step of greater breadth requires less height than that of a less breadth.

The general rule may therefore be as follows;—

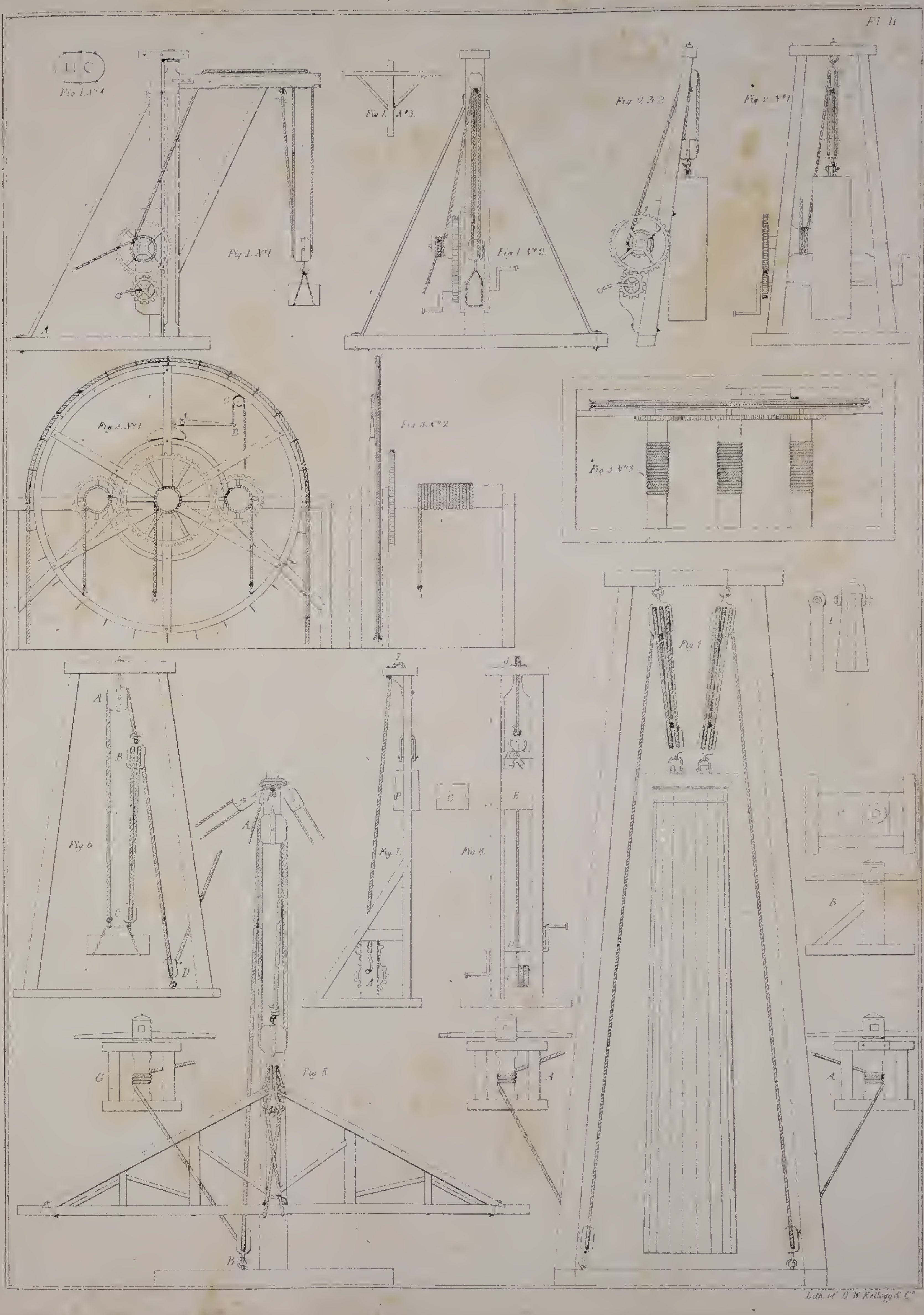
Multiply the breadth and height of a given step together and divide the product by the breadth of the required step, and the quotient will be the answer; or by reciprocal proportion, as the given breadth belonging to the height required, is to the breadth of the given step, so is the height of the given step to the height of the required step.

For example, taking as a standard a step of 11 inches in breadth, and 6 inches in height, we may easily find the height of another of a given breadth, which we shall suppose to be 9 inches. The operation is thus;

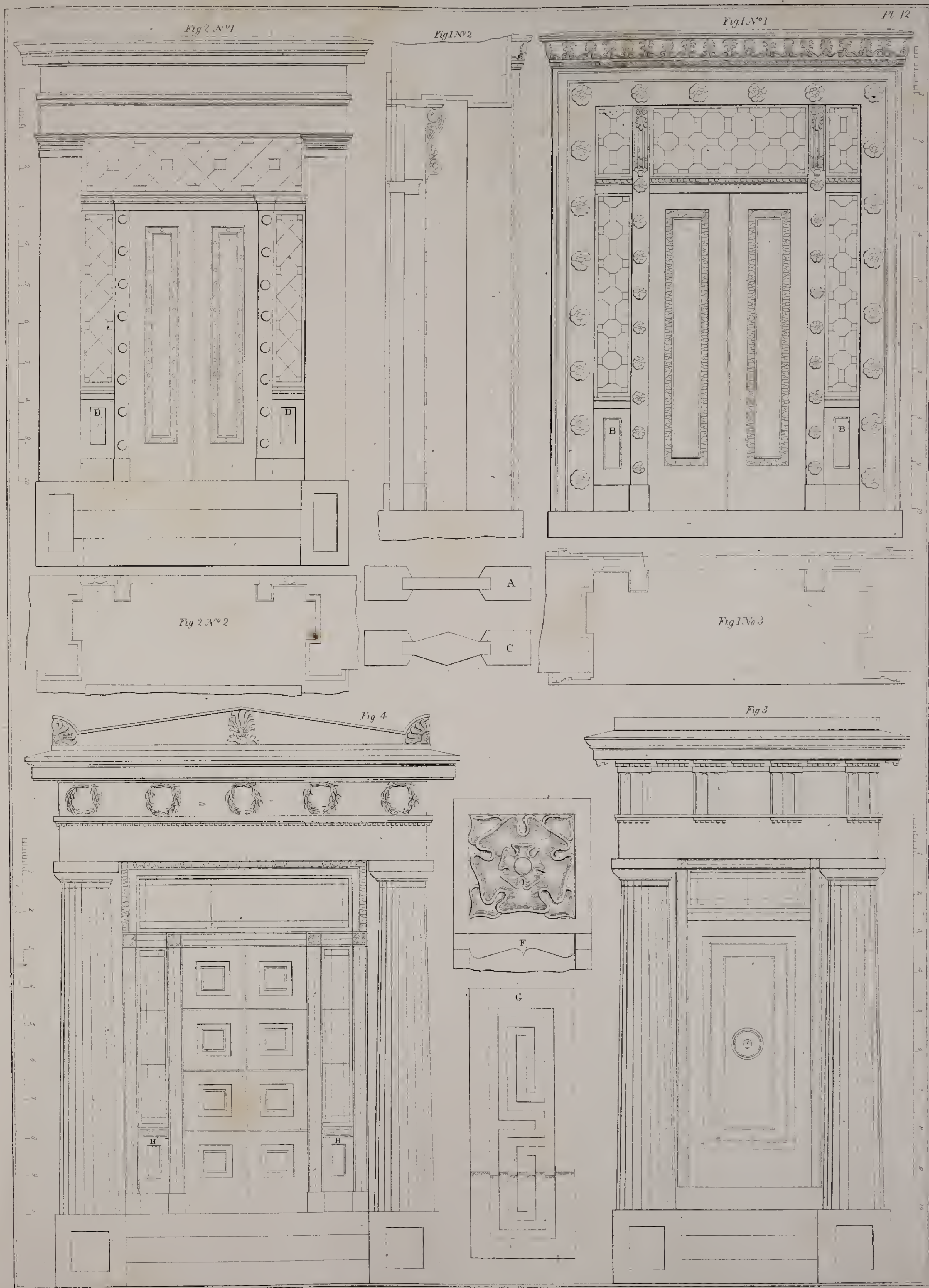
$$
\begin{array}{r}
9 : 6 :: 11 : \\
6 \\
\hline
9)66(7.33 \\
63 \\
\hline
30 \\
27 \\
\hline
30 \\
27
\end{array}
$$

We find it to be $7\frac{1}{3}$ inches for the height, which agrees with what would be allowed in common practice.

Before we lay out the stairs in a building, we must consider the height of the story and determine upon the height of the steps; which being done, we must bring the height of the story into inches and divide the number of inches in the height of the story, by the height of the step. Thus for example, suppose the height of the story to be ten feet three inches, and the height of the step to be seven inches, how many steps will be required in order to ascend to the given height. The operation is



Lith. of D. W. Kellogg & Cº



Lith. of Dr. Kellogg

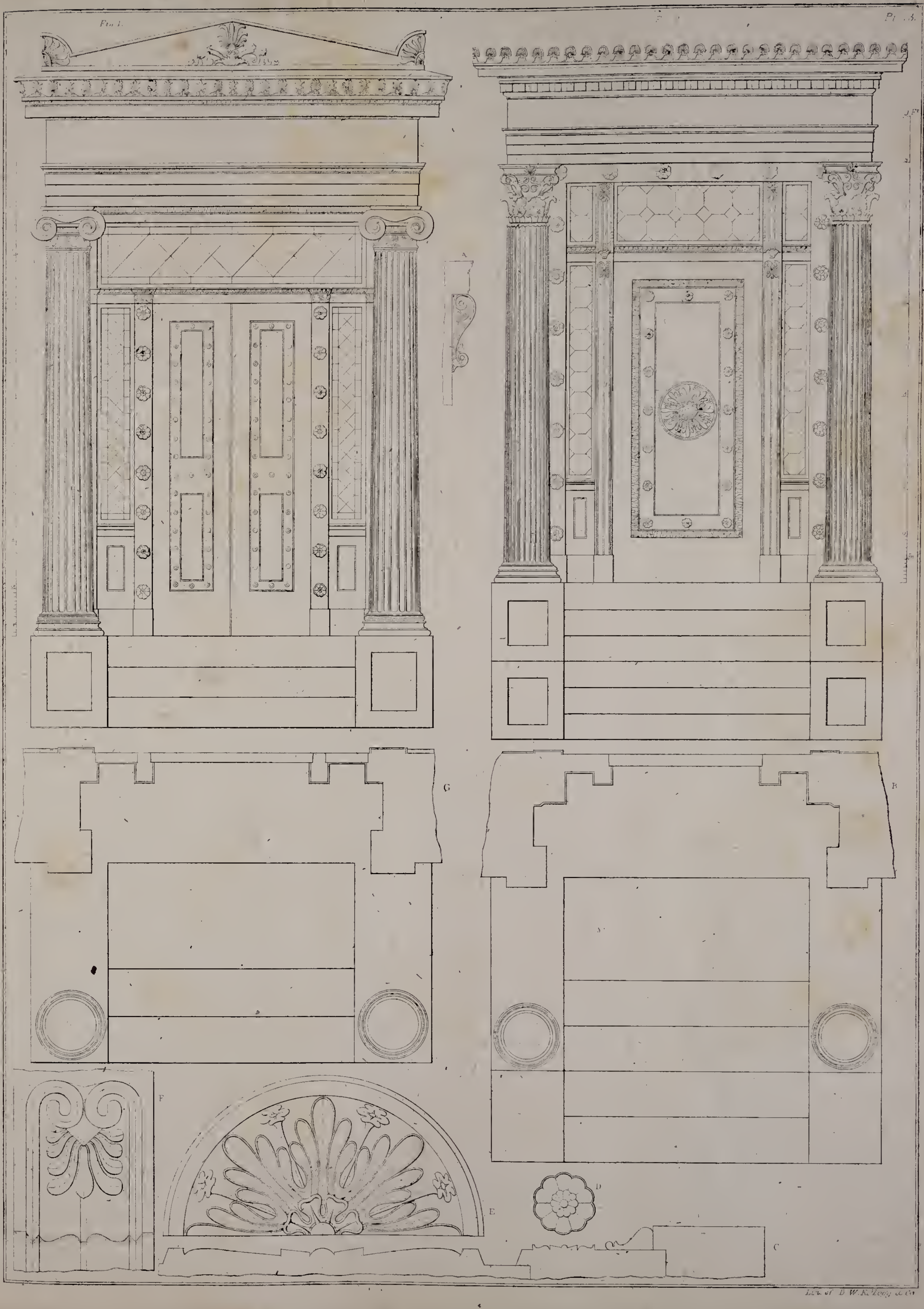


Fig. 1.
Pl. 3.
A
G
B
F
E
D
C

Pl. 14.

Fig. 1.

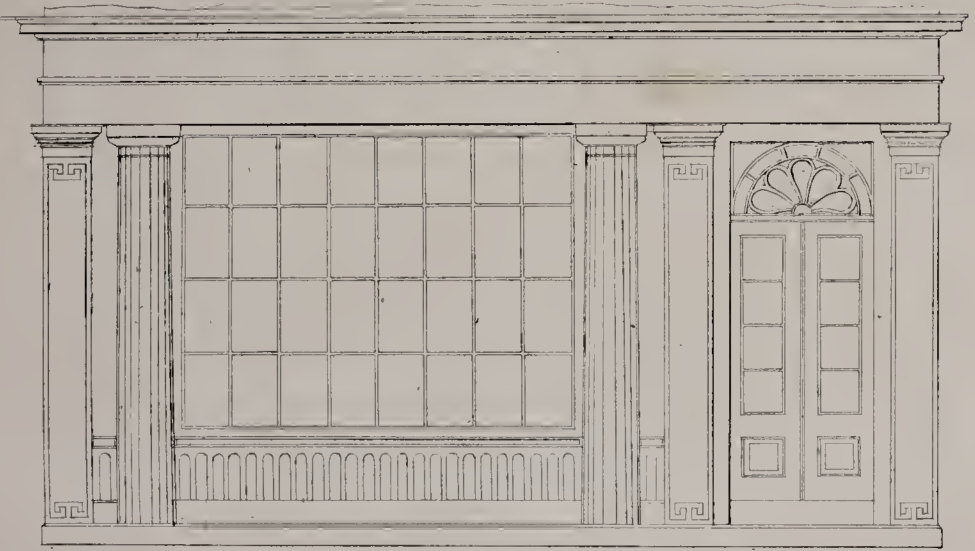


Fig. 2

Fig. 3

Fig. 4.

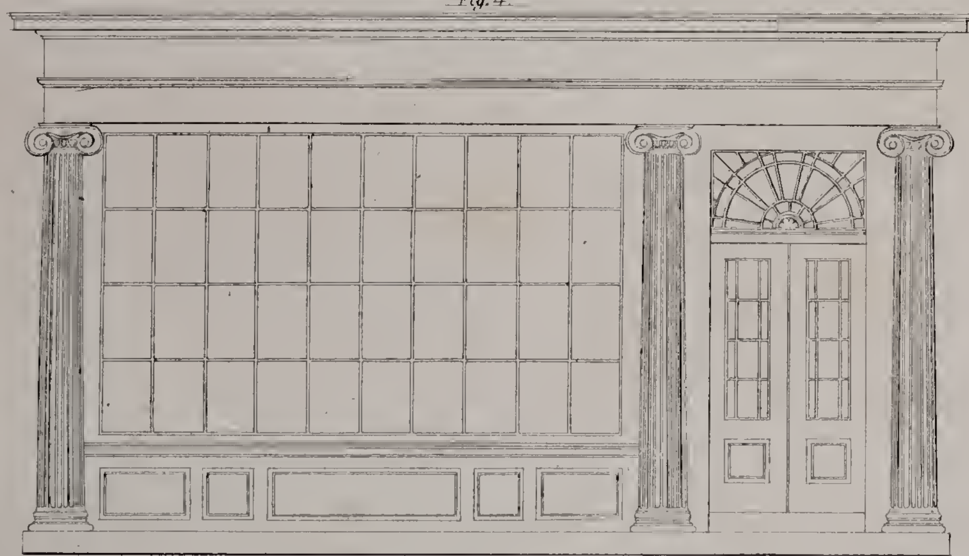


Fig. 5.

Fig. 6.

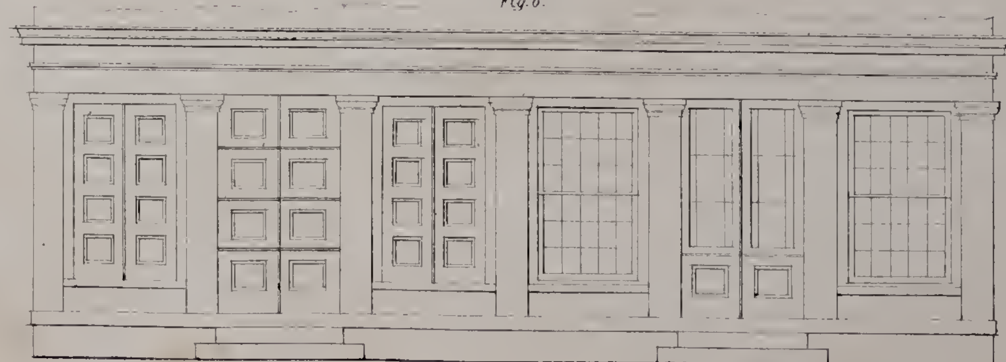


Fig. 7.

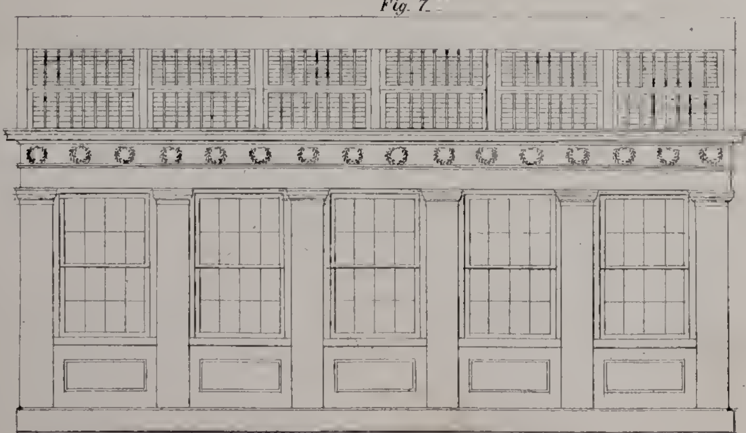
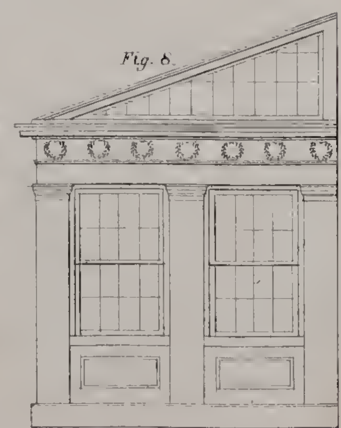


Fig. 8.



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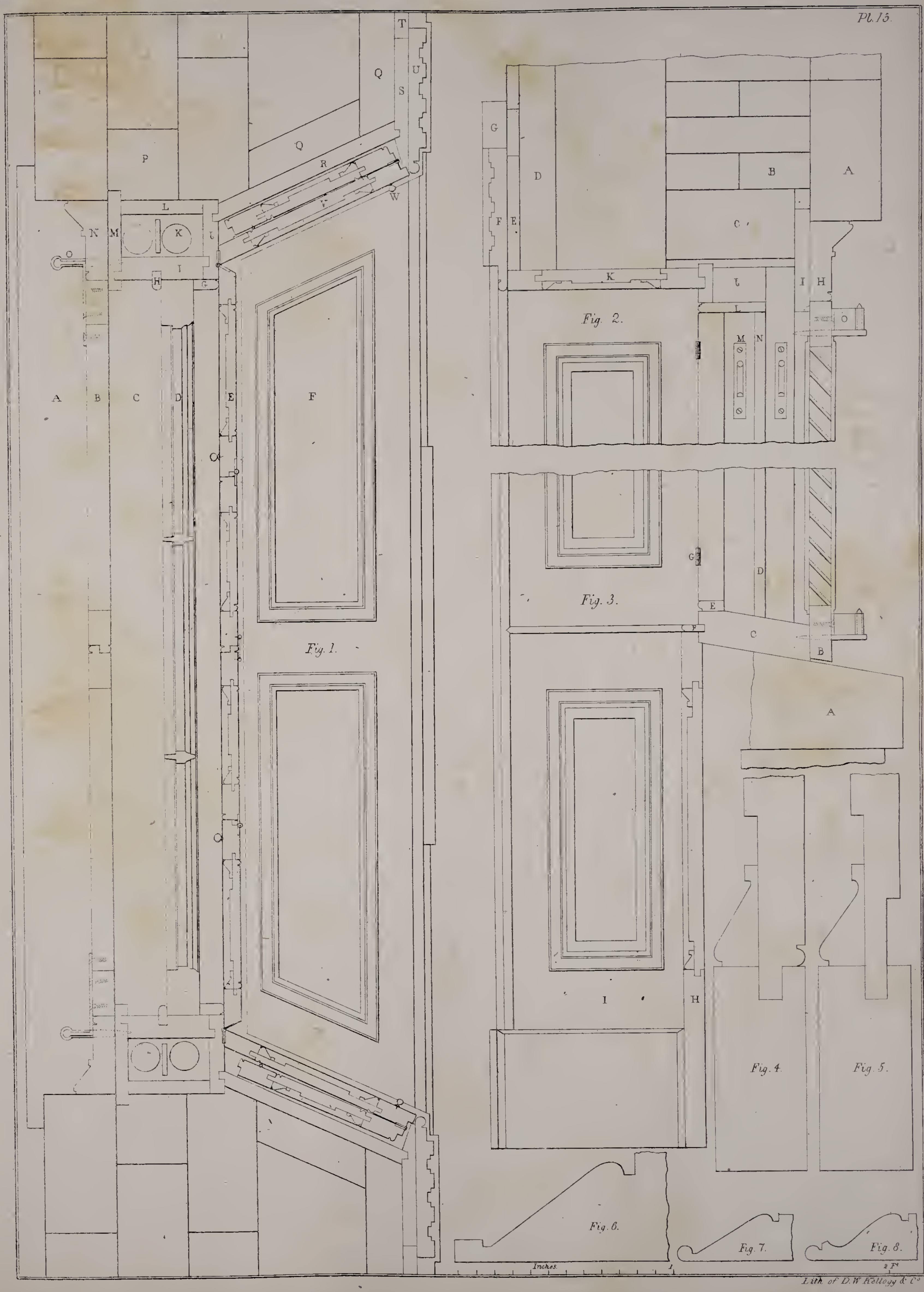

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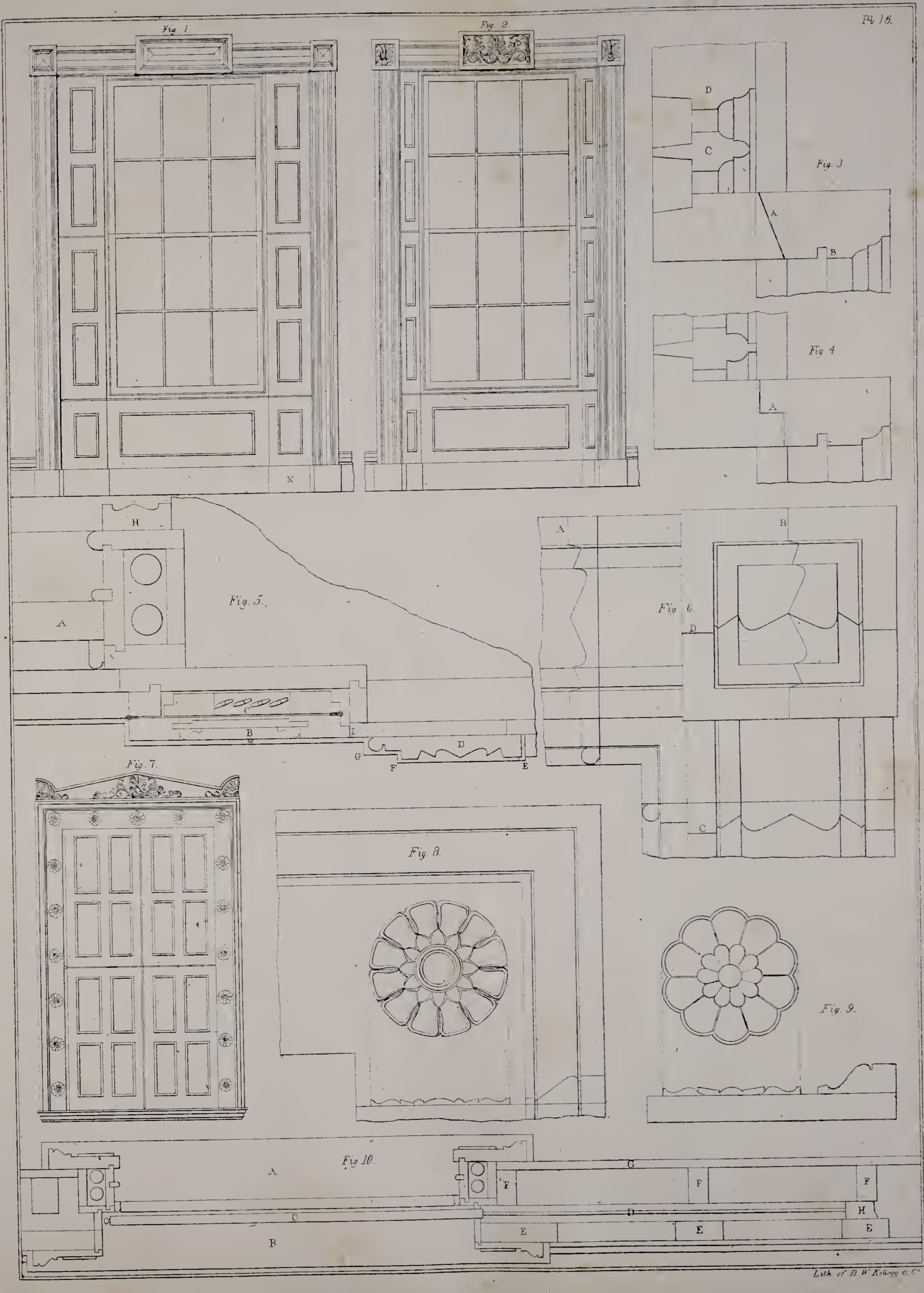

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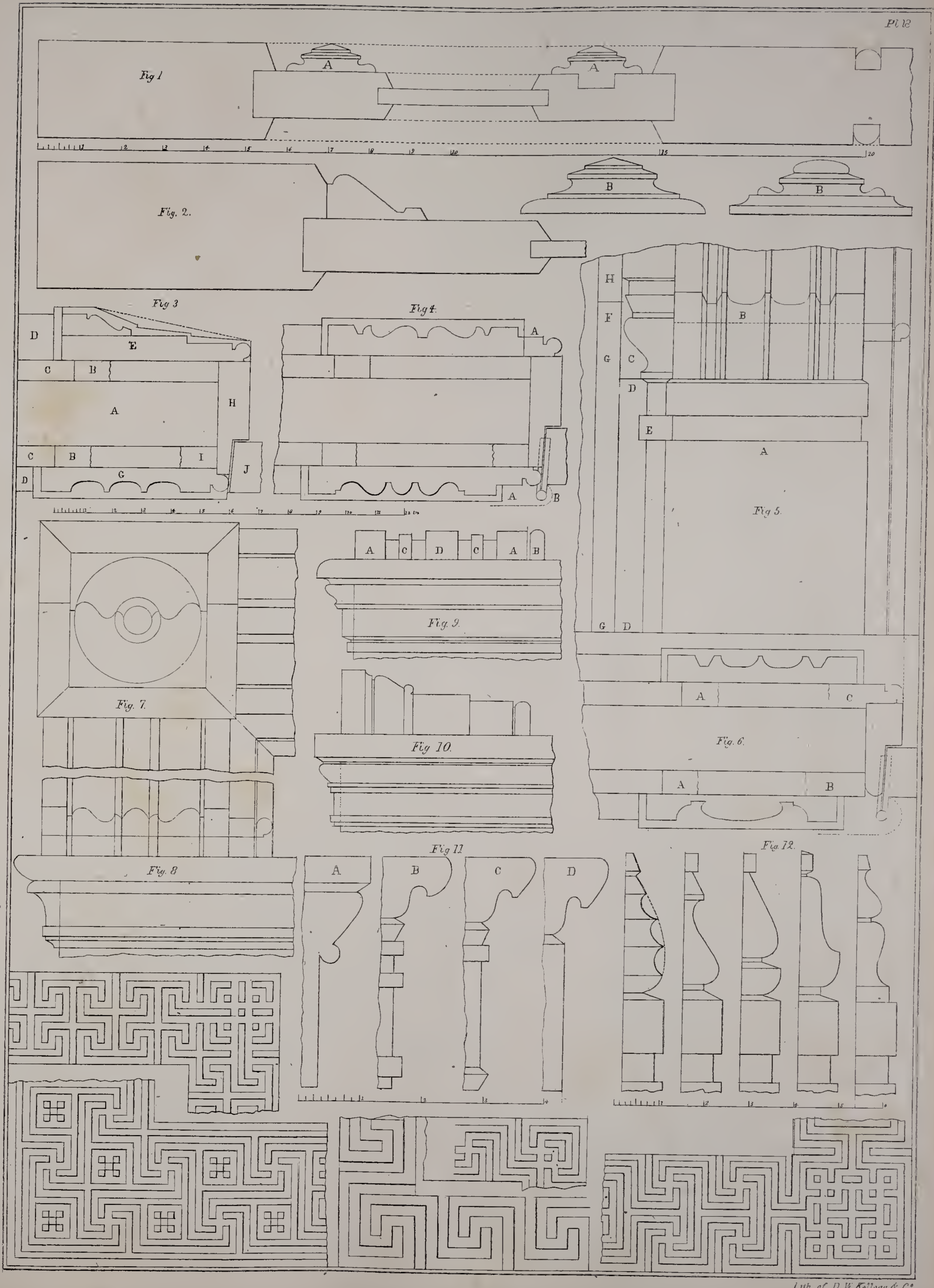

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Pl. 19
Fig. 1.
60 m
Fig. 2.
60 m
Fig. 3
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7
Fig. 8.
Fig. 9
Fig. 10
Fig. 11.
Fig. 12.
Fig. 13.

12	7)123(17
10-3	7
120	53
3	49
123 inches	4

Thus we find the number of steps to be seventeen and a trifle over, but as we must not have a fractional part, therefore the step must be over seven inches in height; we will proeeed thus;

17)123(7.23
119
40
34
60
51

We find the height of the step to be $7\frac{23}{100}$ inches; being a trifle less than $7\frac{1}{4}$. This will answer very well; but, if we are still confined for room on the plan, we may throw the semi-circumference round the newel into winders.

The hreadth of stair-cases may he from four to fifteen feet or more according to the destination of the building; but if the steps be less than two feet six inches in length, they become inconvenient for the passing of furniture, as is generally the case in small houses.

When the height of the story is very high, resting-places become necessary. In high stories, that admit of sufficient head-room and where the plan or area of the stairs is confined, the stairs may make two revolutions in the height of the story; that is the ascendant or descendant may go twice round the newel or well-hole and this becomes necessary; otherwise the steps would he enormously high, or extravagant floor-room must be allowed for the stairs.

In laying out the rise and run for a stair case, there will be one more riser in number than treads, that is by not conuting the upper one or landing which forms that part of the floor; It may not be amiss to give an example here for a principal building, in order to show the numher of steps and risers both in the grand and common stair-case.

For this purpose, suppose the story of a house to he twelve feet one inch high from floor to floor, and we may have sixteen feet ten inches for the run of the grand stair-case, the number of steps of the grand stair case to be twenty in number, that is there will be twenty risers and nineteen treads, what will be the breadth of the treads and risers.

Now the height of the story 12 feet 1 inch being reduced to inches is 145, and the run, which is 16 feet 10 inches being reduced to inches gives 202, and first dividing by 19.—for the run

19)202(10.63	divide by 20)145(7.25
19	140
120	50
114	40
60	100
57	100
3	

We find the breadth of the tread to be $10\frac{63}{100}$ inches and for the riser $7\frac{25}{100}$ inches.

Now for the stair-case of that for the servants, we cannot have for instance but twelve feet nine inches for the run, and of course the height will he the same as in the former; therefore we must have a less number of steps to bring it into proportion.

For example let there be eighteen risers in number and seventeen treads.

The run which is 12 feet 9 inches being reduced to inches is 153 and thus

17)153(9	18)145(8.05
153	144
	100
	90
	10

Which gives 9 inches for the breadth of the tread; and $8\frac{5}{10}$ for the riser being large 8 inches. This will make a good proportion.

In laying out the rise of a stair-case, we may divide the whole height from floor to floor into equal parts on the string boards, then when the strings are set up level at their proper places, scribe off the thickness of one tread on the lower step or curtail, and this will drop it down the thicknes of one at the upper part of the landing, so when the steps are put together they will be all equal in height A farther description of laying out and putting up stair-cases will be described hereafter.

HAND-RAILING.

"*The art of forming Hand-rails* round circular and elliptic well-holes, without the use of the cylinder, is entirely new. Mr Price, the author of '*The British Carpenter*,' is the first who appears to have had any idea of forming a wreath-rail. Subsequent writers have contributed little or nothing towards the advancement of this most useful branch of the Joiner's profession and have contented themselves with the methods laid down by Price, which were uncertain in their application; and, consequently, led to erroneous results in practice.

The first successful method of squaring the wreath, upon Geometrical principles, was invented and published by Peter Nicholson, in 1792 in a work called '*The Carpenter's New Guide*,' a book well known to architects and workmen. No previous author seems to have had any idea of describing the section of a cylinder through any three points in space making a mould to the form of the section, and applying it to both sides of the plank, hy the principles of solid angles so that by cutting away the superfluous wood, the piece thus formed might have been made to range over its plan.

Since the first invention of the method, the author's experience and researches have produced many essential requisites, which were not thought of at first; so that this branch, as here presented, is now much improved.

The principle of projecting the rail furnishes the workman with a method by which he can ascertain, with great precision, the thickness of the plank out of which the rail must be cut. To do this in the most convenient way, the diagram must be made to some aliquot portion of the full size which will supercede the necessity of laying it down on a floor.

It must however, be observed that the thickness of stuff found by this method is what will completely square the wreath or piece. But as the rail is reduced from the square to an oval or elliptic section, much thinner stuff may be made to answer the purpose; so that, generally for rails of common size of 2 or $2\frac{1}{4}$ inches thick, instead of requiring a four inch, when one of two and a half may be made to answer the purpose."

Note.—Since I commenced compiling this work, a number of young builders have requested me to make some of my drawings full size, I have therefore added two double plates, on which a part of the examples are described sufficiently large for practice.

TO DRAW THE SCROLL FOR A HAND-RAIL, AND THE MOULDS FOR EXECUTING THE SAME, AND ALSO THE CURTAIL-STEP (PLATE 20.)

Fig. 1, Shows a method for drawing the scroll to a hand rail, and curtail step, to a full size for practice. The full lines that are drawn represents that part of the scroll, and the dotted lines the curtail step.

To draw the scroll,—Divide the line A B the width of the scroll, into nine equal parts. Draw A D and BC at rightangles with A B, and make each equal in length to one of the nine parts; join C D, and also the diagonal line B D.—Draw E F perpendicular to A B, and at a distance from A of four of the nine parts into which A B is divided. Draw the semi-circle E F O. Thence draw the diagonals E O G and F O H cutting the diagonal B D in the point O. Join G H, H I, I J and J K; Then will E F G H I J K be the centres for describing the scroll and curtail step.

Then from the centre E, with the distance E B, in your compasses, describe the quadrent B L. Then from the centre F, with the radius F L describe the quadrent L D; From the centre G with the distance G D. describe the quadrant D M; From the centre H with the distance H M describe the quadrant M N. From the centre I with the distance I N describe the quadrents N P. From the centre J with the distance J P describe the quadrant P Q. Then from the centre K with the distance K Q describe the arc at R; which finishes the outer spiral of the scroll.

Thence set off the bigness of the rail, from B to S on the line A B.

Again, from centre E with the distance E S in your compasses describe the quadrent S T. From the center F with the radius F T descride the quadrant T U. Then from the center G with the distance G U describe the arc at R, which will complete the inner spiral and terminate in the point R as required.

The curtail step is described in the same manner, and from the same centres, as that of the scroll, which the lines on the face of the curtail terminates at W W, where there is two grooves cut out by running a saw through to let the ends of the veneers which bend around on the face or surface of the riser. as represented by the two dotted lines, in the diagram of the figure.—XXXX Shows the bigness of the newel post: and YYYY the bigness of the mortise that is made through the step, which should also run through the floor, and a mortise made through the tenon of the newel on the underside of the floor and keyed, to draw the step down close to the floor. The circles that are described in the diagram of the figure represent the balusters; The different sections of the remaining part of the curtail step is plainly drawn showing the manner in which it is put together; it will not require any further explanation: therefore we will proceed to describe the face mould for the scroll of the hand-rail.

To Draw the Face Mould, for Squareing the Twisted part of the Scroll. Let A A A in (fig. 2) be the pitch board; and A the angular point as standing over the front edge of the riser of the curtail-step, at the bottom. Let the line ABA be continued to 14 untill it meets the line L 14 in (fig. 1.) Then in (fig. 1) let the line B C be produced until it meets the line L 14 also; and divide this line into any like number of parts: commenceing at B then 1, 2, 3. &c. to 14 as represented in the diagram of the figure. Then draw lines through these several divisions parallel to A B, untill they meet the diagonal line A B A in (fig. 2).—Thence on the diagonal line AB in (fig. 2) draw lines B S, 111, 222, 333, &c at right-angles with A B.

Then take B S the bigness of the rail from (fig. 1) and set it off from B to S in (fig. 2.) take also the distances 111, 222, 333, 444, &c from (fig. 1.) and set them off from 111, 222, 333, 444, &c. in (fig. 2.) When you have got all these distances or ordinates transfered from one to the other; then with a pencil trace out the curve through the several points in (fig. 2) which will give the form of the face mould as required.

To draw the falling mould of the scroll as in (fig. 3.) This figure or mould is drawn on a smaller scale, being only one third of the size of those of (figures 1 and 2.) To draw this mould let B V in (fig. 3) be made equal in length to the stretch out, from B around to V in (fig. 1) the scroll. Then let B V in (fig. 3) be divided into three equal parts; and from the first division at I, lay on the pitch board as represented at AAA. From B raise a perpendicular to C. Thence divide I C and I V each into a like number of equal parts, and form the curve by the intersection of lines. Then set down the thickness of the rail from V to D and from A to E, and join E D. This mould should he made of thick paper, and bent around the scroll to form the curve of the upper edge of the rail. This being done and the upper edge of the rail squared, the curve of the lower edge may be obtained by gauging.

Fig. 4 shows a method of getting a scroll out of a solid piece of wood, having the grain of the wood run in the same direction with the rail; which is far preferable to any of the other methods.

To square out this scroll. Let AAAA be the block of wood of a suitable breadth and thickness, and let the underside of it be faced over and one edge be made square. Then let the block of wood be set up on the bench in a raking position, and place your pitch board under it as represented at EEE, DD represents the line of the bench on which it is placed.

Then with a pair of compasses scribe through from F to G, and work off the corner or the angle AFG on the lower part. This being done and the underside of the scroll FG being faced over true and square, place the pattern of the scroll as in (fig. 1) on the under side and mark all around it, and thence square up from the under side, instead of drawing a face mould, and working by it from the upper side. This method is much quicker and as accurate. When you have got the scroll squared up around perpendicular to the plan. Then bend around your falling mould, so as to have BC in (fig. 3,) stand perpendicular to BC in (fig. 4.) or stand perpendicular to B on the plan in (fig. 1.) If the scroll is required to be worked out of thinner stuff. Let the joint be made and glued together at *aa* in (fig. 1.) And in this case it is not necessary to draw the face mould farther than at, *aa* in (fig. 2,) scrolls are frequently made in this way.

Fig. 5 shows an elevation of a scroll, curtail step, balusters, &c. and two designs for brackets. The rail of stairs should be squared out, and the joints all made and cut to a length, before it is rounded or moulded. This being done and spaced off for the balusters or plummed up, square across upon the under side of the rail, and then lay your pitchboard on as represented at A, and with a pencil mark down upon the side of the rail, which will be a guide to you in boreing, or to morfice by. B shows the manner in which the base makes a finish under the string board of the stairs.

Fig. 6 shows the form of a rail screw, which secures the joints and screws them tight together. The screw is let into the centre of the rail, and the nuts are let in on the under side within about $1\frac{1}{2}$ inches of the joint or shoulders of the rail. The nut at A is made square, and should be let into the mortice without much play. so as not to have it turn around within the rail. B is made round with creases cut into it, for the purpose of turning the nut on or off within the rail. C shows the plan of the same nut.

TO FIND THE MOULDS FOR EXECUTING A HAND RAIL ON A SEMI-CIRCULAR PLAN WITH EIGHT WINDERS; AND ALSO THE CONSTRUCTION OF THE CARRIAGE OF THE STAIR. PLATE 21.

Let ABC (fig. 1) be the plan of the eight winders, as described upon the floor over which the stair is to be built. The full line that is drawn ABC represents the concave side of the string-board. And the dotted line at D E represents the concave side of the hand rail, which projects over the vertical face of the string-board. FF the treads. GG risers. HH the falls or back risers, in which the bearers of the winders are framed. II bearers of the winders. J the middle bearer of a step.

Fig. 2 clearly shows the section of the stretch out of the eight winders as they would naturally appear when erected; and also one of the treads of the flyers both above and below the winders.

But before we proceed to lay out this stair-case and to draw the moulds for executing the hand rail &c; let us suppose it to be made to some aliquot portion of the full size. For instance let the treads of the flyers be equal to $10\frac{1}{2}$ inches in breadth and the rise or risers equal to $7\frac{3}{4}$ inches in height; and the breadth of the treads of those of the winders should be made equal to one half of the breadth of those of the flyers: that is, on the concave part of the well hole; and thus by placing two balusters into a tread of the flyers, and one in the winders of each tread. Then the balusters in the winders and flyers will be at equal distances from each other, which will give a much better appearance. Therefore we will suppose the breadth of the winders to be $5\frac{1}{4}$ inches, on the concave part of the well hole; and the height of the risers of course will be the same as those of the flyers. Now it becomes necessary to know how large the well hole must be in diameter to receive the eight winders; which may be found by the following operation. Thus $5\frac{1}{4}\times8=42$ inches, the stretch out of the well hole, or the semi-circumference.

Then the circumference of the whole circle will be equal to 84 inches, and thus by (Art 13. section 1 of mensuration)

as 22 : 7 : : 84 :
7
———
22)588(26·72
44
———
148
132
———
160
154
———
60
44
———
16

Which we find the diameter to be $26\frac{72}{100}$ inches, being a trifle less than $26\frac{3}{4}$. Now the hand rail generally, projects over the string board from $\frac{1}{2}$ to $\frac{3}{4}$ of an inch; which renders it necessary for us to know what the diameter is, and also the circumference of the concave part of the rail, or the convex circumference of the semi-cylinder, which is called the working cylinder, as represented in (fig. 10.)

Let us suppose the working cylinder of the rail to be $25\frac{1}{2}$ inches in diameter. Then we may find the circumference by (Art. 12 of mensuration.)

7 : 22 : : 25,5 :
22
———
510
510
———
7)5610)80·14
56
———
10
7
———
30
28
———
2

2)80·14
———
40·07

Thus we find the semi-circumference of the cylinder to be $40\frac{7}{100}$ inches.

We will now proceed to describe the falling moulds, and also lay out the run and rise of the eight winders, and find what the length of the hypothenuse lines will be, of the string board, and that of the rail also.

First let a line be drawn upon a floor as ABC in (fig. 2); and make BC equal in length to the semi-circumference ABC in (fig, 1) which is equal to 42 inches. From C raise the perpendicular C D, and make it equal in height to the eight winders; which we suppose takes 62 inches. Draw the hypothenuse line AD. From B draw BE perpendicular to AC. Then will AE and ED be the length of the hypothenuse line of the two quadrants which is the length of the concave side of the string board A D. From A draw A F one of the treads or more; of the flyers draw DG also. And thence set off the width of the string board, from these lines down; and draw the lower line parallel to them and whenever these lines meet in their angular point, they form the curve by the intersection of lines, as represented, (which is called by workmen the *easing of the angle.*)

To construct the falling mould for the hand-rail. In (fig. 2 draw L*m* parallel to AC; and make L,*n*,*m* equal in length to the stretch out of the dotted line ABC in fig. 1,) which is $40\frac{07}{100}$ inches (as calculated heretofore.)

From *m*, draw the perpendicular, *m*J; and equal in height to the eight winders, or C D. Draw the hypothenuse line LJ and from *n*, draw *no*, perpendicular to L*m*. Thence from L draw L*l*N one of the flyers, or the pitch board, at the bottom, and one also at J*h*H, at the upper part of the windows. Then from L set off LM the bigness of the rail, and NO, JK and HI also, and draw OM, MK and KI; and wherever the angular points meet at LM and JK, form the curve by the intersection of lines, which will complete the falling mould.

To find the straight part of the rail, for the lower and upper wreath pieces or quadrants. From L let the line L*m* be continued across the rail until it cuts the upper side at U.

Then U L will be the straight part of the rail for the lower wreath piece, and VK the upper one, which are the same as the straight part AD in (fig. 3) the plan, or AD in (fig. 4)

To find the height of the rail, that is for the lower and upper wreath pieces. At *o*, square across the rail, and divide it into two parts, at the centre Q. Thence draw PQ perpendicular, to L*m*, and continue the line PQ up to R, and draw RX at right angles with RP, cutting the joint of the straight part of the rail at X Then when you come to draw the face moulds as in (figs. 5 and 7,) take the distance from R down to the under side of the rail at S, and set it off on the line R S in (fig. 7); and for the lower wreath piece, take the distance from P up to the upper side of the rail at T, and set it off on the line from P to B in (fig. 5.)

The reader in order to understand what has already been described, should recollect that the hypothenuse A D (fig. 2) (the string board) is longer than LJ of the hand rail.

It has been proved heretofore that the semi-circumference of the well-hole A BC is about two inches longer than that of the rail L*nm*; the difference may not appear so much on the plate.

It may not be amiss to give an example here to show the learner how he may obtain the length of these hypothenuse lines AD and LJ; and also ascertain what their difference will be in the lengths. This may be done by extracting the square root of the sum of the two legs, or by the properties of the *triangle* in *Trigonometry*; or it may be performed by Logarithms which is the same thing. Each method will be fully explained hereafter in their proper place. The operation will be as follows

AC=42 in
42
———
84
168
———
1764

CD=62 in
62
———
124
372
———
3844
1764
———
56·08(74·88
49
———
144)708
576
———
1488)13200
11904
———
14968)129600
119744

L*m*=40·07 in
40·07
———
28049
1602800
———
16056049

J*m*=62 in
62
———
124
372
———
3844
16056049
———
54·49·60·49(73·82
49
———
143)549
429
———
1468)12060
11744
———
14762)31649
29524

The length of A D we find to be $74\frac{88}{100}$ inches. and LJ being $73\frac{82}{100}$ gives nearly $1\frac{1}{16}$ difference.

For further particulars see (Art. 34 of mensuration.)

To find the face mould of the rail. Let ABG (fig. 3) be the plan of the concave side of the rail, and make A G equal in diameter to DE in (fig. 1,) let AF and BC be the thickness of the rail, and let AD the straight part of the rail be equal in length to UL or KV in (fig. 2.) Now it is not necessary to draw or make a pattern of only one of these quadrants, as in (fig. 3) and together with the straight part of the rail AD and EF. The most simple and the most convenient way to draw the face moulds at a full size for practice is as follows.

Take a board of a suitable breadth and length, face over one side and straighten one edge of it, of which the board should be of the breadth of GH in (fig. 4) or wider and equal in length to GG on the edge of the board. From thence square across this board at GH in (fig. 4) and place the pattern of the lower wreath piece (which you have already made as in fig. 3) on the board so as to have the two angular points of the pattern at B and D stand on the line G H. This pattern being tacked on the board, mark around it ; you must be particular in marking the joints at BC, AF and DE when this is done take the pattern off the board, and then divide the quadrant AB, into any number of parts, as 1, 2, 3, &c. the greater the number the more correct will be the operation.

And thence set a guage on the edge of the board GG so as to draw a line from the angular point B through NP up to B in (fig. 5.) Then set it across in A and run it from L through A M up to L in (fig. 5.) Then in F draw JKF to J. Then in D draw DI to D. Then in E draw HE to H also. Thence draw the lines through 1, 2, 3, 4, &c the quadrant part of the rail.

And then square across the board at PH in (fig. 5.) Then take the distance from P up to the upper side of the rail at T in (fig. 2) and set it off from P up to B in (fig. 5.) And draw the hypothenuse line BH, and continue the line from B out to the edge of the board at G. Then take a thin board to make your pattern, and face it over, and straighten one edge, which should be equal in length to GH and HO the breadth. Then place the edge of the board, on the hypothenuse line GH, and tack it down to the other ; and thence square across this board at the several divisions HDJLI 2 3 4 5 B and G. Then take the distance of G C in your compasses in (fig. 4) and set it off on the line from G to C in (fig. 5.) Then take the distance of BN in (fig. 4) and set it off from B to N in (fig. 5) Then take the distances of 555, 444, 333, 222, 111 in (fig. 4) and set them off on the lines at 555, 444, 333, 222, 111 in (fig. 5.) Then take also LAM, JKF, DI and HE, from (fig 4) and transfer them to LAM, JKF, DI and HE in (fig. 5.] Then in (fig. 5 draw the lines BC and DE, which will form the joints of the rail. And draw A D and EF the straight part of the rail. Then to draw the curve tack in brads at the several points AFM and at 11, 22, 33, 44, 55, BN and C and bend a thin slip of wood around, with a pencil trace out the curves which will complete the face mould for the lower wreath piece of the rail as required.

The face mould of the upper wreath piece of the rail, may be found in the same manner, as described in (figs. 6 and 7) only the pattern as in (fig. 4) must be turned over so as to have the straight part of the rail, come on the edge of the board as represenred in the diagram of (fig. 6.) Then take the distance from R down to the under side of the rail at S in (fig. 2) ; and set it off from R to R in (fig. 7); and draw the hypothenuse line BT. Thence you may transfer the several ordinates from (fig. 6) to (fig. 7) as heretofore described in (figs. 4 and 5,) and complete it in the same manner,

Figs 8 and 9 shows the method of cutting the rail out of a plank ; the plank should be from 3 to $3\frac{1}{2}$ inches thick, to square out the wreath pieces of the rail, when the rail is from $2\frac{1}{4}$ to $2\frac{1}{2}$ inches in thickness. To do this let (fig. 8) be the plank ; lay your pattern on the face, so as to have the angular points stand upon the edge at BD, and mark around it, then take the pattern off and turn the plank up, upon its edge as represented in [fig. 9.] Then lay your pitch board [of the winders] on at BLG, and draw the vertical line BL ; and lay it on also at AD the straight part of the rail, and draw the vertical lines AE and DC. Thence turn your plank down upon the other side, and turn the pattern of the face mould over also, so as to have the angular points at BD in [fig .8] stand on the edge of the plank at LC in (fig. 9]. When the pattern is laid upon the face as above described, mark around it, as was done in [fig. 8]. And then cut away the superfluous wood on both sides of the rail, which is generally done by screwing the plank into a vice ; then set a narrow frame saw in on the line BL in (fig. 9) ; two men are required to work it, one on each side of the plank, so that they may govern the saw, and work to the lines which are drawn on both sides of the plank. This being done and the rail trimmed down smooth to the lines. take the pattern of the falling moulds as in (fig. 2) which should be made of a thin board or thick paper, and bend it around on the concave side of the rail, and draw the upper side of the mould, and then the thickness of the rail may be obtained by gaugeing, after the upper side is squared around perpendicular to the plan, or the working cylinder.

It will be well however to leave the rail a trifle thicker at the ease-offs, that is where the straight part of the rail joins on to the wreath pieces. Then when the rail is screwed together, and set to its proper place ; you may in a short time, with a coarse file work it down, so as to have the curve appear natural and easy. And the string board of the stair should also be worked in the same manner, since by thus doing you will be less liable to make cripples.

The greatest art of hand railing, depends on finding the section of a cylinder to pass through three given points on its surface, as heretofore described ; therefore the reader is requested to understand this part thoroughly, before he actually commences his study upon hand rails ; for if the principles are not comprehended he will always be in difficulties and liable to spoil his work.

Fig 10 shows, a section of the hand rail as formed to the *prismatic mould* or the *working cylinder*.

This method which was formerly practised for forming the wreaths and fitting the rail together, is now of no other use than merely to help the conception of the learner ; therefore it may be better for the inexperienced to form his work to a working cylinder. To do this let the cylinder be made equal in length to the height of ten steps as represented in (fig. 10) : and the convex circumference equal to ten also, that is the semi-circumference shall be equal to the eight winders, as ABG in (fig. 3) together with the two fliers AH and GI, or HAB in (fig. 10) shows one half of the plan of the vertical section ; and CED and FGI the two fliers and JHLMNOPF the winders.

Fig. 11 shows the method of laying out the notch board of the stair case, in which the treads and risers enter.

Let AB and CD be the width of the board or plank, and from the under side AC gauge on one inch at the line EF ; then lay your pattern or pitch board on as represented at GHI, and draw GH and HI ; then slip the pattern along to O NP and draw ON and NP, and so on until the whole are drawn.

Then with a pair of dividers set off GM and HL the thickness of the riser, and draw LM; then also set off HK and IJ, the thickness of the tread, and draw JK &c. The nosing of the steps may be drawn with dividers, or work a small piece in wood for a pattern, and lay it on at its proper place and mark around it ; the pattern should be made a trifle less in size than that of the step so as to have it fill up tight. If the treads and risers are calculated for wedging let the steps be laid out as above described. then from P set off a suitable thickness for the wedges at PP and PR, and run them out to a point at NN.

In regard to this method of wedging the steps into the notch board, there is not the necessity of wedging them in an open stair case as in a tight one, that is where the stair is carried between two partitions. Wherever there is to be a nice stair case built of this description, the steps should be let into the notch boards, or strings. upon both sides ; which is done by letting the step boards and risers in on the under side of the strings or stairs, the step boards and risers should also be tongued and grooved, and put together with glue, and also the wedges, as described in the diagram of the figure.

The notch boards in which the steps are let in, are generally from $1\frac{1}{2}$ to 2 inches in thickness, they should not be less than $1\frac{1}{2}$; then lowering $\frac{3}{4}$ for the thickness of the lath and plastering, there will be $\frac{3}{4}$ left for the thickness of the base ; or if the plank be $1\frac{3}{4}$ inches thick, lowering $\frac{3}{4}$ for plastering, there will be 1 inch left for the thickness of the base, which generally is as thick as required. If the plank be $1\frac{1}{2}$ inches thick, the steps should be let into them at least 1 inch ; and to have firm steps they should not be less than $1\frac{1}{4}$ in thickness.

To put up a tight stair case of the above description ; first lay out your notch-boards, or strings as heretofore described, and then nail them to their proper places, and be careful also to drive the nails upon the under side of the steps so they may not be seen. Then if your stair be all the way of equal width in the opening, square and saw off all the treads and risers, and the hollows of the nosing to their right length and also have the wedges made, and the square blocks of wood which is glued into the internal angles of the risers and step boards ; then commence at the bottom by letting the lower riser into the floor, or scribe it down tight to it. We must remember, that this riser can neither be let in upon the under side or wedged, nor tongued into the first step board; therefore we must cut it one inch shorter than the rest, and so slip one end into the notch board, the depth of one inch, then draw it back into the other so as to divide them equal, then nail it down tight to the floor, and the notch boards by driving the nails obliquely on the backside of the riser. Then put in the step boards, and when you have got them all in and wedged firm, and glued, commence putting in the risers ; the tongues of the risers and step boards should be glued into the grooves.

The hollow of the nosing must however be put into its place before the riser. When the risers are put in and wedged ; nail them strong to the step boards, upon the backside taking care the nails do not go through the face.

Then glue in the square blocks in the internal angles of the risers and step boards, as represented at NN, in (fig. 11,) which will strengthen the work and prevent them in a great measure from becoming ricketty ; and though this stair be done with the utmost care, it can never be made so firm as not to yield to the passenger ; especially those of an open stair case.

Fig. 12, Shows the method of diminishing and traceing one bracket from another, in a stair case ; A B being the breadth of those of the fliers : and A C those of the winders.

To find the Falling and Face-Moulds for an Eliptical planed Stair (Plate 22.)

The plan and section of the rail being laid down as in (fig. 1, No. 1.)

The reader will observe, that the ends of the steps are equally divided off, around the eliptical wall and also at the rail or well hole. In order to cut the rail out to the best advantage, it should be made in three lengths, as represented at A B C.

The plan of this rail must be divided into as many equal parts, as there are steps; then take the treads of as many steps as you please ; suppose there be eight in A, and six in B and C ; let *h h* at (fig. 1, No. 2.) be the tread of eight steps, that is the stretch out of the eight steps at A, and let H *h*, be the stretch out of the six steps at B and C. Thence draw the perpendiculars *h m* and H*k*, and make *h m* equal in height to the eight risers at A, and H *k*, equal in height to the six at B and C ; and then draw the hypothenuse line *h m*, which will be the underside of the falling mould. Then set up the thickness of the rail *m n* and *kl*, and draw *n l* parallel to *k m*, which completes the falling mould. This falling mould, will be a straight line, excepting a little turn at the landing, and at the scroll where the rail must bend, in order to bring it level with the landing, and to the scroll.

To construct the face mould. Draw the chord line for each piece to the joints at A B C ; also, draw lines parallel to the chords, to touch the convex side of the plan of the rail, as *h h* at A, and H *h* at B and C ; and then from every joint draw perpendiculars to their respective chords. Then take the distance of *h n* in (fig. 1, No. 2.) and set it off from *h* to *n* in F ; and draw the hypothenuse line *n, i* and then take the distance of H *l* in (fig. 1. No. 2.) and set it off from H to *l* in D and E, also draw the hypothenuse line *l i*. From thence you can transfer the several ordinates in A B C to D E F and complete the face mould as in the foregoing plate.

Fig. 2, No. 1, and fig. 2, No. 2. Shows a plan and elevation of a semi-circular well-hole for a stair and fig. 3, No. 1, and fig. 3, No. 2, shows the method of drawing the face mould of the hand-rail, and fig. 4, No. 1, shows a method of describing a vertical scroll appertaining to the same. All of these figures are

drawn at one third the size for practice. This stair-case is calculated for a straight run; and the well-hole is inserted at the landing of the floor.

To construct the well-hole as in (fig 2, No. 1,) let the semi-circle A B C that part of the string board be worked out of two pieces of plank; and let the grain of the wood run vertically; and spring them by taking the angular corners off at D C K, E K J and F H I &c. Then let a groove be run in them at H, and glue them together by putting in a false tongue. Each piece should however be worked or rounded out to a pattern before they are glued which can be done with more facility and convenience, than when they are put together. The dotted lines at R S shows the projection of the outer edge of the upper step-board of the well-hole, and P Q the hollow under the step-board, or the nosing; MN is the opening between the rough string board and the trimmer joist.—W is the rough string.—X the upper riser which mitres into the string-board of the well-hole at A. The step-board of this well-hole should be worked in one piece, that is, from S where it mitres to the upper riser let it be continued across to T at the centre of the trimmer joist, and let R T be a square joint, where it finishes to the level flooring; then from T let it be squared across to the centre of the trimming joist at U. Thence let it run from U on the centre of the joist across the stair into the notch-board. The hollow of the nosing, which is worked around in the well-hole should be worked out of a square piece of board. This may be done by cutting the well-hole off that part of the string-board, on a plane at the under side of the hollow of the nosing.

When the well-hole of the string is set in atits proper place, take a board the thickness of your hollow, and fit it in close between the rough string and trimmer joist, as represented by the dotted lines at L M N O. Then set a pair of dividers of the projection of the hollow and from P scribe around the well-hole to Q, and this part after it is sawed out, must be worked out around with a gouge; and let the joint at P O be square, where the hollow on the level part ends, and the joint also at L M left square, so return the hollow on the piece from Q to L, and let the hollow on the upper riser end square to it.

The upper step-board may be worked around the well-hole in the same manner as above described.

To describe the falling mould or the ease-off of the well-hole as in (fig. 2, No. 2.)

The ease-off should be made where it can be at the centre, as represented at E. In order to describe it in this manner; let A B C in (fig. 2 No. 2.) be equal in length to the stretch out of A B C in (fig. 2, No. 1,) and C D in No. 2, be equal to the height at the level part of the landing: Then to square around within the well-hole, let a pattern of the pitch-board be made of thick paper as represented at F G H; and place the square side of this pattern on the vertical line or edge at C D, so as to have the square corner at H come upon D, and bend it around on the plane of the well-hole. With a pencil draw the line D E to the centre; then turn the pattern over upon the other edge as represented at F G H and draw the line F E cutting the line D E at the centre E. Form the curve by intersecting lines, or trace the curve around with a pencil, which two inches each way from the angular point E will be sufficient to form the curve. The string-board of the stair in which the risers mitre into and finishes with that part of the well-hole at F I; the width of this string may be ascertained by drawing out one step at full size from F I.

To draw the face mould of the handrail as in (fig. 3, No. 1, and fig. 3, No. 2.) which stands over this semi-circular well-hole.

The reader will observe, this rail is not worked on a spring as is sometimes done; the quadrant part as in No. 1, lies on a perfect plane every way; that of No. 2, also lies on a plane one way, and the other, which, by running up on its true raking position brings the rail high enough upon the level landing at B D where the joint of the rail is made.

Patterns of this face mould may be made of a thin board or thick paper.—In No. 1, draw the quadrant E A B the inside of the rail; from A set off the bigness of your rail to C, and draw the quadrant C D; Then from A C draw so much of the straight part of the rail as is necessary, and draw the line E D, which will form the joint of the rail at B D, which completes the pattern for the level part of the rail.

To draw the pattern or the face mould for the raking part of the rail as in (fig. 3, No. 2.) First place your pitch-board of the stair, as represented at E F G so as to have the lower angular point come at E, and draw the raking line from E to G. Take the pitch-board off the paper or board, then continue the line G E to H, and draw the lines E A C and H I J at right angles with G H. Then take the distance of E A in No. 1, and set it off from E to A in No. 2, and from H to I also and draw the straight line A I. Then take the bigness of the rail, A C in No. 1, and set it off from A to C in No. 2, and at I J also, and draw the straight line C J; which will complete so much of the straight part of the rail as will be necessary. Then divide the line E D in No. 1 into any like number of parts as 1, 2, 3, &c. and through these several divisions, draw the line 1 *a b*, 2 *c d*, 3 *e f*, &c, parallel to C E, until they meet the line G H in No. 2. Then through 1, 2, 3, &c, in No. 2, draw the lines 1 *a b*, 2 *c d*, 3 *e f*, &c. parallel to E C, or at right angles to G H. Thence take the distances 1 *a b*, in No. 1, and transfer them to I *a b* in No. 2. Then take 2 *c d* in No. 1, and set them off at 2 *c d* in No. 2, and so on until you have got all of the several ordinates transfered from No. 1, to No. 2. Then with a pencil draw the curve A B in No. 2, through the points at *a c e* &c. then at C D through *b d f* &c, which completes the face mould.

In order to work this rail out, let us suppose No. 2, to be the plank, and the pattern of the face-mould being laid on upon the face of it as represented in the diagram of the figure; mark around on the inside of the pattern at B A I, and on the outside also at D C J. Take the pattern off, and let the plank be turned up upon its edge as represented on the plate, at the right of No. 2, then place your pitch board on the edge as represented at A B C and draw the vertical line A D; slip the pitch-board along at E, and draw E F, to G also, and draw G H.—Then let the plank be turned down upon the other side, and turn the pattern of the face mould over so as to have the edge E B D, lie upon the edge at D F H, and mark around it as above described. Then let this plank be screwed into a vice, and set your compass-saw in on the line E F upon the edge of the plank as represented at the right of No. 2, and saw the quadrant part around to A D, by following the lines which are drawn upon both sides of the plank. Then set the saw in on the line G H and let this be worked around in the same manner.—Thence if your rail be round, draw the circle as represented at E F, G H, which does not require the plank to be thicker than that of the level part as represented at the left of No. 1.

To draw the vertical scroll of the hand rail. This is generally made about eight inches in height; therefore we will suppose the height on the line A B as in fig. 4. No. 1, to be eight inches. Then let the line A B be divided into eight equal parts, then each division will be equal to one inch, and bisect the fifth division at the centre 1, from A down; from 1 draw 1, 2, at right angles to A B, which make 1, 2, equal in length to one inch or one of the eight parts (see fig. 4, No. 2, on which the centres are drawn to the full size;) bisect A B at the centre H.

Draw the lines B G C and H E D, at pleasure, and at right angles to A B.—Take the distance of B H in your compasses, and set it off from B to G and draw the line E F G parallel to A B. Bisect E G at the centre F.—Then take the distance of G F and set it off from G to C and draw C D parallel to G E.—Then will A B C D and E be the centres as represented at 1, 2, 3, 4 and 5 in No. 1.

To describe the scroll as in No. 1. Take the distance of 1 A in your compasses and on 1 as a centre draw the quadrant A C. Then on 2, as a centre with the distance of 2 C draw the quadrent C D. Then on 3 as a centre with the distance of 3 D draw the quadrant D E. Then on 4 as a centre with the distance 4 E draw the quadrant E F. Then on 5 as a centre with the distance of 5 F draw F G. Then from A set down the thickness of your rail at H; and then in 1 as a centre with the distance of 1 H draw the quadrant H G, cutting the former at G: which completes it.

To draw the ease-off of the falling-mould; let a line be drawn from A at I, at right angles to the perpendicular A B. From A set off $2\frac{1}{2}$ inches to J. Then place your pitch board on as represented at I J K, and draw the raking line J K and set off about 3 inches from J to L and form the curve by the intersecting lines.

In putting up a continued rail of the above description there should be one or more iron balusters inserted near the well-hole at the landing, to support the rail and prevent it from trembling; or you may let an iron run from the under side of the raking rail off into the trimmer joist, as represented at A B C in Fig. 6.

To describe a scroll for a hand-rail as in fig. 5, No. 1. The reader will observe the scroll that has already been described on this Plate, is suitable for one of 8 inches; and the one that has been described on Plate 20, is suitable for one of 10 inches; which is drawn at full size And the one that I am now about to describe, will be suitable for one of 12 inches; though either of the above mentioned examples, may be increased, or diminished, as choice may direct.

To describe this scroll as in fig. 5, No. 1, suppose the line A B the width of the scroll to be twelve inches. Then divide the line A B into eleven equal parts, and from 1, at the sixth division from A, draw 1, 2, at right angles to A B and make 1, 2, equal in length to one of the eleven divisions. We will now turn to fig. 5, No. 2, where the centres are drawn at full size. Let A B be equal in length to 1, 2, or one of the eleven divisions as in No. 1. Bisect A B as in No. 2, at the centre K; from K draw K H at right angles to A B and make K H equal to K A or K B; join the diagonals H A and H B. From B draw B J C at pleasure, and at right angles to A B; from H draw H J parallel to A B; and bisect H J at the centre G; then bisect H G at the centre I; and from I draw I L parallel to B C, which make I L equal in length to I J; from L draw L C parallel to I J; produce the line C L to D and make L D equal in length to L C. From D draw DE parallel to L I or B C until it meets the diagonal HA at E; From E draw E F parallel to A B, until it meets the diagonal H B at F; from F join F G.—Then will ABCDEF and G be the centres, which is the same as 123456 and 7, in No. 1. Thence in No. 1, take the distance of 1 A in your compasses, and one foot in 1 as a centre draw the quadrant A C. Then in 2, as a centre, with the distance of 2C draw the quadrant C D. Then in 3 as a centre with distance of 3D draw the quadrant D E. Then in 4 as a centre with the distance of 4E, draw the quadrant E F. Then in five as a centre, with the distance of 5F draw the quadrant F G. Then in 6, as a centre with the distance of 6 G draw the quadrent G H. Then in 7 as a centre with the distance 7H draw the arc to I.

Then set off the bigness of your rail from A to J. And again, with one foot of your compass in 1 as a centre with the distance of 1J draw the quadrant J K.

Then in 2, as a centre with the distance of 2K draw the quadrent KL. Then in 3 as a centre with the distance of 3L draw the arc at I, intersecting the former at I, which completes it.

Fig. 7, Shows a section of a *dog-legged stair* with newel-posts.

The proportions are figured off in feet and inches.

The shortest newel, as represented at the landing stands opposite to the other. The hand-rail of the stair finishes with a *knee* at the bottom, and is mitred into a cap, which is turned on the upper part of the newel, the cap being turned in the form of the rail. And the rail miters into the cap of the newel-posts at the upper part, and is ramped up in the form of a *Swan-neck*, so called, being concave below and convex on the top, and terminating at the newel, so as to be parallel to the horizon.

This kind of stair, is substantial when finished, but its erection requires double the labor necessary for the completion of continued rails, as described on this plate.

DESIGNS FOR CHIMNEY-PIECES. (PLATE 23.)

Fig. 1 represents a design for a Grecian Chimney-piece, suitable for a Dining Room. The ornaments in the pilasters represents ripe fruit; the tymphan of the pediment, an antique wreath, with ribbons; the extremities of which are terminated by Grecian heads, taken from models in the British Museum. The design is by Mr. Elsam, taken from an English work.

Fig. 2 represents a design for a Grecian Chimney-piece, with antae, imitated from the Choragic Monument of Trysallus.

Fig. 3 represents a design with pilasters, suitably adapted for a modern size room.

Fig. 4 is a design with columns after the Grecian style. The tablet and trusses of the frieze both in this example and also in that of fig. 3, are finished with diamond pannels; and the opening between the trusses and tablet are pannelled; they may be sunk about half an inch, and the form of them upon the face may be made like those at AC (pl. 12.) The trusses and entablature may project $\frac{1}{2}$ to $\frac{3}{4}$ of an inch from the pannels of those which are sunk into the frieze.

The plans of these designs are not described; but the columns of that in fig. 4 should stand clear from the face of the wall or the stiles of the frame; that is, let the abacus be perfectly square so as to have the capital of the column stand clear.

The pilasters of that of fig. 3 may project equal to one fourth of their breadth, which will be about two inches.

In fig. 2, let the antae project equal to one half of their breadth or diameter, which will be $4\frac{1}{2}$ inches; and then let one of the antique wreaths be cut in the centre, and place the halves on the returns of the ends of the frieze.

That of fig. 1 should project so far as to have so much of the head show on the ends as it does in front. This example and those of figures 2 and 4, are calculated for large size rooms; there is a scale of feet and inches applied for each example, though their general outlines are expressed in feet and inches, by figures. A is a section of the dentil band as in fig. 2, drawn at full size for practice.—B the antae capital at full size also.—C is a section through the front of the cap or shelf of the Chimney-piece.—D the bed-mould under the shelf which is recessed into the soffit, the height *e, e*.—E a section of the capital of the column at full size, as in fig. 4.—F and G a section of the cap and bed-mould at full size also.—*g, g,* shows so much of the bed-mould as is recesssed into the soffit.

Plans and *Elevations* for *Country Villacs*. (*Plate* 24.)

Fig. 1 exhibits the plan of the main body of the house, and elevation of the same, with a portico in front; the *order* is of the *Grecian Doric*.

An addition to the main body, as represented at AAA is usually erected for a *kitchen, wash-room* &c. The doors at DD enter the kitchen from the *dining room* and *hall*. The door at C in the *hall*, leads out into an open colonnade, and the two windows also at BB in the *setting room*, come down near to the level of the floor, forming a step out. The sash frames should be so constructed as to make the sashes go higher up than the top of the window, in order that a person may walk out without stooping. The rooms are calculated to be heated with coal, the grates are set at E,E,EE and the flues go up within the walls as represented on the plan.

There is a scale of feet applied, on which the different parts may be measured; or the dimensions are as follows:—The whole length of the building from the extremities of the antae is 48 feet 4 inches. The breadth of the portico at the top of the columns is 32 feet, and its height, from the surface of the ground to the top of the crowning mould of the cornice, 27 feet 10 inches; the entrance is raised 2 feet above the level of the ground, and ascended by three steps. The columns are 20 feet in height, and 2 feet 11 inches in diameter, being a trifle less than seven diameters; which we may consider a suitable proportion for a private dwelling. The Entablature is two diameters in height, which is 5 feet 10 inches. The height of the Architrave and Frieze are each made 44 minutes, equal to 2 feet $1\frac{3}{4}$ inches for the height of each. The Triglyph's are made 23 minutes in breadth, nearly 16 $\frac{3}{4}$ inches. The breadth of the metopes is made 42 minutes, equal to $24\frac{1}{2}$ inches. The height of the cornice is 32 minutes, equal to $18\frac{3}{4}$ inches, and the projection is $27\frac{1}{2}$ minutes, equal to $16\frac{1}{4}$ inches.

The breadth or diameter of the antae are 55 minutes, being equal to 2 feet $7\frac{1}{2}$ inches, and they project from the face of the wall 8 inches, or the length of one brick. The capitals of the antae are taken from the Choragic Monument of Trysallus, as described in (*fig.* 6. *pl.* 9, *vol.* 1 *of Orders.*) The face of the antae is on a vertical line with the face of the architrave; and the breadth of the soffit of the architrave or epistylium, is equal to the breadth of the antae or 55 minutes: that is, in the pediment part. The glass of the windows in the first story are 18 by 12 inches; and those of the second story 16 by 12.

Note.—Among all the entablatures of the Five Orders, the Doric is the most difficult to distribute: it is on account of the intervals between the centres of the triglyphs, which will not admit of being increased or decreased, without materially injuring the symmetry and characteristic beauty of the composition. And whenever the composer doubts in such a case, it will be better to leave them off entirely, and only employ the dentil-band, as represented in (*figures* 3, 4 an 5, on *Plate* 26.) For we may conceive by comparing this plan and elevation to that of fig. 2, their proportions of the piers or spaces are not so correct, neither upon the exterior part, nor in the interior part of the rooms.

Fig. 2 represents a plan and elevation of a *Country Villa*, with a portico in front, in the *Ionic Order;* taken from the little *Ionic Temple* on the river *Ilyssus* near *Athens;* as described on (pl. 10, vol. 1 of Orders.) The dimensions of this building are nearly the same as the other, or as follows. The whole length from the extremities of the antae is 48 feet 6 inches. The breadth of the portico at the top of the columns is 30 feet 2 inches. The whole height from the surface of the ground to the top of the crown moulding of the cornice 27 feet 6 inches. The three steps that ascend is equal to 2 feet. The columns are 20 feet 2 inches in height including the base and capital; and their diameter is 2 feet 4 inches, which is equal to $8\frac{1}{2}$ diameters or nearly. The entablature is 2 diameters and 17 minutes, which is equal to 5 feet 4 inches. The height of the architrave is $55\frac{3}{4}$ minutes, and equal to 2 feet 2 inches. The frieze $49\frac{3}{4}$ minutes or 23 inches. The cornice 32 minutes or 15 inches. The projection of the cornice 39 minutes or $18\frac{1}{4}$ inches. The breadth of the antae is 54 minutes, which is equal to 25 $\frac{1}{4}$ inches; and the soffit of the epistylium of the portico, the same. The antae project from the face of the wall 6 inches, and the face of them stand on a line perpendicular to the face of the architrave of the entablature. The glass in the windows of the first story are 18 by 12 inches, and those of the second story 16 by 12.

Plate 25 exhibits the plans and elevations for Country Villas, after the Castellated or Gothic style; or more properly called, British Architecture. The designs are by M. A. Nicholson.

Fig. 1. *Elevation of a Castellated Gothic Villa*, with buttresses, &c. The whole length of this building is 78 feet; and its height from the surface of the ground to the top of the battlements, 34 feet 2 inches. The battlements are continued all around the building, and the height of them is 2 feet 6 inches. The buttresses are 29 feet 3 inches high, with two water tables, on the top of which is a cornice. The cornice is continued all round the building. The windows on the ground floor are 4 feet 3 inches from the ground, their height 9 feet, and width 4 feet. The top of each window is crowned with a tablet, which reaches a little below the top of the window, on each side. The chamber-floor windows are 19 feet from the ground, their height is 8 feet, and width 4 feet, and they are crowned with a tablet, as below. The entrance is on the flank to the left, raised 1 foot 6 inches above the level of the ground, and ascended by three steps; it is enclosed within a porch of 12 feet in front, and 8 feet deep; the openings of the front and sides of the porch are 8 feet, and 4 feet 10 inches. The height to the springing of the arches is 8 feet, and to the top of the arch 3 feet 10 inches, over which runs a band, and is of the same height as that in the octagonal front of the building; on the right flank is a green-house, which will have a very beautiful effect on entering, as seen at one extremity of the passage through a sash door.

Fig. 2.—*Ground Plan of the Principal Story. A Porch*, 8 feet by 6 feet; B *Passage*, communicating with the different apartments, 74 feet long by 6 feet wide; C *Staircase* to the Bed-chambers, steps 3 feet 6 inches long, treads 11 inches, risers rather more than 6 inches; *Breakfast-room*, 20 feet by 17 feet 6 inches; *Dining-room*, 20 feet by 17 feet 6 inches; *Drawing-room*, 30 by 30 feet; *Library*, 20 feet by 17 feet 6 inches; E *Parlour*, 20 feet by 17 feet 6 inches; D *Waiting-room* or *Dressing-room*, 19 feet by 15 feet; F *Water-closet*, which is entered by a door under the staircase; *Green-house*, 42 feet 6 inches by 12 feet 6 inches. The *Servants' apartment* &c. are on the basement, which is entered by the staircase, C.

Fig. 3.—*Elevation of a Castellated Gothic Villa*, with Buttresses and Pinnacles, on a straight Front. The extent of this building, from the extremity of one wing to that of the other, is 60 feet; extent of each of the wings, 11 feet 10 inches. The body of the building, 36 feet 4 inches. The entrance, 3 feet 4 inches wide, with a Gothic head, receding from the central part of the front, 3 feet 6 inches, forming a Porch, and raised above the level of the ground 1 foot 6 inches; ascended by 3 steps of 6 inches rise. The entrance to the Porch is 4 feet 10 inches wide, and it rises 6 feet to the springing of the arch; the arch is 4 feet high and is ornamented with mouldings and crockets on each side. The windows of the ground floor on each side of the Porch, are 6 feet 6 inches high by 4 feet 6 inches wide; those in the bed-chamber 4 feet 3 inches high and 4 feet wide.

Fig. 4.—*Ground Plan of the Principal Story.* A flight of three steps to tho Porch, L, 7 feet 7 inches by 2 feet 6 inches. I *Hall*, 9 feet 6 inches by 7 feet 7 inches. N *Staircase*, steps 3 feet 3 inches, their rise is $6\frac{1}{2}$ inches, and the tread $10\frac{1}{2}$ inches. PP *Passage* to the different apartments, 57 feet 8 inches by 4 feet. B *Breakfast-room*, 14 feet 3 inches by 12 feet. A *Dining-room*, with folding doors, 14 feet 3 inches by 12 feet. F 12 feet 3 inches by 10 feet 7 inches. D *Parlour*, 12 feet by 10 feet 4 inches. C *Library*, 12 feet by 10 feet 4 inches. H *Servants Waiting-room*, 7 feet 6 inches by 6 feet 3 inches. E *Kitchen*, 12 feet 3 inches by 10 feet 7 inches. G *Wash-house*, 10 feet by 7 feet 6 inches. O *Water-closet*, 4 feet 2 inches by 3 feet.

Plate 26.—Elevations for Villas. Fig. 1 Elevation of a plain Country Villa. The dimensions are as follows:—breadth 24 feet, its height from the surface of the ground to the top of the crown-mould of the entablature, 25 feet. The entrance is raised 2 feet 10 inches above the level of the ground, and ascended by four steps. The height of the entablature 3 feet 11 inches. The architrave 1 foot 6 inches; frieze 1 foot 6 inches; cornice 11 inches. The glass in the lower windows are 16 inches by 11. Those of the upper 15 by 11.

Fig. 2. Elevation of a Villa with antae and an entablature. The breadth of the building 24 feet 4 inches; its height from the surface of the ground to the top of the crown-mould of the entablature, 25 feet 8 inches. The entrance is raised 2 feet 10 inches from the level of the ground, and ascended by four steps. The height of the antae is 18 feet 2 inches; their breadth or diameter 2 feet 4 inches or the length of $3\frac{1}{2}$ bricks; and their projection from the face of the wall 4 inches or the breadth of one brick. The height of the entablature 4 feet 8 inches, or two diameters. The height of the architrave is 22 inches; the frieze $19\frac{1}{2}$ inches cornice $14\frac{1}{2}$ inches. The Antae Capitals are taken from (fig. 2 on pl. 11 vol. 1 of Orders.)

The entablature may be taken from pl. 10 or from pl. 1; or the entablatures from plates 24 or 25 may be used if a dentil cornice is required. Let the breadth or diameter of the antae be divided into 60 minutes, and proportion the mouldings &c. from the scale. The glass in the lower windows are 16 inches by 11, and those in the upper, 15 by 11.

Fig. 3.—Elevation of a Villa with antae and an entablature. The dimensions of this building are nearly the same as the one last described, or as follows: Its breadth 24 feet 4 inches; the height from the surface of the ground to the top of the crown-mould of the cornice, 25 feet 4 inches; the height of the antae, 18 feet 2 inches; their breadth 2 feet 4 inches, or the length of $3\frac{1}{2}$ bricks; projection 4 inches, or the breadth of one brick. The height of the entablature is 4 feet 4 inches, or 111 minutes; the architrave is 21 inches, or 45 minutes; the frieze 17 inches or 36 minutes; and the cornice 14 inches or 30 minutes. The antae capital is taken from (fig. 6, pl. 9, vol. 1 of Orders,) the height of which is $27\frac{1}{4}$ minutes, or equal to $12\frac{3}{4}$ inches. The proportions of the entablature are from (pl. 3,) only the mutules are left off, and the frieze is made 4 minutes higher. Figures 1 2 and 3 are drawn from the scale of feet which is exhibited at Fig. 2.

Fig. 4.—Elevation of a Villa, with a *Belvedere* turret, or an observatory, which if the building be erected on elevated ground, will afford a favorable situation for viewing the surrounding country.

The *Order* or style of this building is the same as the one last described; and the dimensions are as follows; The extent of the front of this building to the extremities of the antae, is 46 feet; its height from the surface of the ground to the top of the crown-mould of the cornice, 29 feet 5 inches. There are four steps 8 inches each, which are equal to the height of 2 feet 8 inches from the level of the ground. The height of the antae 21 feet; their diameter or breadth 3 feet, or the length of $4\frac{1}{2}$ bricks; they project 8 inches, or the length of one brick. The height of the entablature is 5 feet 9 inches, or 1 diameter and 55 minutes. The architrave is 2 feet $4\frac{1}{4}$ inches nearly, or 47 minutes; frieze 1 foot 8 inches, or 37 minutes; cornice 1 foot $6\frac{3}{4}$ inches, or 31 minutes. The ledge or belt above the cornice, is $10\frac{1}{4}$ inches in height, or 18 minutes. The gutters of the roof should lie on the back side of this belt, and they may be made of tin, copper, or any other metal. The face of this belt stands out about 4 inches from the vertical face of the entablature, and the chimney-shafts stand a little within so as to have the water run freely in the gutter, betwixt the belt and chimney. The roof should be covered with tin or some other metal, especially that part below the belt. The roof rises 8 feet from the top of the crown-mould of the cornice, to the under side of the belvedere. The belvedere is 10 feet in breadth; and its height from the under side of the plinth or pedestal to the top of the cornice, is 15 feet. The plinth on which the antae sets is 1 foot 5 inches in height. The height of the antae is 10 feet, and their breadth or diameter is 1 foot $10\frac{1}{2}$ inches; and they project 6 inches. The height of the entablature is 3 feet 7 inches, or 1 diameter and 55 minutes. The architrave is 1 foot $5\frac{1}{2}$ inches, or 47 minutes nearly; the frieze is 1 foot $1\frac{3}{4}$ inches, or 37 minutes; and the cornice is $11\frac{3}{4}$ inches, or 31 minutes. The windows are of the Grecian style, the breadth of them are 3 feet 10 inches, that is, the sashes; their height is 7 feet. The architraves are 9 inches in breadth, and are similar to those of (fig. 8 or 9, on pl. 16.) The sashes should be made in one part, and so constructed as to run higher up above the top of the sash-frame into the entablature. The glass in the windows of the principal story may be from 18 to 20 inches by 12; and those of the chamber windows 16 to 18 by 12.

Fig. 5, *shows* an *Elevation* of a building suitably constructed for a City block, or where the building is continued from one street to another or around a square, and is divided into a number of tenements. The dimensions and parts of this building may be taken from the scale of feet. The basement story is supposed to be finished, and the windows in front consist of six panes of glass, and the size of them are 20 inches by 12. Those in the principal story consist of twelve panes each, the size of which is 20 inches by 12. And those in the first story of the bed-chambers are 18 inches by 12; and in the next, 16 by 12. The *attic*story is designed to be finished also, with frieze windows, in the frieze of the entablature. These windows are generally concealed with some kind of ornament, either foilage or a fret as represented. The walls of the basement and principal story are calculated to be built of stone, and that of the principal story rusticated. The thickness of the basement wall is 20 inches, and that of the principal story 16. Above the principal story the walls are supposed to be built of brick, with antae; the antae project 8 inches, their diameter or breadth is 3 feet, or the length of $4\frac{1}{2}$ bricks. Their height is 20 feet; and the entablature is 6 feet or two diameters. Above this entablature is a *Balustrade*; the height of which is 5 feet, or 100 minutes. The height of the base or plinth is 1 foot $2\frac{2}{3}$ inches, or 24 minutes. The die of the pedestal is 3 feet in height, or 60 minutes; and their breadth is also the same; the height of the cornice is $9\frac{1}{2}$ inches or 16 minutes.

Plate 27.—*Ground Plan* and *Elevations* of a *Church* in the *Grecian Style*.—The design is by M. A. Nicholson.

References to the plan. The Portico (*a*) of four columns, projects out from the front of the wall, 8 feet 6 inches. The *Vestibule* (*b*) leads to the body of the chapel and side stair-cases; diameter, 19 feet 9 inches within the columns.

Stair-cases of an elliptical form (*cc*) leading to the gallery, 22 feet 8 inches by 20 feet 9 inches; length of treads, 4 feet 10 inches; breadth in the middle, 11 inches; risers rather more than 6 inches. Side Entrances, (*ff*.)

The body of the Church may be 89 feet long and 54 feet 3 inches broad, and it will contain sixteen hundred sittings, (including free seats,) exclusive of seats for children in front of the organ.

hhhh, represents Pews 3 feet wide; seats 1 foot; book-desk $5\frac{1}{2}$ inches; *oo*, *larger Pews*; *pp*, spaces between the free-seats. The *Pulpit* (*n*) of an hexagonal form; *i Stair* ascending to it. *Reading-desk*, (*m*) with clerk's seat in front; stair (*i*) ascending to it. The *Communion-place* of a circular form, with four three-quarter columns, and two antae; betwixt the columns are two niches and a window in the centre.

The *Vestry-room* (*s*) is 22 feet 2 inches by 15 feet 4 inches. The entrance to Circular Stair (*y*) leading to library above. The *Anti-room* under Portico (*u*) 10 feet 5 inches by 8 feet. The *Robing-room* [*t*] 22 feet 2 inches by 15 feet 4 inches. The *Anti-room* [*v*] 10 feet 5 inches by 8 feet. X *Entrance* to the Catacombs. W *Back-Portico* of four columns, projecting out from the wall 5 feet 6 inches.

Description of the *Front Elevation*. The extent of the front of this building to the extremities of the antae, is 64 feet 9 inches; the breadth of the portico at the top of the columns, is 37 feet. The columns are raised 1 foot $7\frac{1}{2}$ inches above the level of the ground, and are designed from the Monument of Lysicrates, as described on [Pl. 23 Vol. 1 of Orders,] their diameter is 3 feet, and height, including the base and capital, 29 feet. The height of the entablature is 7 feet 4 inches; the architrave 2 feet 9 inches; the frieze, 1 foot 10 inches; and the cornice, 2 feet 9 inches. The ornament which stands on the top of the cornice is 13 inches high, and is continued all round the building. The antae are of the same width as the top of the upper diameter of the columns, and do not diminish. The capitals of the antae are a composition, as there are no antae to be found in this style of Grecian architecture. The principal entrance is ascended by three steps in front of the portico, of $6\frac{1}{2}$ inches rise; tread 1 foot. Its width at the bottom is 6 feet 11 inches; it diminishes to the top, and its height is 14 feet. The side entrances are each 6 feet 7 inches at bottom, and diminish to the top; their height 12 feet 9 inches, with an architrave round them and a cornice at the top, supported at each extremity by a console. The niches on each side of the principal entrance are 4 feet 3 inches wide and 9 feet 10 inches high, and diminish on each side parallel to the sides of the columns. The Attic, which stands over the cornice of the entablature, is 5 feet 9 inches high, with a dentil cornice and three fascias below. The height of the pediment is 7 feet 10 inches from the top of the cornice on the attic. The height of the pedestal, from the bottom of the pediment to the top of the columns round the belfry, is 7 feet 10 inches. The columns and entablature round the belfry are 20 feet 10 inches high, and are similar to those in the portico; the wall which is seen between the columns, is rusticated above the two plinths. The apertures in the belfry for letting out the sound, are 4 feet 2 inches wide, and 11 feet 3 inches high.

The part where the dials of the clock are placed is of an octagonal form; its height, including the two circular steps from the top of the cornice, round the entablature of the belfry, to the top of the cornice, above the dials, is 9 feet 10 inches. There are four dials in it at right angles to each other, and four small apertures in the diagonal faces, each 3 feet wide and 4 feet high, filled in with perforated luffer boarding in the form of scales.

The part over the dials above the two circular steps, is of an octagonal form; with eight columns supporting an entablature. Its height, including the two circular steps at the top of the entablature, is 15 feet 8 inches. The diameter of each column is 1 foot $5\frac{1}{2}$ inches and 11 feet 7 inches high; the entablature 2 feet 7 inches. The height of the small pediments above the entablature is 1 foot 9 inches, with a honey-suckle betwixt each.

The height of the spire above the top of the pediments to the top of the cross, is 44 feet $9\frac{1}{2}$ inches, and this potion is ornamented with scales to the height of 28 feet 10 inches. The whole height of the steeple, from the ground to the top of the cross, is 152 feet.

Description of the *Flank Elevation*.—The whole extent of this front, including the projecting porticos on the bottom line of the entablature, is 166 feet 3 inches. That between the two extreme half antae, on each side of the bows, is 146 feet 8 inches; and the plain part between the bows, is 88 feet 2 inches. Each of the bows is 26 feet 3 inches. The height, from the top of the steps to the top of the sills of the lower windows, is 3 feet 8 inches. The lower windows are 5 feet 2 inches wide, diminishing a little at the top, and their height is 4 feet 10 inches. The height between the under side of the lintel of the lower windows and the top of the sill of the upper windows is 6 feet 7 inches. The height of the windows above is 9 feet 6 inches; and the breadth, at the bottom, 5 feet, diminishing to the top about $3\frac{1}{2}$ inches. The height from the under side of the lintel of the upper windows, to the lower line of the entablature, is 4 feet 5 inches. The height from the ground to the top of the roof is 50 feet $7\frac{1}{2}$ inches. The frames of the windows to be of metal. All the ornaments on the exterior of this building may be of terra-cotta, or of stone, if built in a country where both labour and stone are cheap.

Plate 28.—*Ground Plan* and *Elevations* of a *Chapel*.—A, represents the *Porch*, recessed within two columns, 26 feet 4 inches by 4 feet 6 inches. B, an elliptical *Vestibule*, with pilasters and niches, lighted from the top, 27 feet 3 inches by 16 feet 10 inches. D and C, *Side Staircases* to gallery, 25 feet by 13 feet; with a circular staircase in one corner, leading to the children's gallery and tower.

The size of the interior body of the chapel is 83 feet by 58 feet. The principal passage, representing the free seats, is 8 feet within the clear of the pew-doors. The side passages are each 3 feet to the front of the seats next the walls. The pews are 3 feet wide; the seats 1 foot; the book-desk $5\frac{1}{2}$ inches; and the doors 1 foot 7 inches.

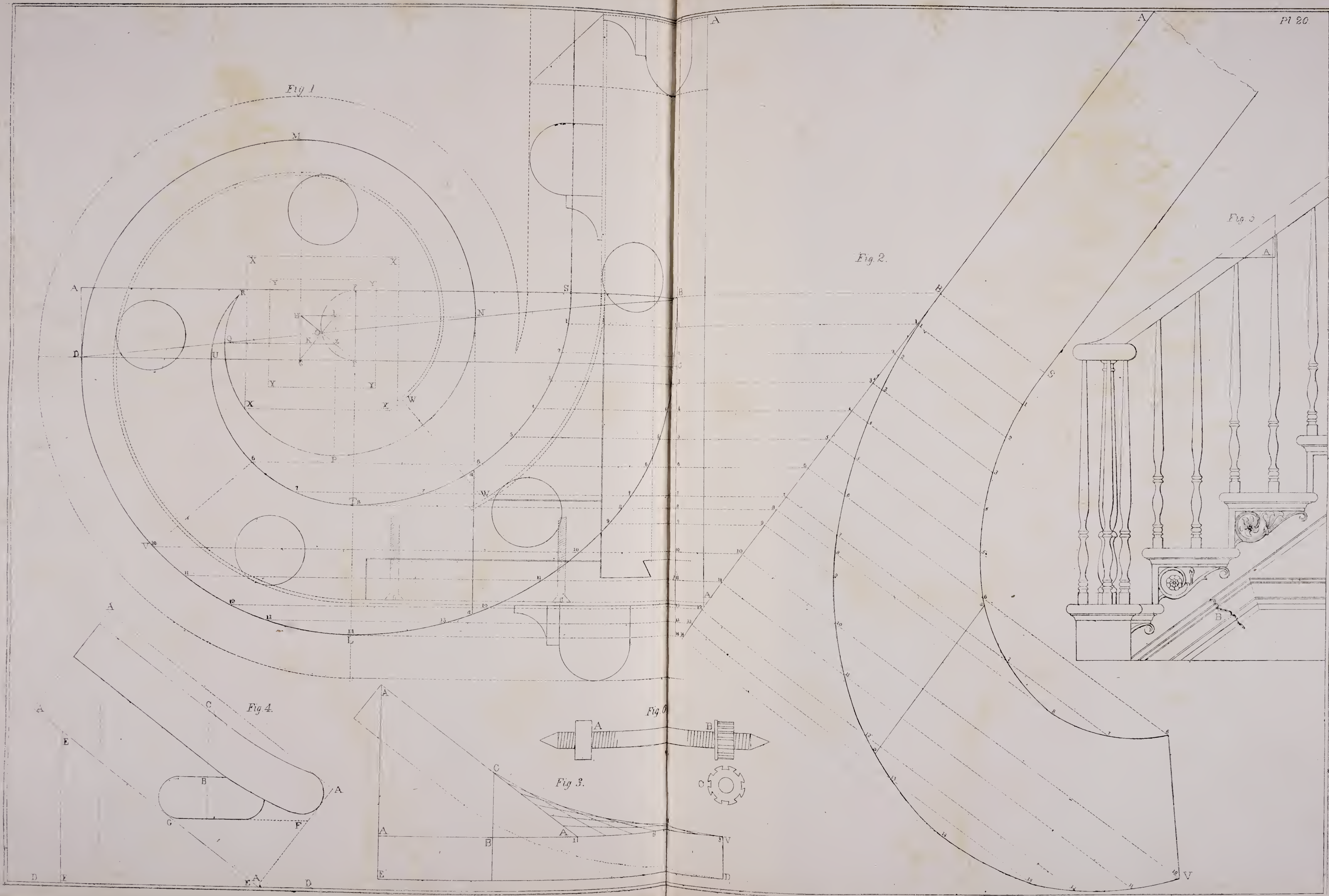
The *Pulpit*, [*n*] is of an hexagonal form, with stairs ascending up to it. *o*, *Reading-Desk*, with clerk's seat in front.

The little black circles represent the columns which support the gallery. The *Communion-Place* [H] is of an elliptical form, and raised one foot high. E, *Vestry-Room*, 18 feet 6 inches by 13 feet, with a fire-place, and small closets in the angles. The *Strong Closet*, [*e*] 6 feet 2 inches by 5 feet 2 inches. *Water-Closet* [*g*] 6 feet by 3 feet 9 inches, the mean prpportion. F, *Robing-Room*, with fire-place, and closets of the same dimensions as Vestry-Room. G, entrance to the vaults.

Description of the *Front Elevation*.—This building is in the style of the *Grecian Doric*. The extent of its front, to the extremities of the pilasters, is 66 feet. Its height, from the ground to the top of the cross, is 112 feet. The entrance, or door, is raised 2 feet 8 inches above the level of the ground, ascended by 5 steps of rather more than 6 inches rise, which are continued all round the building. The opening of the door is 7 feet 3 inches in the clear at bottom, and 6 feet 10 inches at the top; diminishing about one-seventeenth part of the breadth. The door is of oak; it is divided into eight panels, and opens in two halves, to the height of the bead betwixt the third and fourth panel; and is hinged to a vertical bead, which runs up by the side of the architrave of the door. The architrave is about two-ninths of the breadth of the door. That part of the architrave which extends across the top of the door, is a little less. Over the architrave is a Cornice and Pediment, with an ornament at each corner, supported at each extremity of the cornice by a console. Over the door is a pancl, which may be filled in with bas-relief, or an inscription.

The columns are 31 feet 5 inches high; their diameter at bottom 5 feet $2\frac{1}{2}$ inches, and at top 4 feet. The pilasters are of the same width as the top of the column. The mouldings of the caps are similar to those of the Monument of Th asyllus. The lower windows betwixt the pilasters, are 5 feet 5 inches wide at bottom, and at top 5 feet $1\frac{1}{4}$ inches. The windows above are of the same width at bottom as those below, and at top 5 feet $3\frac{1}{2}$ inches. The architraves are 1 foot $1\frac{1}{2}$ inches with a break at top, of about 2 inches. The bars of the windows are of metal. The sills of the windows are $11\frac{1}{2}$ inches.

The height of the Architrave, Frieze, and Cornice, is 8 feet. The breadth of the triglyphs is 2 feet 5 inches. The height of the Pediment, from the top of the



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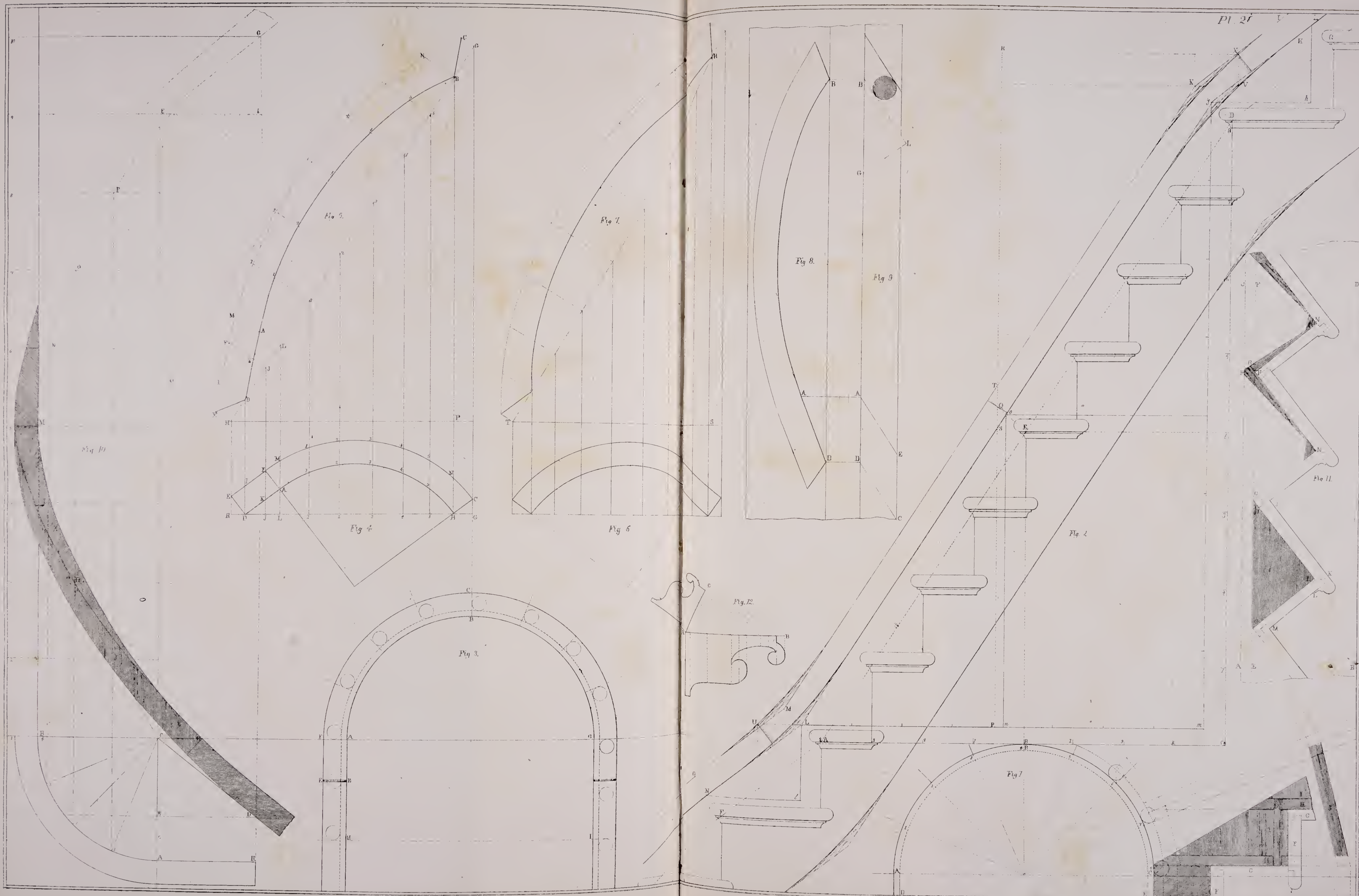
Pl. 21



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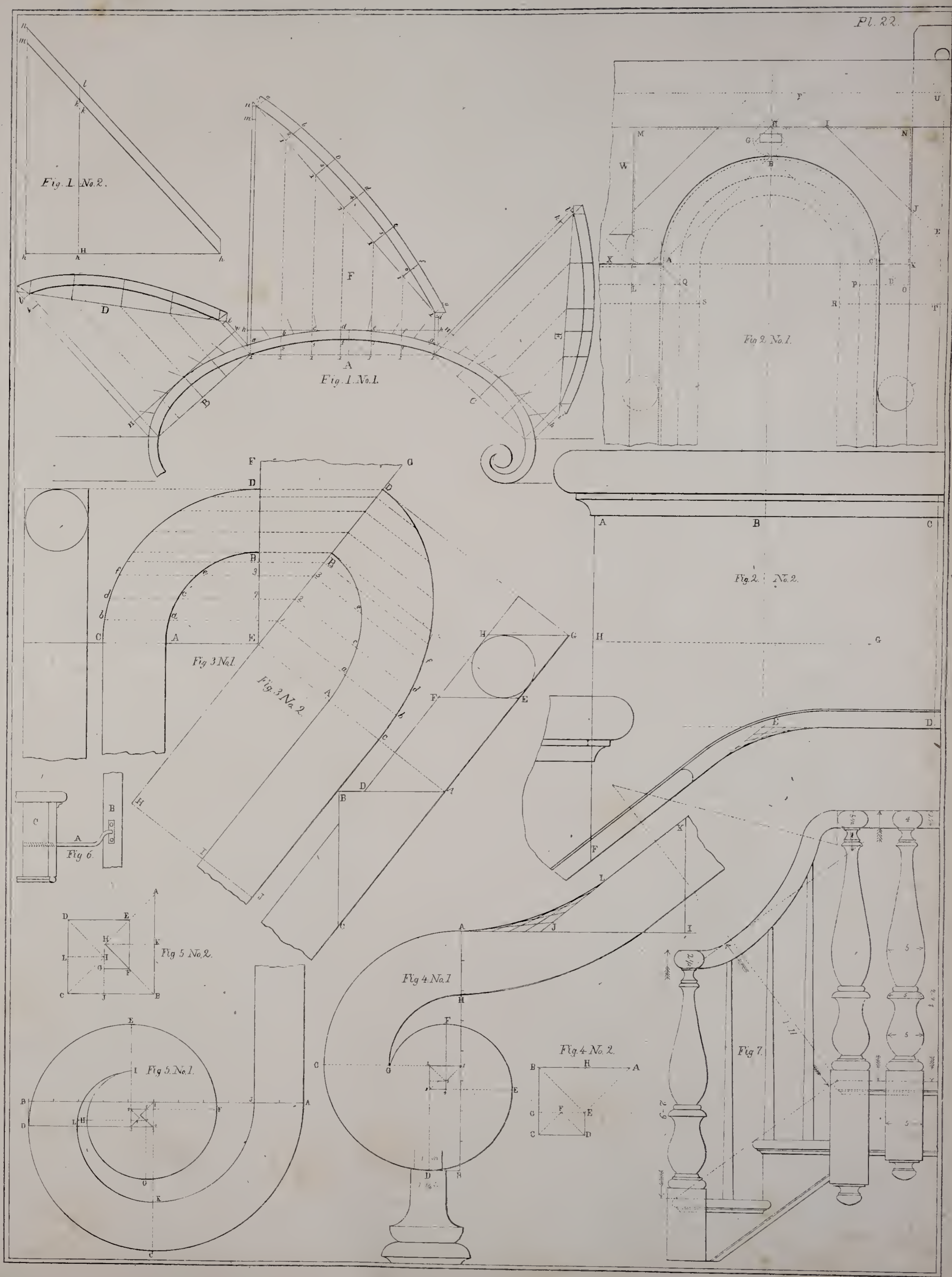
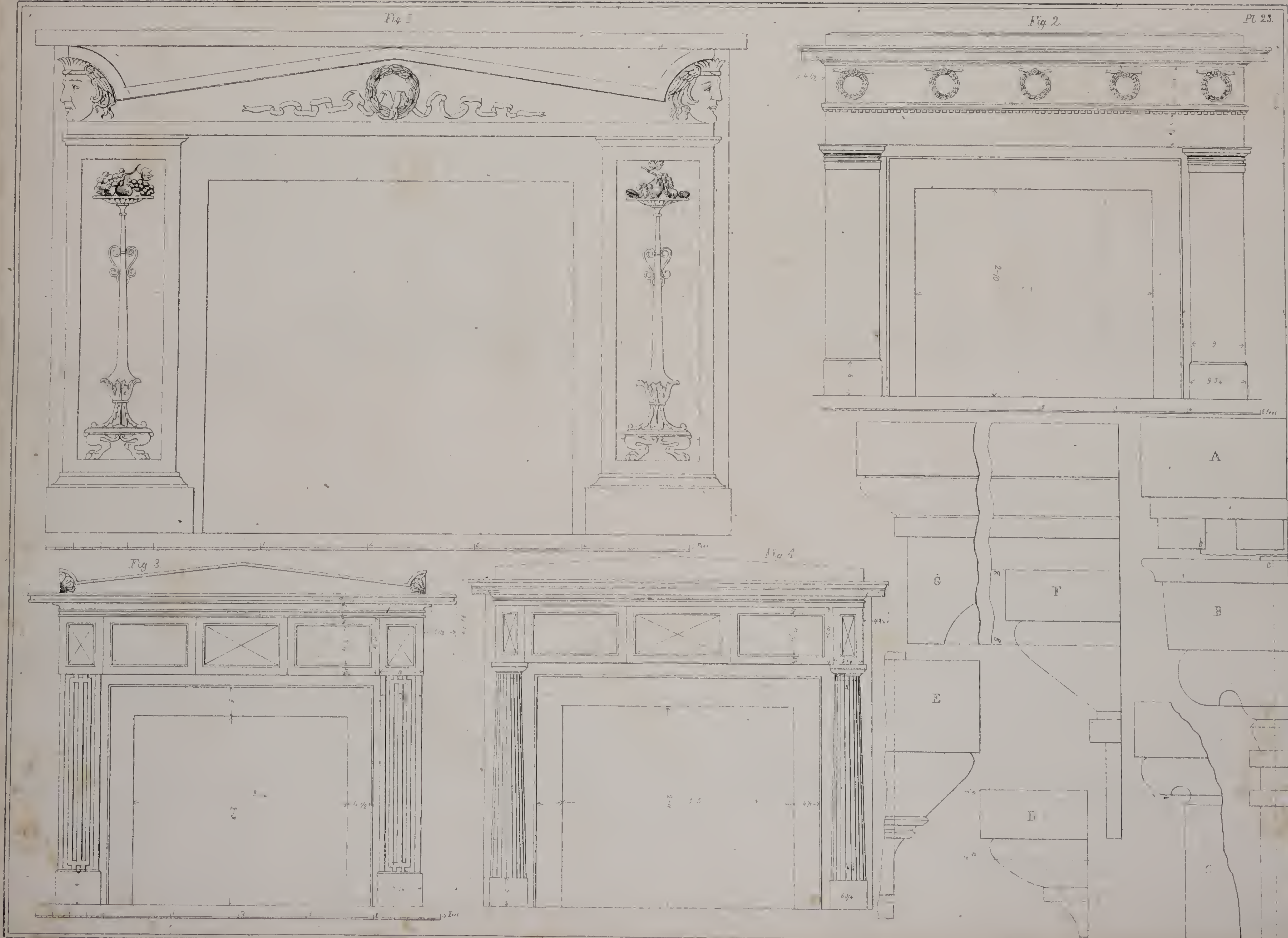


Pl. 22.
Fig. 1. No. 2.
Fig. 1. No. 1.
Fig. 2. No. 1.
Fig. 2. No. 2.
Fig. 3 No. 1.
Fig. 3 No. 2.
Fig. 4 No. 1.
Fig. 4 No. 2.
Fig. 5 No. 1.
Fig. 5 No. 2.
Fig. 6.
Fig. 7.



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Pl 24.

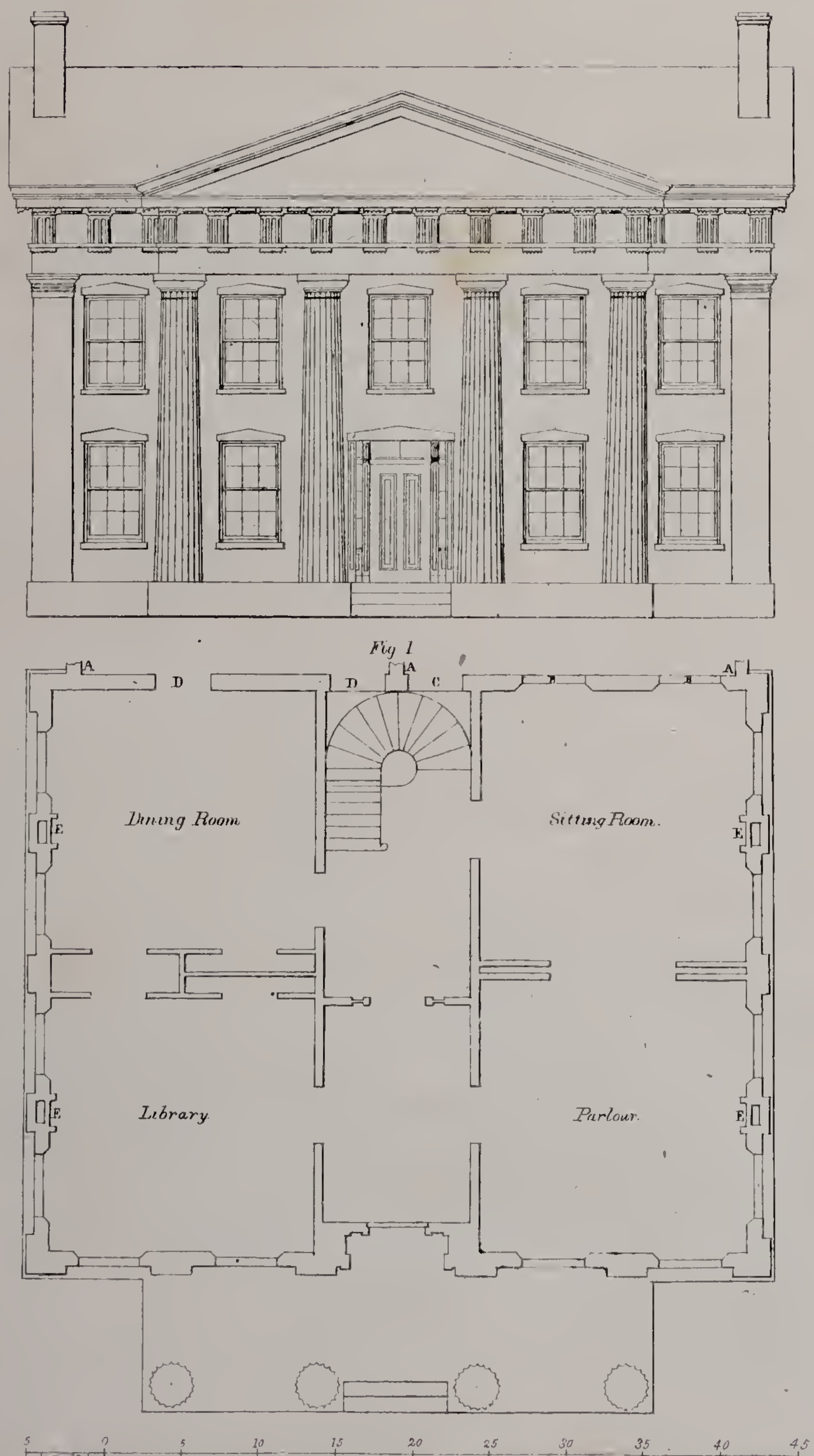

Fig 1

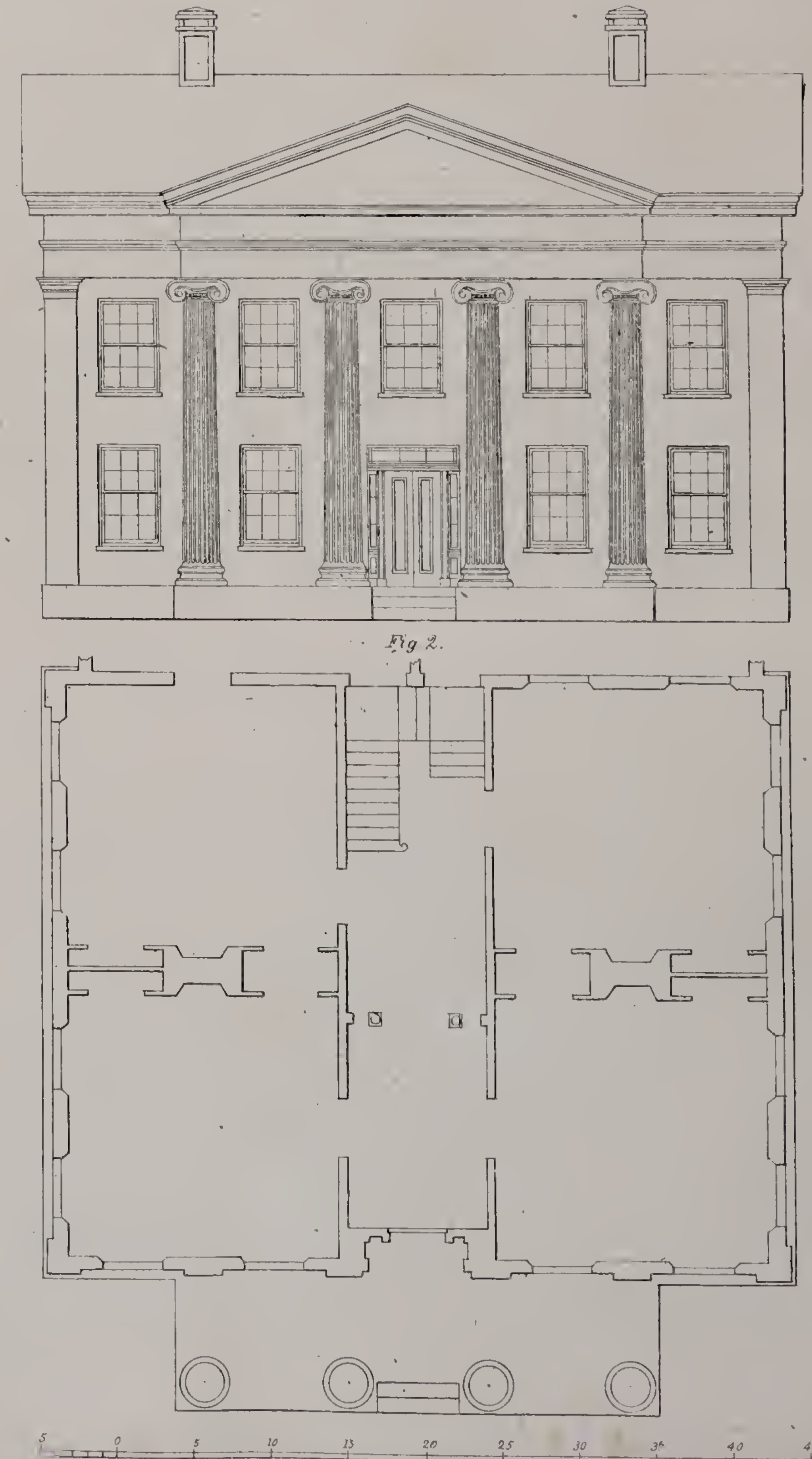

Fig 2.

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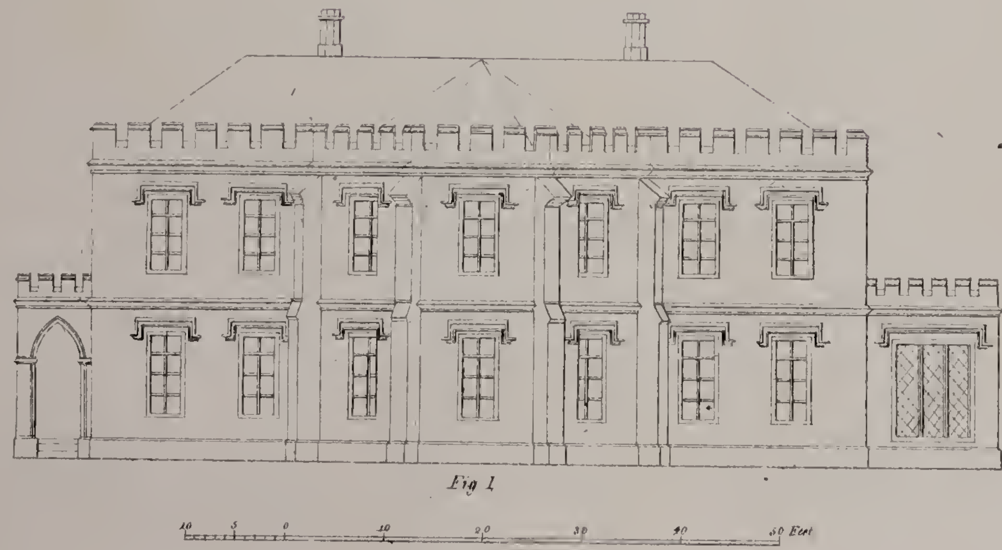


Fig. 1

Fig. 3

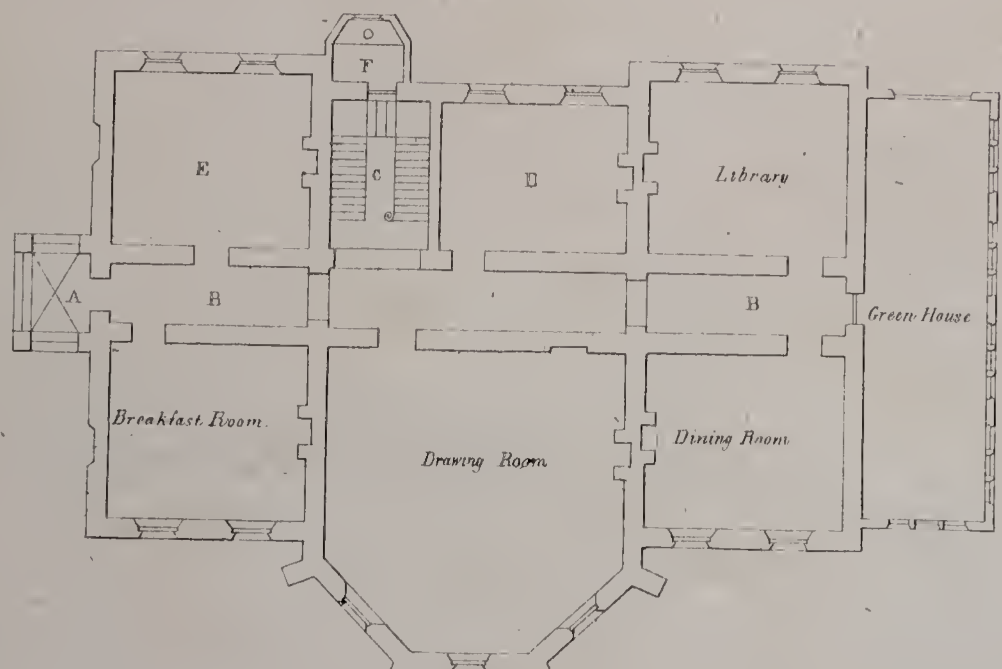


Fig. 2

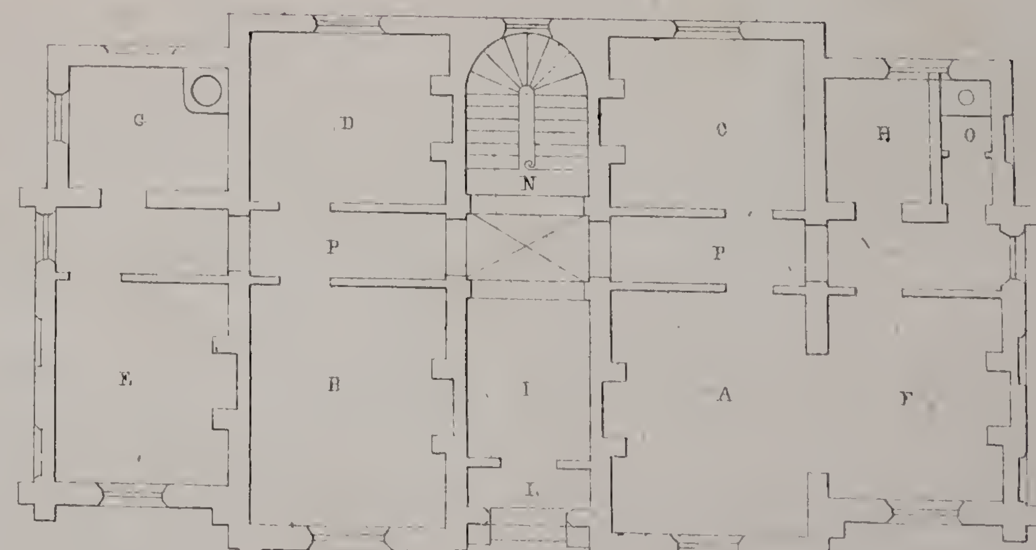


Fig. 4

Fig. 1.

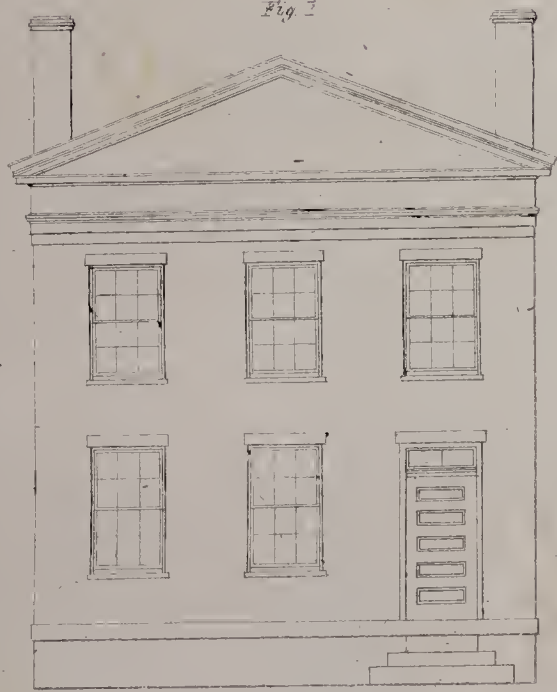


Fig. 2.

Fig. 3.

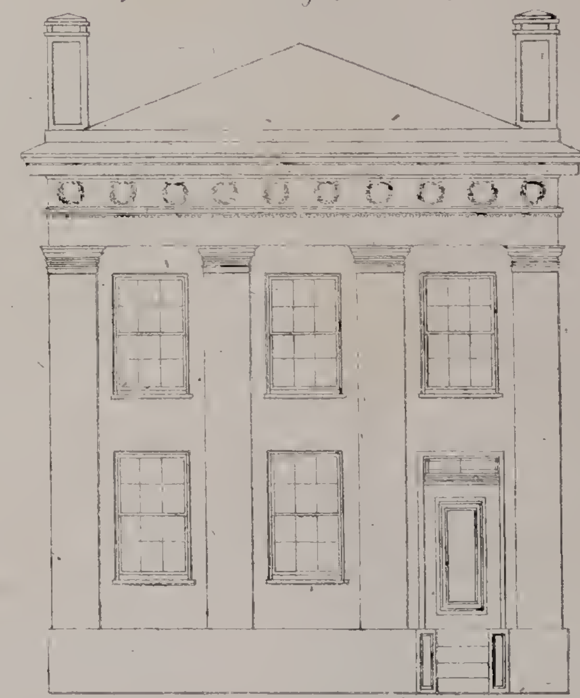


Fig. 4.

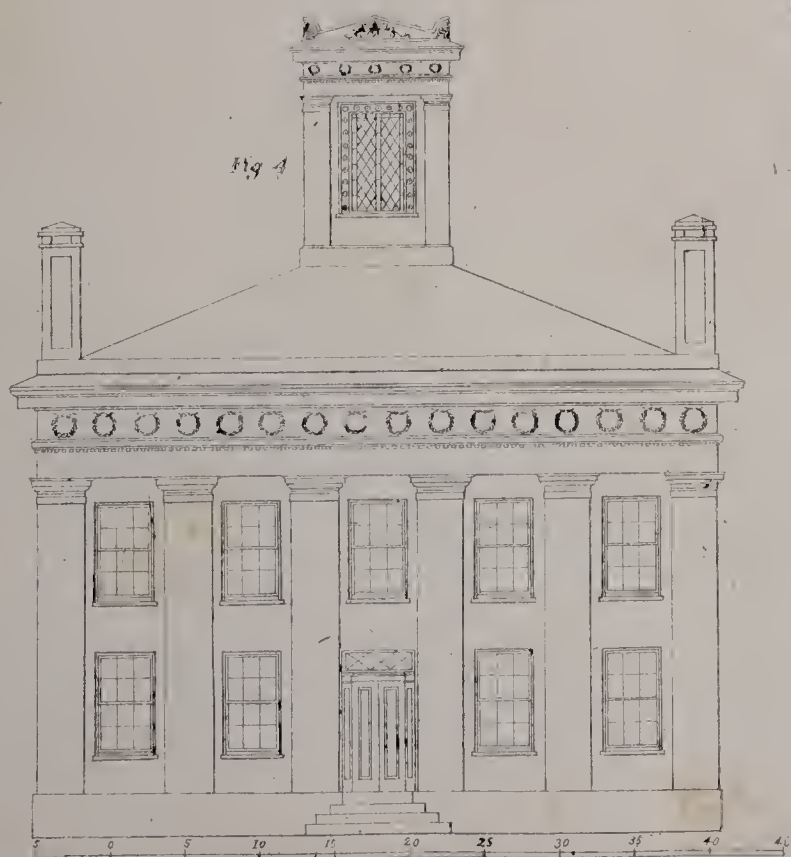
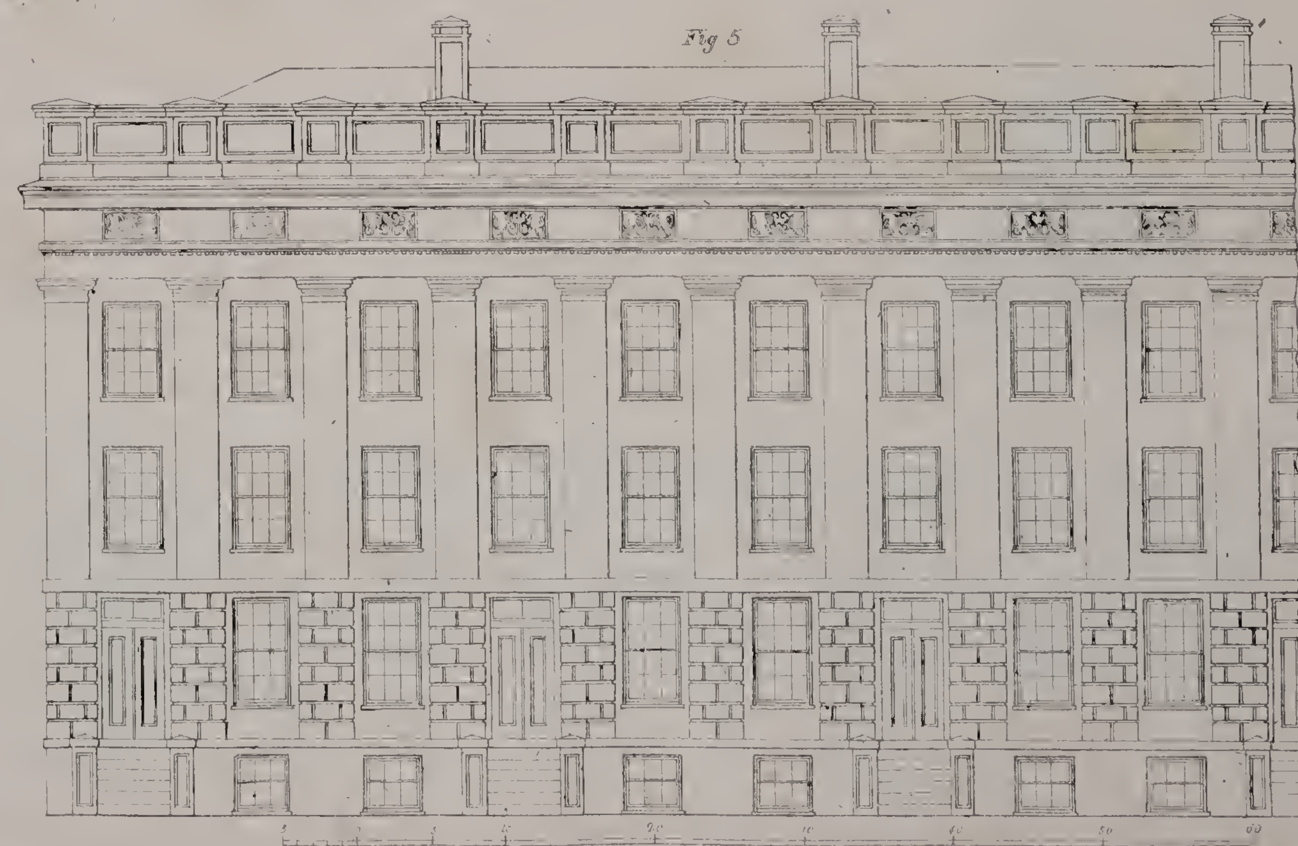


Fig. 5.



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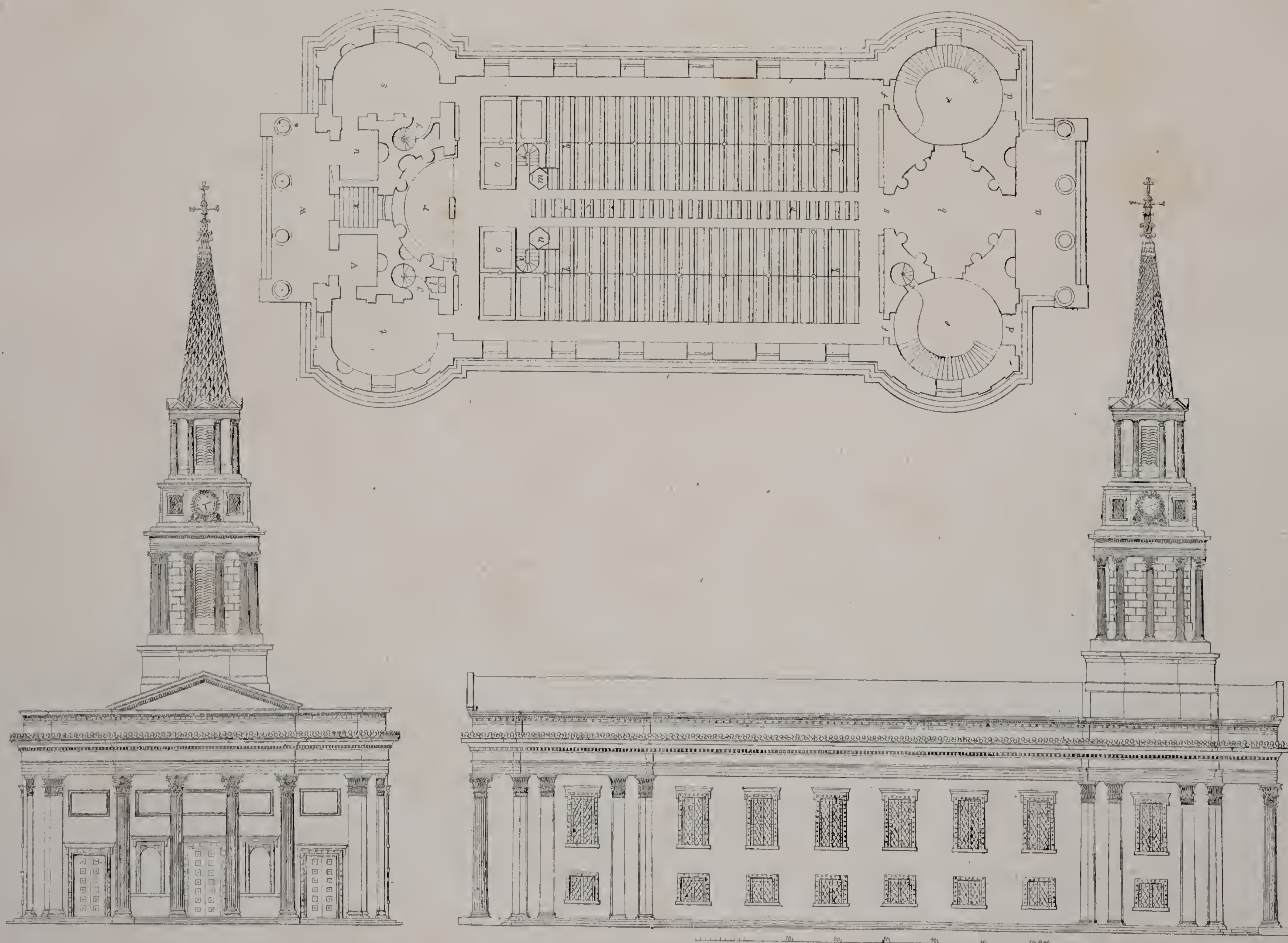
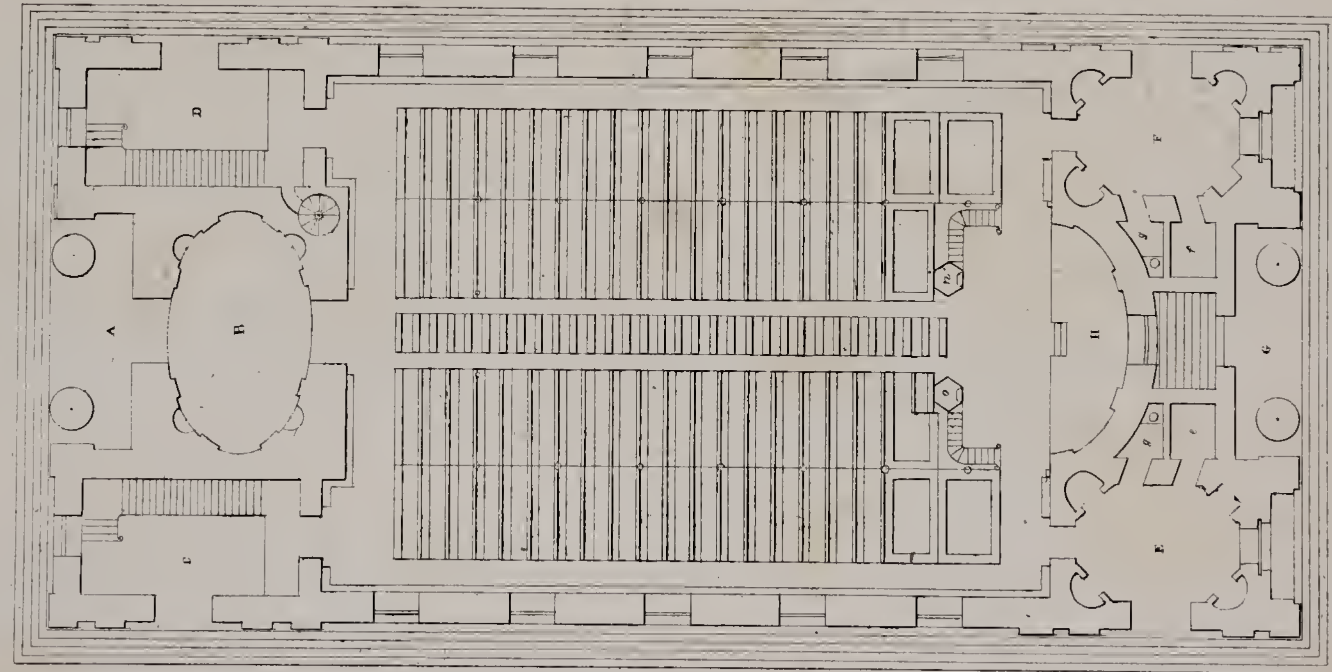
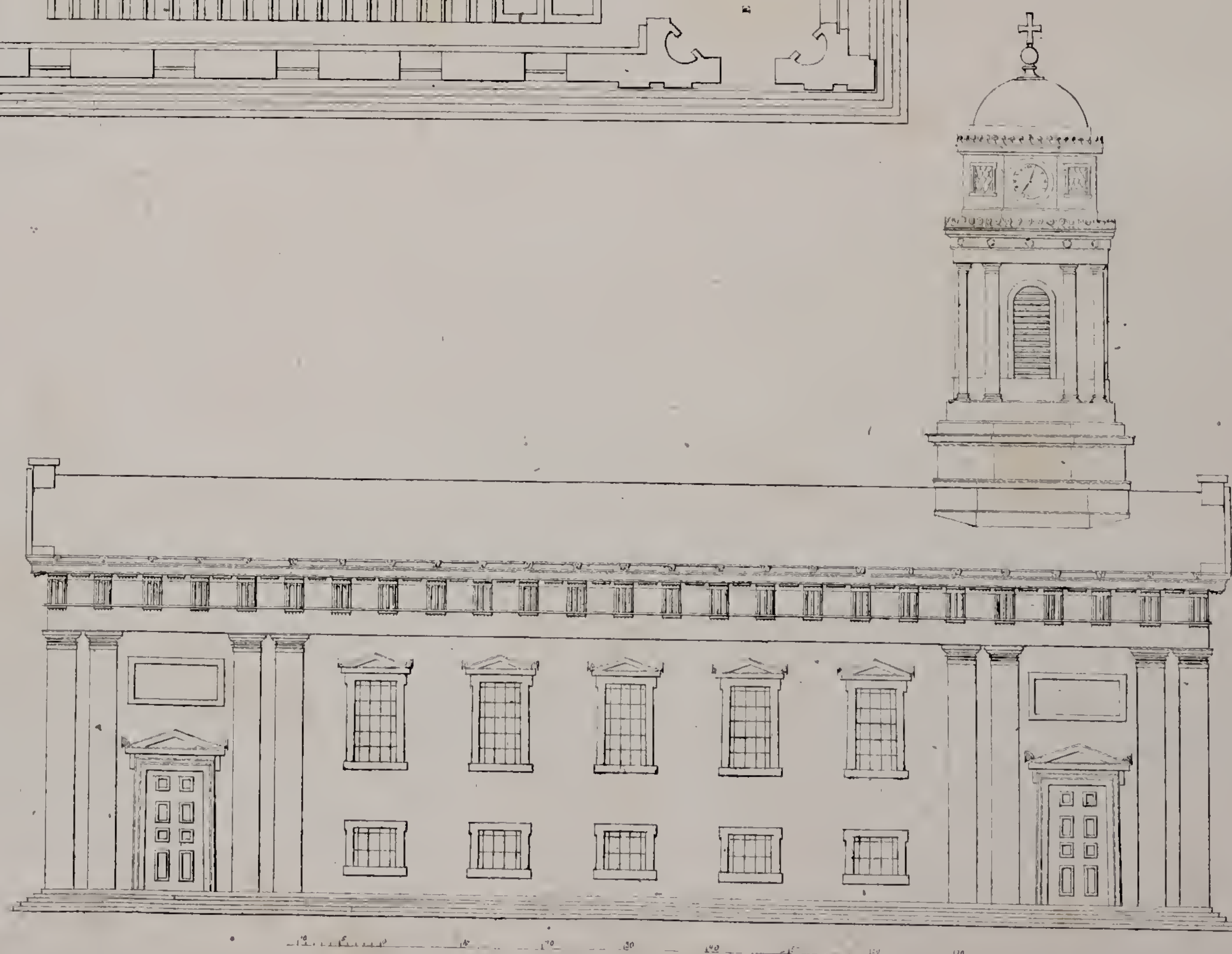
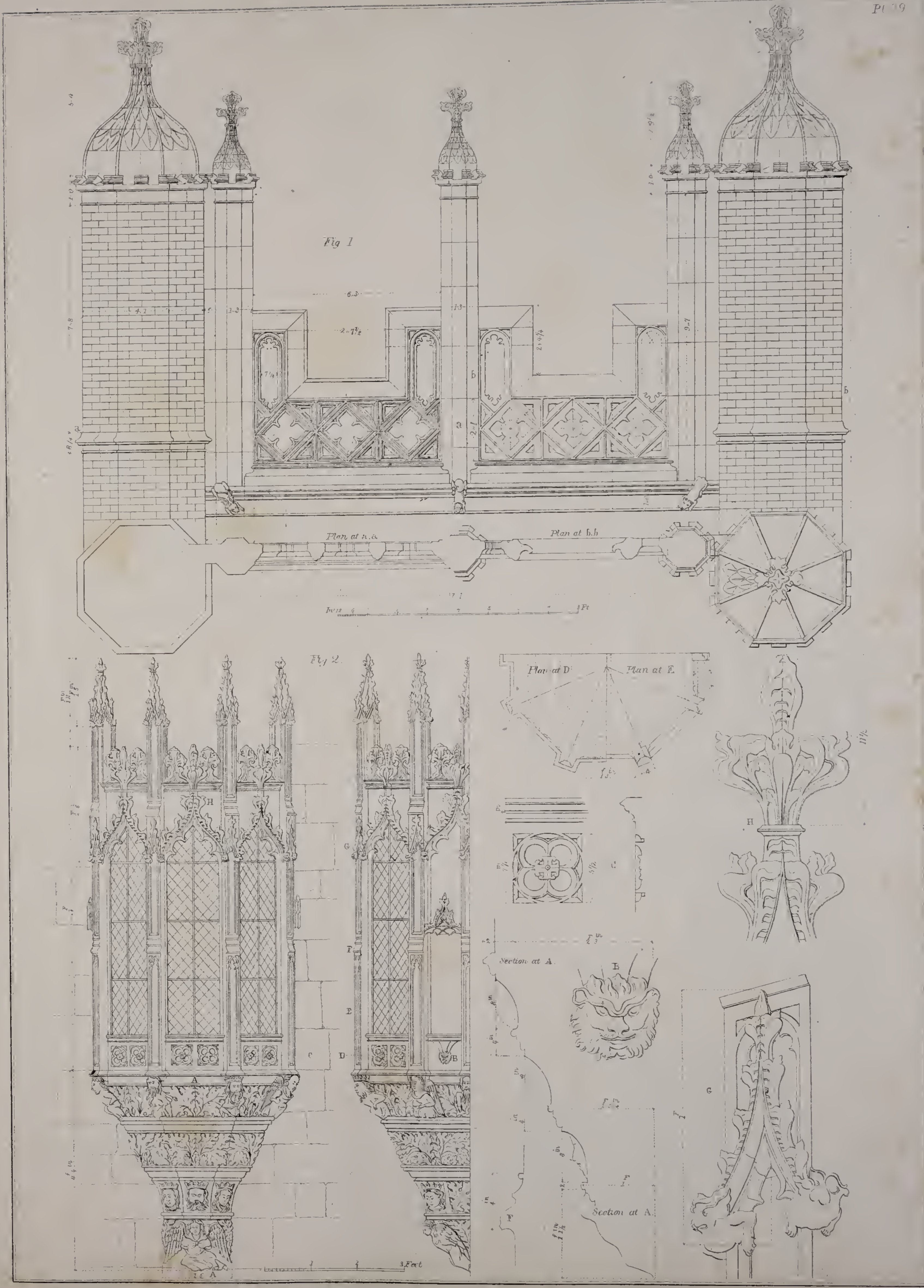


Pl.28





Pl. 39



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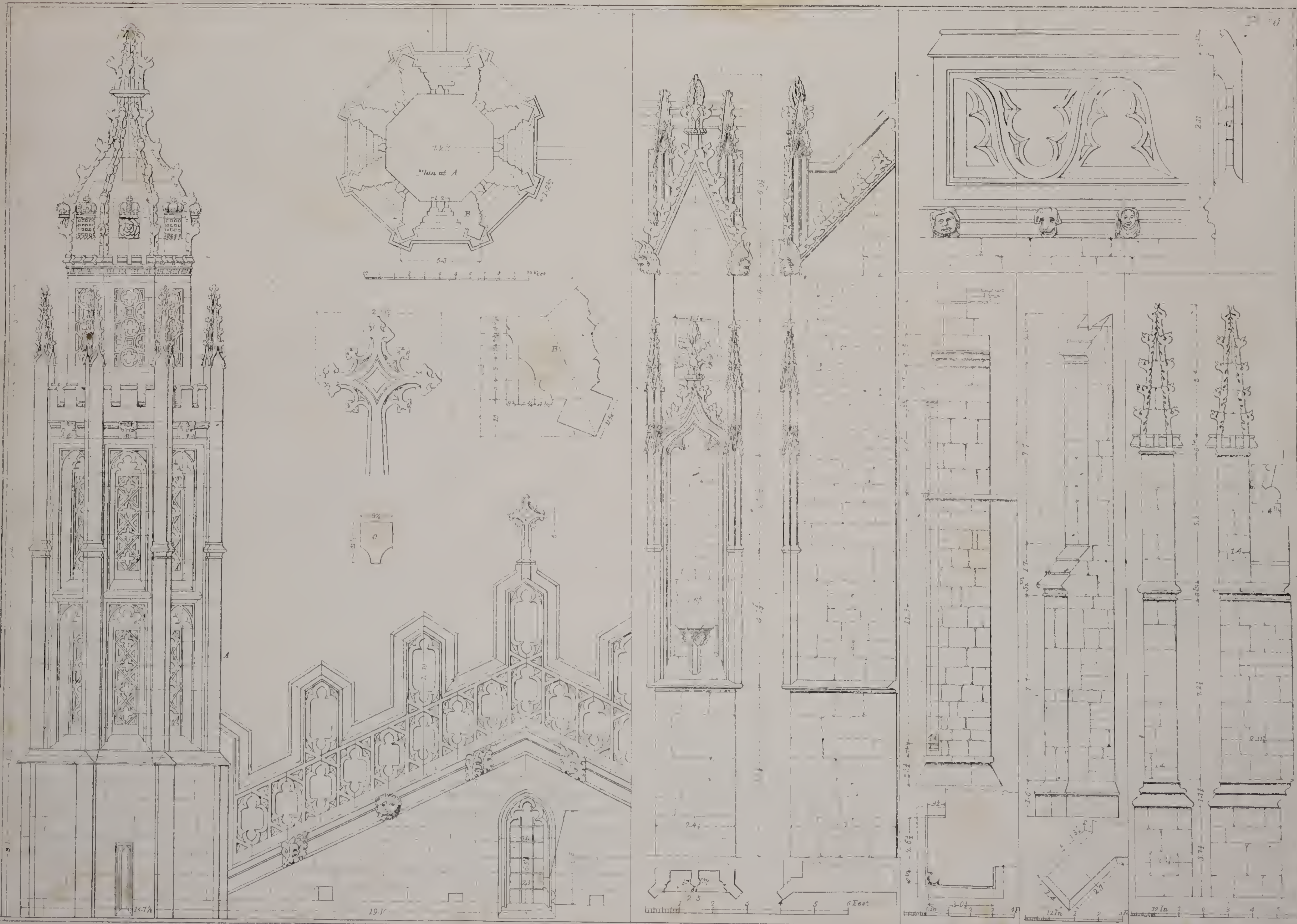

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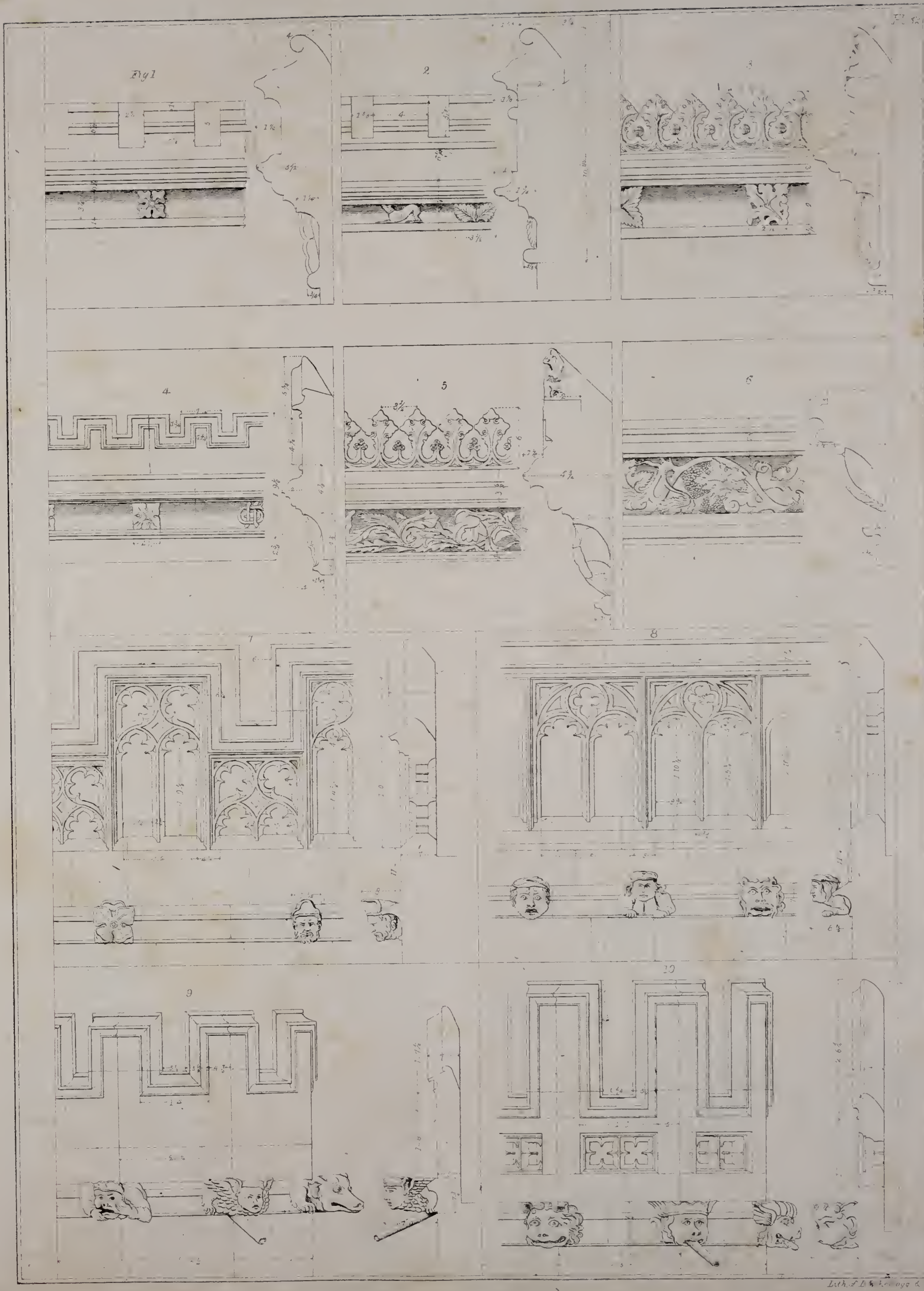
Fig 1
2
3
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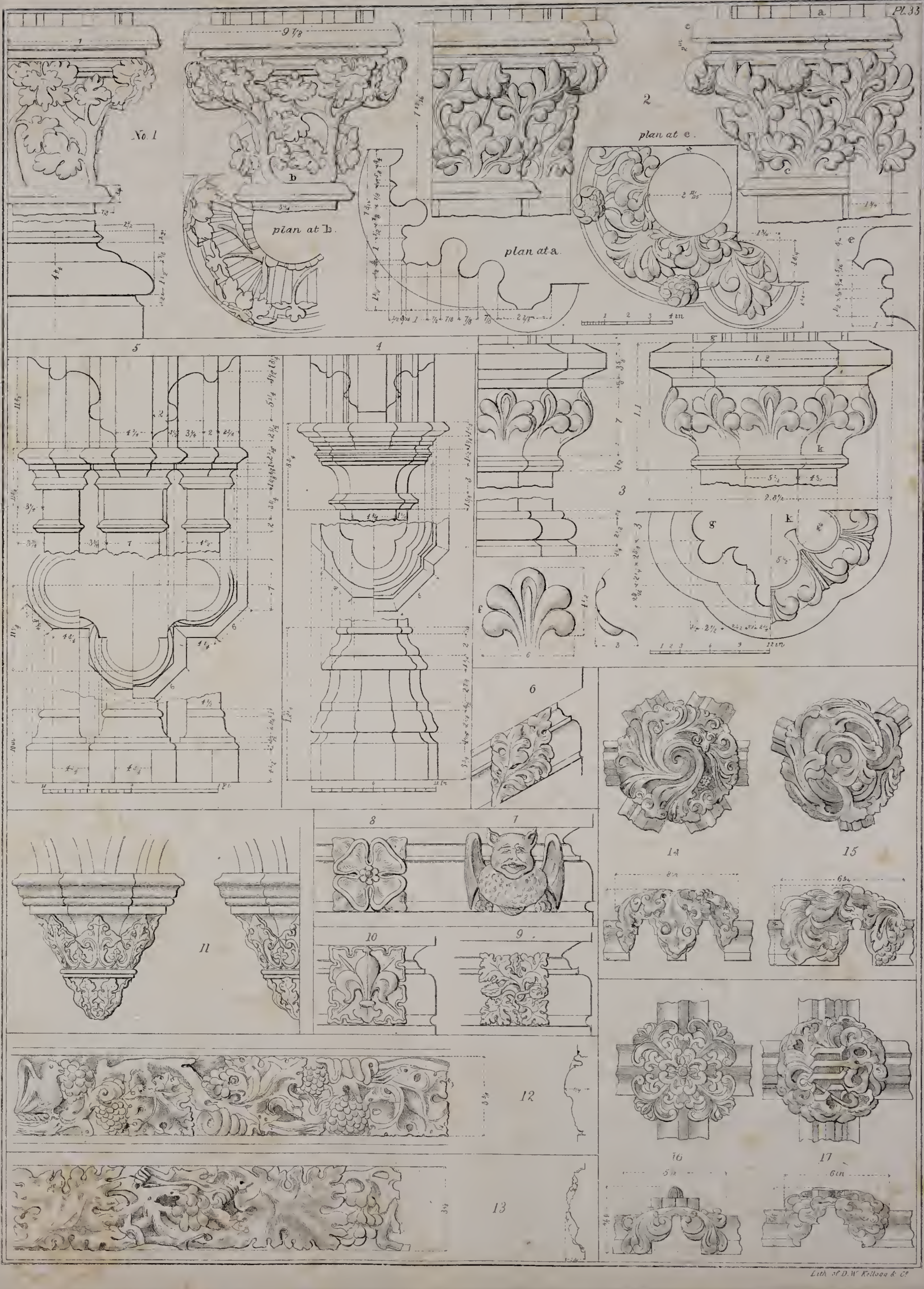

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cornice to the top of the Cymatium, is 11 feet 6 inches: on the top, and at each extremity of which are placed *Acroteriæ.*

That part which projects beyond the bottom of the Cupola, is to admit light into the vestibule by means of six small windows in the faces of the pedestal of the Cupola, which is concealed within it. The windows in the belfry are 4 feet 5 inches wide, and 11 feet 6 inches high, to the top of the arch. The aperture of the latter is filled in with horizontal luffer-boarding. The pilasters round the belfry are 16 feet 6 inches high, and 1 foot 11 inches wide; the moulding in the caps, are the same as those in the front; the bars are similar to the attic base; the height of the entablature is 4 feet 2 inches, with wreaths in the frieze, and ornaments above the cornice.

The part above the belfry, which contains the clock-work, is of an octagonal form, with a cornice and continued ornament above, similar to that on the top of the cornice of the Monument of Lysicrates. (*Orders Plate* XXIII.)—The faces of the octagonal part is filled in with four dials, at right angles to each other, and four small windows, 3 feet 6 inches wide, and 3 feet high: the apertures of which are filled in with luffer-boarding, in the form of scales. Above the octagonal part is a circular dome raised upon a step, with a ball and cross over it.

Description of the *Flank Elevation.*—The whole length of this elevation is 142 feet between the two outer pilasters. That part between the pilasters, wherein the windows are, is 79 feet. The heights of the doors, pilasters, entablature, and cupola, are the same as those in the front elevation. The lower windows are 5 feet 4 inches wide, and 4 feet 7 inches high, and diminish at the top one inch and a half: the windows above these are 5 feet 4 inches wide, and 9 feet 8 inches high, and diminish 3 inches and three quarters at top. The architraves are 1 foot 1 inch and a half, with a break at top of about 2 inches on each side: over the top of the architrave is a cornice and a pediment, with a honey-suckle at each extremity.

GOTHIC ARCHITECTURE.

Although much has been said against this style of building, yet it must be acknowledged that we are indebted to Gothic Architects for many improvements in our present mode of construction. We find a lightness in Gothic designs, and a boldness in their execution, which the Greeks and Romans never attained, or the moderns duly appreciated till within the last century. Formerly every design which did not perfectly accord with Grecian or Roman models was censured as barbarous and unworthy the attention of modern architects: but an abatement of that enthusiastic zeal for classic structure which for a time universally prevailed in Europe, opened the way for a revolution in architecture.—Within the last century, many Gothic buildings have been erected in Great Britain that are admirable both for the art with which they were designed, and the taste with which they were executed. The English Architect has studied the antiquities of his own country as well as those of Greece and Rome; and the Gothic abbeys, cathedrals, and baronial castles of his ancestors, are no longer considered as void of ornament or convenience. The ancient architecture of England is at the present period held in high esteem, and though the edifices of "olden times" are fast falling to ruin, yet a remembrance of them is preserved in the beautiful buildings which are erected in every part of the country, for worship; education, benevolence, and the accommodation of mankind.

It would afford me much satisfaction to speak of Gothic Architecture at considerable length, but the limits of the present work will not permit me to enter its history. If any person desires to be thoroughly acquainted with this style of building, he should examine the works of A. Pugins, and where the subject is extensively treated of, and from which, the specimens here given were taken.

PLATE 29.—FIG. 1.—PLAN AND ELEVATION OF THE OPEN PARAPET AND TURRETS OVER THE WESTERN ENTRANCE OF HAMPTON COURT PALACE.

A succession of three gates with towers over them, leads from the western front to the interior of the place, where king William's buildings join to the ancient courts.

The embattled parapet here represented, has a very light airy effect; the tracery being all pierced, as is shown on the plan. The pinnacles, formed into slender copies of the turrets, instead of shooting up into pointed spires, as in earlier buildings, are peculiar to the latest period of Gothic taste. The same sort of pinnacle is seen upon the battlements of the hall, and in other parts of the palace.

Fig. 2.—Plan and Elevation of an Oriel Window, John of Gaunt's Palace, Lincoln.

The curious investigator of domestic antiquities will not fail to appreciate this remnant of a once splendid habitation. In delineating its form and enrichments, most scrupulous care has been taken to give a full and exact portrait: such an interesting *specimen* being very rarely seen. The elevations of the front and profile exhibit no more than what actually exists, except the tops of the pinnacles, which being broken off level with the foilage between them, are here restored in style corresponding with the ornaments; it may be also proper to notice, that three lights, which, no doubt, were once *cloised well with roiall glass.*—[Old Romance of the Squire of Low Degree] are now blocked up, and the mouldings partly obscured by plastering. The bracket which sustains the frame of the window, is covered with sculpture, divided by plain mouldings into four tiers.—The lowest of these consists of a single figure, representing an angel, serving as a bracket. The next has three marks, or faces; viz. at the right, a queen, in front, a king; on the left, a bearded man, rather defaced. Above these runs a course of foilage, displayed in large leaves. The uppermost division has six figures, one beneath each of the little abutments, which guard the angles of the window. Against the wall on the right hand, is a man covered with hair, and with a long beard, holding a bird in one hand, in the other a branch; next to him, an angel playing upon a cithern: Then a king with a long beard, on his left hand, an old man clothed in a mantle; beyond this figure, a youth in a close robe; and lastly, against the wall, a bearded man, rather disfigured. A plan, or horizontal section, taken at two different heights, is drawn in the upper part of the Plate, D, E; below is an enlarged section of the bracket, showing the projection of all its mouldings, with their several measurements. These details are also represented separately, with letters referring to the elevations B. Head upon the little bracket of one of the niches, in the two blank lights. CA panel, with section, of those beneath the lights. F loping of a buttress. H Finial rising from the crockets over every light. All examination of the interior of the oriel is unfortunately obstructed by a modern chimney, built up within it.

PLATE 30.—TURRET AND GABLE OF KING'S COLLEGE CHAPEL, CAMBRIDGE.

The chapel of King's College, Cambridge, has been as much celebrated as any Gothic building in Europe, so that nothing need be said here respecting the general character of its architecture. At the left of this Plate represents the upper part of one of the four lofty turrets which adorn its angles; with a portion of the adjoined gable. The turrets are corniced up without any ornament as high as the battlements of the roof, above which they are beautifully decorated, as is shown in the Plate. The character of these decorations deserves a particular examination; the projections and recesses are bold and decisive, producing a clear and distinct effect even at the great height they are placed. The fretted compartments in the sides are pierced quite through the walls, giving light to the interior, and making the turrets appear very rich on the outside. The armorial badges and crowns refer to Henry VII., who contributed very largely to the completion of the structure, though it was not effected in his day. A, Plan, taking in the lower compartments of the elevation. B, one corner of the same on a larger scale. C, Mullion. An enlarged elevation of the Cross upon the crest of the gable.

To the right of this Plate are designs of buttresses and one battlement, as taken from Oxford.

Plate 31.—Exhibits a number of designs of windows, as taken from different Churches and other Edifices from Oxford.

Plate 32.—Figures 1, 2, 3, 4, 5 and 6, are Cornices from Westminster Abbey and Henry VII's Chapel.

The sections of these six specimens are drawn on a larger scale than the front views, the better to show the turns of the mouldings.

Figs. 1, 2, and 4 have *crests* of small battlements above the cornices, and their casements are studded with small ornaments of *entail,* set at intervals.—Figs. 3 and 5 have *crests* of leaves, arranged to a pattern of great elegance, and which was very frequently used in the 15th century. The *crest* of Fig. 6, appears to have been broken off. This specimen being of wood, the *entail* is worked on a thin piece, inserted afterwards into the *casement.*

Figures 7, 8, 9 and 10, are specimens of Parapets from St. George's Chapel, Windsor.

The upper roof of this magnificent structure is guarded by a straight parapet pierced in compartments; whilst the aisles have an embattled parapet, which is also pierced. These last four examples that are exhibited on this Plate, the cornices is studded with heads, grotesque and ludicrous, agreeably to the fashion of the age in which the building was erected, when exhibitions of masques and mummeries entertained the gravest and most polished characters, no less than the lowest classes of society.

The elevation and corresponding section of each of these *specimens* seem to require no explanation.

Plate 33.—Exhibits a number of designs for Capitals and bases of Pillars, Brackets and sculptured Ornaments, &c.

No's. 1, 2 and 3, shows three specimens of foilated capitals, with their respective bases. It may be useful to observe, that in designing a capital of this sort, the *corps* or solid part, ought to be proportioned before any ornaments of leaves, flowers, &c. are applied; a small *neck-mould* is required to distinguish the capital from the shaft, and over the leaves a *hood-mould* such as that marked *e* in the second specimen. By comparing the letters on the sections with the corresponding ones on the elevations, the whole will be clearly explained. Nos. 1 and 2 are of the latter end of the 14th century; No 3 of the early part of the same, or the end of the 13th.

Nos. 4 and 5.—Exhibit two specimens of Capitals and Bases, of a plain description, they being finished with mouldings only, without foilage. The mouldings are expressed in feet and inches by figures, and they clearly show the manner in which the arches are set upon the pillars, and will be found carefully marked, and the size and form of each pillar.

Nos. 6, 7, 8, 9 and 10, are enrichments of cornices.

No. 11, a design of a corble; which is sculptured with leaves, after the form of the capital of a column.

Nos. 12 and 13, are specimens of running foilage, fruit, &c.

Nos. 14, 15, 16 and 17, are specimens of *knots* on the intersections of ribs, in roofs, these are all shown in profile, as well as in front. In 17, the letters IHS, an abbreviation of the sacred name Jesus, are wrought amongst the foilage.

APPENDIX—DECIMAL FRACTIONS.

DECIMAL FRACTIONS.

Fractions or *Vulgar Fractions* are expressions for any assignable part of an unit; they are usually denoted by two numbers, placed one above the other, with a line between them : thus, $\frac{1}{4}$ denotes the fraction one-fourth, or one part of four of some whole quantity, considered as divisible into four equal parts. The lower number 4 is called the *denominator* of the fraction, shewing into how many parts whole or integer is divided ; and the upper number 1, is called the *numerator*, and shews how many of those equal parts are contained in the fraction. And it is evident that if the numerator and denominator be varied in the same ratio, the value of the fraction will remain unaltered : thus if the numerator and denominator of the fraction $\frac{1}{4}$ be multiplied by 2, 3, or 4, &c. the fractions arising will be $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, &c. which are evidently equal to $\frac{1}{4}$.

Decimal Fraction is a fraction whose denominator is always an unit with some number of ciphers annexed, the numerators of which may be any numbers whatever ; as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{15}{1000}$, &c. And as the denominator of a decimal is always one of the numbers 10, 100, 1000, &c. the inconvenience of writing these denominators may be avoided, by placing a point between the integral and the fractional part of the number ; thus $\frac{3}{10}$ is written 3 ; and $\frac{14}{100}$ is written 14 ; the *mixed* number 3 $\frac{14}{100}$, consisting of whole numbers and fractional ones, is written 3.14.

In setting down a decimal fraction, the numerator must consist of as many places as there are ciphers in the denominator ; and if it has not so many figures the defect must be supplied by placing ciphers before them ; thus, $\frac{16}{100}=.16$, $\frac{16}{10000}$ .0016, &c. And as ciphers on the right hand side of integers increase their value in a tenfold proportion, as .2, 20, 200, &c. so when set on the left hand of decimal fractions. they decrease their value in a tenfold proportion, as .2, .02, .002. &c. but ciphers being set on the right hand of these fractions can make no alteration in their value, neither in their increase or their decrease ; thus .2 is the same as .20 or .200. The common arithmetical operations are performed the same way in decimals, as they are integers ; regard being had only to the particular notation, to distinguish the integral from the fractional part of a sum.

ADDITION OF DECIMALS.

Addition of decimals is performed exactly like that of whole numbers, placing the numbers of the same denomination under each other, in which case the decimal separating points will range straight in one column.

EXAMPLES.

	Miles.	Feet.	Inches.
	34.6	4.30	4.53
	45.18	3.44	127.05
	126.306	12.25	.047
	.004	1.346	35.6
Sum	206.090	21.336	167.227

SUBTRACTION OF DECIMALS.

Subtraction of decimals is performed in the same manner as in whole numbers, by observing to set the figures of the same denomination and the separating points directly under each other.

EXAMPLES.

From	35.345	54.46	2.340	1246.2
Take	3.23	.032	.286	25.164
	32.115	54.428	2.054	1211.036

MULTIPLICATION OF DECIMALS.

Multiply the numbers together the same as if they were whole numbers, and point off as many decimals on the right hand as there are decimals in both factors together ; and when it happens that there are not so many figures in the product as there must be decimals, then prefix as many ciphers to the left hand as will supply the defect.

EXAMPLE I.

Multiply 4.35 by 5.6
4.35
5.6
2.610
21.75
Answer 24.360

In one of the factors is one decimal, and in the other two, their sum 3 is the number of decimals of the product.

EXAMPLE II.

Multiply 0.6 by 0.8.
0.6
0.8
Answer 0.48

EXAMPLE III.

Multiply 4.33 by .04.
4.33
.04
Answer 17.32

EXAMPLE IV.

Multiply .12 by .09
.12
.09
Answer .0108

In each of the factors are two decimals, the product ought therefore to contain 4, and there being only 3 figures in the product I prefix a cipher.

EXAMPLE V.

Multiply .16 by 26.
.16
26.
96
32
Answer 4.16

EXAMPLE VI.

Multiply 33.2 by 2.5.
33.2
2.5
16.60
66.4
Answer 83.00

DIVISION OF DECIMALS.

Division of Decimals is performed in the same manner as in whole numbers ; only observing that the number of decimals in the quotient, must be equal to the excess of the number of decimals of the dividend above those of the divisor.—When the divisor contains more decimals than the dividend, ciphers must be affixed to the right hand of the latter to make the number equal or exceed that of the divisor.

EXAMPLE I.

Divide 14.625 by 3.25.
3.25)14.625(4.5
1300
1625
1625

In this example there are 2 decimals in the divisor, and 3 in the dividend, hence there is one decimal in the quotient.

EXAMPLE II.

Divide 0.35 by 0.7
7).35(.5
.35

EXAMPLE III.

Divide 3.1 by .0062.

Previous to the division I affix a number of cyphers to the right hand of 3.1, which does not alter its value.

.0062)3.100000(500.00
310
0000

Therefore the answer is 500.00 or 500.

EXAMPLE IV.

Divide 9.6 by .06.
06)9.60
Answer 160

Here by affixing a cypher to 9.6 it becomes 9.60, and has then 2 decimals in it which is the same number as is in the divisor, therefore the quotient is an integer number.

EXAMPLE V.

Divide 17.256 by 1.16
1.16)17.25600(14,875
116
565
464
1016
928
880
812
680
580
100

REDUCTION OF DECIMALS.

If you wish to reduce a vulgar fraction to a decimal, you may add any number of cyphers to the numerator, and divide it by the denominator, the quotient will be the decimal fraction; the decimal point must be so placed that there may be as many figures to the right hand of it as you added cyphers to the numerator: if there are not as many figures in the quotient, you must place cyphers to the left hand to make up the number.

EXAMPLE I.

Reduce $\frac{1}{5}$ to a decimal.

5)1.0

Answer .2

EXAMPLE II.

Reduce $\frac{3}{8}$ to a decimal.

8)3.000

Answer 375

EXAMPLE III.

Reduce 3 inches to the decimal of a foot.

Since 12 inches=1 foot, this fraction is $\frac{3}{12}$

12)3.00

Answer. .25.

EXAMPLE IV.

Reduce $3\frac{1}{2}$ inches to the decimal of a foot.

$3\frac{1}{2}=\frac{7}{2}$. this divided by 12 is $\frac{7}{24}$.

Answer, 24)7.000(.291
```
 48
 ——
 220
 216
 ——
  40
  24
  ——
  16
```

EXAMPLE V.

Reduce 1 foot and 6 inches to the decimal of a yard.

Here 1 foot 6 inches 18=inches.

And 1 yard = 36 inches, therefore this fraction is $\frac{18}{36}$;

36)18.0(.5 Answer.
180

If you have any decimal fraction, it is easy to find its value in the lower denominations of the same quantity; thus if the fraction was a decimal of a yard, by multiplying it by 3 we have its value in feet and parts; if we multiply this by 12, the product is its value in inches and parts; and in the same manner the values may be obtained in other cases.

EXAMPLE VI.

Required the value of 3.25 yards.
```
3.25
   3
————
 .75
  12
————
9.00
```
Answer 3 yards, 0 feet, 9 inches.

EXAMPLE VII.

Required the value of 7.231 days.
```
 7.231
    24
——————
   924
  462
——————
 5.544
    60
——————
32.640
    60
——————
38.400
```
Answer 7 days, 5 hours, 22 minutes, and 33 seconds.

MULTIPLICATION OF FEET, INCHES AND PARTS; OR DUODECIMALS.

The multiplication of *feet* and *inches* is generally called *duodecimals*, because every superior place is 12 times its next inferior in this scale of notation. This way of conceiving an unit to be divided, is chiefly in use among *artificers*, who generally take the linear dimensions of their work in *feet* and *inches*: It is likewise called *cross multiplication*, because the *factors* are sometimes multiplied crosswise.

RULE I.

1. Under the multiplicand write the corresponding denominations of the multiplier; that is, feet under feet, inches under inches, parts under parts, &c.
2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier; write each result under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.
3. In the same manner multiply every term in the multiplicand by the inches in the multiplier, and set the result of each term one place removed to the right-hand, of those in the multiplicand.
4. Work in a similar manner with the parts in the multiplier, setting the result of each term removed two places to the right-hand of those in the multiplicand. Proceed in like manner with the rest of the denominations, and their sum will be the answer required.

EXAMPLE 1. Let 7 feet 9 inches be multiplied by 3 feet 6 inches.

	F.	I.	
Multiplicand	7	9	
Multiplier	3	6	
	23	3	Parts,
	3	10	6
Product	27	1	6

First multiply 9 inches by 3, saying, 3 times 9 is 27 inches, which make 2 feet 3 inches; set down 3 under inches, and carry 2 to the feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the feet.

Then begin with 6 inches, saying, 6 times 9 is 54 parts which is 4 inches and 6 parts; set down 6 parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 inches, which is 3 feet 10 inches; which set down, and, add all up together, and the product is 27 feet 1 inch 6 parts.

EXAMPLE 2. Let 7 feet 5 inches 9 parts be multiplied by 3 feet 5 inches 3 parts.

	F.	I.	P.		
Multiplicand	7	5	9		
Multiplier	3	5	3		
	22	5	3	S.	
	3	1	4	9	T.
		1	10	5	3
Product	25	8	6	2	3

In this example, I first begin with 3 feet, and thereby multiply 7 feet 5 inches and 9 parts: First, I say, 3 times 9 is 27 parts, that is 2 inches and 3 parts; set down 3 under the parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 foot 5 inches; set down 5 inches, and carry 1, and say, 3 times 7 is 21, and 1, I carry is 22; set down 22 feet: Then begin with 5 inches, saying, 5 times 9 is 45, which is 45 *seconds*, which makes 3 parts and 9 seconds; set down 9 seconds a place towards the right hand, and carry 3 parts, saying 5 times 5 is 25, and 3 I carry is 28, which is 2 inches and 4 parts, set down 4 parts and carry 2, saying 5 times 7 is 35, and 2, I carry is 37, which is 3 feet 1 inch; set down 3 feet 1 inch; and begin to multiply by 3 parts, saying 3 times 9 is 27 *thirds*, that is 2 *seconds* and *thirds*; set down 3 thirds, and carry 2, saying 3 times 5 is 15, and 2 I carry is 17, that is, 1 part and 5 seconds; set down 5 seconds, and carry 1, saying 3 times 7 is 21, and 1 I carry is 22, which is 1 inch and 10 parts, which set down and add all up, and the product is 25 feet 8 inches 6 parts 2 seconds 3 thirds.

RULE II.

When the feet in the Multiplicand are expressed by a large number.

Multiply first by the feet in the multiplier, as before. Then, instead of multiplying by the inches and parts, &c. proceed as in the Rule of Practice, by taking such aliquot parts of the multiplicand as correspond with the inches and parts, &c. of the multiplier. Then the sum of them all will be the product required.

EXAMPLE 3. Let 75 feet 7 inches be multiplied by 9 feet 8 inches.

In			F.	I.	
4 }			75	7	Multiplicand.
4 }	$\frac{1}{3}$		9	8	Multiplier
			680	3	
			25	2	4
			25	2	4
		Product	730	7	8

Multiply by 9 feet first, as above directed; then instead of multiplying by the 8 inches, let them be divided into aliquot parts of a foot, as 4 and 4, because 4 is the third part of 12. So if you take the third part of 75 feet 7 inches, and set it down twice, and add all together, the sum will be 730 feet 7 inches 8 parts. To take the third part, say, how often 3 in 7, which is twice; set down 2; then because twice 3 is 6, say, 6 out of 7, and there remains 1, for which you must add 10 to the 5, and it makes 15; then the threes in 15 are 5 times; set down 5; and because three times 5 is 15, there is 0 remains. Then go to the 7 inches, saying, the three in 7 are twice; set down 2 in the inches; and because twice 3 is 6, take 6 out of 7, and there remains 1 inch, which is 12 parts; threes in 12 are 4 times, and 0 remains. So the third part of 75 feet 7 inches is 25 feet 2 inches 4 parts; which set twice over, and add them together as in the example.

EXAMPLE 4. Let 37 feet 6 inches 5 parts, be multiplied by 4 feet 8 inches 6 parts

I.	P.			F.	I.	P.		
4	0 }	$\frac{1}{3}$		37	7	5	Multiplicand.	
4	0 }			4	8	6	Multiplier.	
				150	5	8	S.	
I.	P.			12	6	5	8	
0	6	$\frac{1}{8}$		12	6	5	8	T.
				1	6	9	8	6
			Product	177	1	5	0	6

In this example I first multiply by 4 feet as usual. Then for the 8 inches I say 4 inches is the third of a foot, therefore I take the third part of 37 feet 7 inches 5 parts, which is 12 feet 6 inches 5 parts 8 seconds, and set it down twice. Then for 6 parts, I say, 6 parts are the eight of four inches, because 12 parts make 1 inch; hence it follows, that whatever be the value, or product, by 4 inches, the

value of 6 parts will be one-eighth thereof; therefore I take one-eighth of 12 feet 6 inches 5 parts 8 seconds, and find it to be 1 foot 6 inches 5 parts, 8 seconds, 6 thirds; so that the sum of the whole is 177 feet 1 inch 5 parts 6 thirds.

EXTRACTION OF THE SQUARE ROOT.

If a square number be given;

To find the Root thereof, that is to find out such a number, as being multiplied into itself, the product shall be equal to the number given; such operation is called *The Extraction of the Square Root*; which to do, observe the following directions.

1. You must point your given numbers; that is, make a point over the unit's place, another over the hundred's, and so over every second figure througout.

2. Then seek the greatest square number in the first period towards the left hand, placing the square number under that point, and the root thereof in the quotient, and subtract the said square number from the first point, and to the remainder bring down the next point, and call that the resolvend.

3. Then double the quotient, and place it for a divisor on the left hand of the resolvend; and seek how often the divisor is contained in the resolvend (reserving always the unit's place) and put the answer in the quotient, and also on the right hand side of the divisor; then multiply by the figure last put up in the quotient, and subtract the product from the resolvend (as in common division) and bring down the next point to the remainder (if there be any more) and proceed as before.

A Table of Squares and their Roots.

Root	1	2	3	4	5	6	7	8	9
Square	1	4	9	16	25	36	49	64	81

EXAMPLE 1. Let 4489 be a number given, and let the square root thereof be required.

```
 4489(67
 36
 ————
127)889  Resolvend.
    889  Product.
    ———
    ...
```

First, point the given number, as before directed; then by the little table foregoing, seek the greatest square number in 44 (the first point to the left-hand) which you will find to be 36, and 6 the root; put 36 under 44, and 6 in the quotient. and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other point 89, placing it on the right-hand, so it makes 889 for a resolvend; then double the quotient 6, and it makes 12; which place on the left-hand for a divisor, and seek how often 12 in 88 (reserving the unit's place) the answer is 7 times; which put in the quotient, and also on the right hand side of the divisor, and multiply 127 by 7, as in common division, and the product is 889, which subtracted from the resolvend, there remains nothing; so is your work finished; and the square root of 4489 is 67; which root if you multiply by itself, that is 67 by 67, the product will be 4489, equal to the given square number, and proves the work to be right. Had there been any remainder it must have been added to the square of the root found.

EXAMPTE 2. Let 106929 be a number given, and let the square root thereof be required.

```
 106929(327
 9
 ————
62)169   Resolvend.
   124   Product.
 ————
647)4529 Resolvend.
    4529 Product.
    ————
    ....
```

First point your given number, as before directed, putting a point over the units, hundreds, and tens of thousands; then seek what is the greatest square number in 10 (the first point) which by the little table you will find to be 9, and 3 the root thereof; put 9 under 10, and 3 in the quotient; then subtract 9 out of 10, and there remains 1; to which bring down 69, the next point, and it makes 169 for the resolvend; then double the quotient 3, and it makes 6, which place on the left hand of the resolvend for a divisor, and seek how often 6 in 16; the answer is twice, put 2 in the quotient, and also on the right hand of the divisor making it 62. Then multiply 62 by the 2 you put in the quotient, and the product is 124; which subtract from the resolvend, and there remains 45; to which bring down 29, the next point, and it makes 4529 for a new resolvend. Then double the quotient 32, and it makes 64, which place on the left side of the resolvend for the divisor, and seek how often 64 in 452, which you will find 7 times; put 7 in the quotient, and also on the right hand of the divisor, making it 647, which multiplied by the 7 in the quotient, it makes 4529, which subtracted from the resolvend, there remains nothing. So 327 is the square root of the given number.

NOTE. The root will always contain just so many figures, as there are points over the given number to be extracted: And these figures will be whole numbers or decimals respectively, according as the points stand over the whole numbers or decimals. The method of extracting the square root of a decimal is exactly the same as in the foregoing examples, only if the number of decimals be odd, annex a cypher to the right hand to make them even before you begin to point. The root may be continued to any number of figures you please, by annexing two cyphers at a time to each remainder, for a new resolvend.

EXTRACTION OF THE CUBE ROOT.

To extract the Cube Root, is nothing else but to find such a number, as being first multiplied into itself, and then into that product, produceth the given number; which to perform, observe the following directions.

1. You must point your given number, beginning with the unit's place and make a point, or dot, over every third figure towards the left-hand.

2. Seek the greatest cube number in the first point, towards the left-hand, putting the root thereof in the quotient, and the said cube number under the first point, and subtract it therefrom, and to the remainder bring down the next point, and call that the resolvend.

3. Triple the quotient, and place it under the resolvend; the unit's place of this under the ten's place, of the resolvend; and call this the triple quotient.

4. Square the quotient, and triple the square and place it under the triple quotient; the units of this under the ten's place of the triple quotient, and call this the triple square.

5. Add these two together, in the same order as they stand, and the sum shall be the divisor.

6. Seek how often the divisor is contained in the resolvend, rejecting the unit's place of the resolvend (as in the square root,) and put the answer in the quotient.

7. Cube the figure last put in the quotient, and put the unit's place thereof under the unit's place of the resolvend.

8. Multiply the square of the figure last put in the quotient into the triple quotient, and place the product under the last, one place more to the left-hand.

9. Multiply the triple square by the figure last put in the quotient, and place it under the last, one place more to the left-hand.

10. Add the three last numbers together, in the same order as they stand, and call that the subtrahend.

11. Subtract the subtrahend from the resolvend, and if there be another point, bring it down to the remainder, and call that a new resolvend, and proceed in all respects as before.

NOTE. To square a number is to multiply that number by itself. And,
To cube a number is to multiply the square of the number by the number itself.

A Table of Cubes and their Roots.

Roots	1	2	3	4	5	6	7	8	9
Cubes	1	8	27	64	125	216	343	512	729

EXAMPLE I. Let 314432 be a whole number, whose root is required.

```
314432(68 Root.
216
————
98432 Resolvend.
————
  18 Triple quotient of 6.
 108  Triple square of the quotient 6.
————
1098 Divisor.
————
  512 Cube of 8, the last figure of the root.
 1152 The square of 8, by the triple quotient.
 864  The Triple square of the quotient 6 by 8.
————
98432 The subtrahend.
————
.....
```

After you have pointed the given number, seek what is the greatest cube number in 314, the first point, which, by the little table annexed to the rule you will find to be 216, which is the nearest that is less than 314, and its root is 6; which put in the quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next point, 432, and annex it to 98; so will it make 98432 for the resolvend. Then triple the quotient 6, it makes 18, which write down the unit's place, 8, under 3, the ten's place of the resolvend. Then square the quotient 6, and triple the square, and it makes 108, which write under the triple quotient, one place towards the left-hand; then add those two numbers together, and they make 1098 for the divisor. Then seek how often the divisor is contained in the resolvend, (rejecting the unit's place thereof) that is, how often 1098 in 9843, which is 8 times; put 8 in the quotient, and the value thereof below the divisor, the unit's place under the unit's place of the resolvend. Then square the 8 last put in the quotient, and multiply 64, the square thereof, by the triple quotient 18; the product is 1152; set this under the cube of 8, the units of this under the tens of that. Then multiply the triple square of the quotient by 8, the figure last put up in the quotient, the product is 864; set this down under the last product, a place more to the left-hand. Then draw a line under these three, and add them together, and the sum is 98432, which is called the subtrahend; and being subtracted from the resolvend, the remainder is nothing;

which shows the number to be a true cubic number, whose root is 68 ; that is, if 68 be cubed, it will make 314432.

For if 68 be multiplied by 68, the product will be 4624 ; and this product, multiplied, again by 68, the last product is 314432, which shows the work to be right.

Example 2. Let the cube root of 5735339 be required.

After you have pointed the given number, seek what is the greatest cube number in 5, the first point, which by the little table, you will find to be 1, which place under 5, and 1, the root thereof, in the quotient ; and subtract 1 from 5, and there remains 4 ; to which bring down the next point, it makes 4735 for the dividend. Then triple the 1, and it makes 3 ; and the square of 1 is 1, and the triple thereof is 3 ; which set one under another, in their order, and added, makes 33 for the divisor. Seek how often the divisor goes in the resolvend, and proceed as in the last example.

5735339(179 Root.
1

4735
3 Triple of the quotient 1, the first figure.
3 The triple square of the quotient 1.

33 The divisor.

343 The cube of 7, the second figure of the root.
147 The square of 7, multiplied in the triple quotient 3.
21 The triple square of the quotient multiplied by 7.

3913 The subtrahend.

822339 The new resolvend.

51 The triple of the quotient 17, the two first figures.
867 The triple square of the quotient 17.

8721 Divisor.

729 The cube of 9, the last figure of the root.
4131 The square of the 9, multiplied by the triple quotient 51.
7803 The triple square of the quotient 867 by 9.

822339 The subtrahend.

In this example, 33, the first divisor, seems to be contained more than seven times in 473, the dividend, after the unit's place has been rejected ; but if you work with 9, or 8, you will find that the subtrahend will be greater than the dividend.

MENSURATION

Of Superfices and Solids.

Section I. of superfices.

The areas or superfices of any plane figure is estimated by the number of squares contained in its surface ; the side of those squares being either an inch, a foot, a yard, a link, or a chain according to the measures peculiar to different artists.

Our common measures of length is given in the tables below the right hand table, it is the square measure as taken from the other by squaring the several numbers.

1 Lineal Measure.		2 Square Measure.	
12 Inches	1 Foot.	144 Inches	1 Foot.
3 Feet	1 Yard.	9 Feet	1 Yard.
6 Feet	1 Fathom	36 Feet	1 Fathom.
$16\frac{1}{2}$ Feet / $5\frac{1}{2}$ Yards	1 Pole or Rod	$272\frac{1}{2}$ Feet / $30\frac{1}{4}$ Yards	1 Pole or Rod
40 Poles	1 Furlong	1600 Poles	1 Furlong.
8 Furlongs	1 Mile	64 Furlongs	1 Mile.

Land is generally measured by a chain of 4 rods or 66 feet in length, called Gunters chain, from the name of the inventor.

This chain is made up of 100 links, and every tenth link, from either end, is marked by a small brass plate attached to it and notched to designate its number from the end. This chain being divided into one hundred equal parts is a convenient one, since the divisions, or links, are decimals of the whole chain and in the calculations may be treated as such.

The length of the chain being 4 poles or 66 feet, is equal to 792 inches which being divided by 100, gives 7,92 inches for the length of each link a mile being equal to 320 rods, therefore 80 chains is equal to 1 mile, and 40 chains is equal to half a mile, and 20 chains is equal to one fourth of a mile. And ten square chains or ten in length and one in breadth, make an acre, or 160 square poles or 100,000 square links each being the same in quantity. Forty perches or square poles make a rood, and 4 roods, make an acre.

The length of lines which are measured with a chain, are in general way set down in links, as whole numbers, every chain being 100 links in length. Therefore after the dimensions are squared or the superfices found, it will be in square links, when this is the case it will be necessary to cut off five of the figures on the right hand for decimals, and the rest will be acres. These decimals must be then multiplied by 4, for roods, and the decimals of these again, after five figures are cut off by 40 for poles, and the five decimals again by $272\frac{1}{4}$ for feet.

Article I. To measure a square, having equal sides.

Rule.—Multiply the side of the square into itself, and the product will be the area or superficial content, of the same name with the denomination taken, either in inches, feet, or yards, respectively.

Let A B C D (fig. 1 pl. 34, of Mensuration,) which represents a square whose sides are 12 inches, or 12 feet. Multiply the side 12 by itself thus :—

12 inches, 12 feet.
12 inches, 12 feet.

Area,-144 inches. 144 feet.

Art. II. To measure a Parallelogram or long square.

Rule.—Multiply the length by the breadth and the product will be the area, or superficial content.

Let A B C D, (fig. 2,) represent a parallelogram, whose length is 16 feet, and breadth 10 feet

Length 16
Breadth 10

Area, 160 Ans.

To demonstrate these examples let two sides of the given squares, be divided into equal parts as represented in the figures, and draw lines through the several divisions parallel to A B and A D which will divide the figures into as many little squares as it contains inches, feet, respectively.

Art. III. To measure a Rhombus.

Rule.—Multiply the side by the length of a perpendicular let fall from one of the obtuse angles to the opposite side.

What is the area of a rhombus A B C D (fig. 3) the length A B being 5 feet 9 inches, and the perpendicular height A E 5 feet, 9 inches.

By duodecimals.		By decimals.
5-9	5-9	5-75
5-9	5-9	5-75
4-3-9	28-9	2875
28-9	4-3-9	4025
		2875
Ans. 33-0-9	Ans. 33-0-9	Ans. 33,0625.
f i ii	*f i ii*	

Art. IV. To find the area of a Rhomboid.

Rule.—Multiply one of the longest sides by the perpendicular let fall from one of the obtuse angles to the opposite side.

What is the area of a rhomboid A B C D (fig. 4) whose length A B is 10f. 9 and the perpendicular height A E 4f.-6i.

10-9	10-9	10-75
4 6	4 6	4-50
5-4-6	43-0	53750
43-0	5-4-6	4300
Ans. 48-4-6	48-4-6	48,3750
f, i ii	*f i ii*	

Demonstration of the two last figures ; let B F in (fig. 3 and 4) be drawn perpendicular to A B, and produce the line C D to F, then will the angles A D E be equal to the angles B C F and A B E F will be a square in fig. 3, and fig. 4, will be a rectangle.

Art. V. To find the area of a Trapezoid.

Rule.—Multiply the half sum of the parallel ends, by the length and the product will be the area.

What is the area of the trapezoid A B CD in (fig, 5) the end A D being 4f-6 and the end B C 3f-8 and 12f 3 inches long.

End A D= 4-6 12-3 12-3
End B C = 3-3 4-1 4-1

divide 2)8-2 1-0-3 49-0
49-0 1-0-3
4-1-half sum
Ans. 50-0-3 50-0-3
f i ii

Art. VI. To find the area of a Triangle.

Rule.—If it be a right angled triangle multiply the base by half the perpendicular, or half the base by the perpendicular, and the product will be the area ; but if it be an oblique, obtuse, or acute angled-triangle multiply the base by half of the perpendicular let fall on the base from the angle opposite to it, and the product will be the area.

What is the area of the triangle A B C (fig. 6.) whose base A B is 24f. and the perpendicular B C 18f. 6i.

Base A B=24-0	Or perp, B C=18-5
½ Perp. B C= 9-3	½ Base A B=12-0
6-0-0	3700
216-0	185
Ans. 222-0-0	222.00

To demonstrate this figure let A G be drawn perpendicular to A B; at E the half of the perpendicular height, draw E F G parallel to A B, draw C H parallel to E G, and make equal in length to E F, and through F draw H F D parallel to B C (Geometry, Theorem, 1.) and thus, the triangle C E F is equal to the triangle A G F, and if the trangle C E F be taken and placed upon the angle A G F, then will A B E G become a rectangle which proves the first method. Again let the triangle ADF be taken away and placed upon the angle CHF then BCDH becomes a rectangle which proves the second method.

And the proceeding figures may also be demonstrated in the same manner.

To find the area of the triangle ABC (fig. 7) whose base AB is 16 *f*, and the perpendicular CD is 10 *f* 2 *i*.

Base AB=16.0	or Perp CD=10.2
½ Perp CD= 5.1	½ Base AB= 8.0
1.4.0	14.0
80.0	80
81.4.0	81.4.0
f. i. ii	

Art. VII. There is another method of finding the area of triangles the three sides being given.

Rule.—Add the three sides together, then take the half of that sum, and out of it subtract each side severally; and multiply the half of the sum and these remainders continually, and the square root of this product will be the area of the triangle.

What is the area of the triangle ABC (fig. 7) the side AB being 16 *f.* and the side AC 14 *f.* and the side BC is 12 *f.*

side AB=16	21	21	21
side AC=14	16	14	12
side BC=12	5 first diff.	7 sec. diff.	9 third diff.

2)42 sum
21=half the sum
×.5
105
×.7
735
×.9
6615

6615(81.33= area
64
161)215
161
1623)5400
4869
16263)53100
48789

If any two sides of a right angled triangle be given the third side may also be found; this part will be fully explained hereafter in trigonometry.

Art. VIII. To find the area of a trapezium.

Rule.—Draw a diagonal line from one of the angles to the opposite angle as AC in (fig. 8) and then will the trapezium be divided into to two triangles of which the diagonal is the common base; then letting fall perpendiculars from the opposite angles as BF and DE to the diagonal line AC: and then add these perpendiculars BF and DE together, and multiply half that sum into the diagonal, or half of the diagonal into the perpenciculars, and that product will be the area of the trapezium.

What is the area of the trapezium ABCD (fig. 8) the base AC is 18 *Ch* 74*Li*: the perpendicular BF 6*Ch* 90*Li* and DE 7*Ch* 56*Li*.

Perp BF=6.90	Base AC=18.74
Perp DE=7.56	½ Perpendicular= 7.23
Divide by 2)14.46	5622
	3748
7.23	13.118
	acres.=1354902
	4
	roods=2.19608
	40
	rods=7.84320
	272
	168640
	590240
	168640
	feet=229.35040

Art. IX. To find the area of any irregular figure.

Rule.—Divide the figure into triangles, by drawing diagonals from one angle to another, then measure all the triangles by either of the rules, already taught, at Arts. 6, 7, or 8, and the sum of the several areas of all the triangles will be the area of the figure.

What is the area of the irregular figure ABCDEF in (fig. 9.) Draw the diagonal FC, and let fall the perpendiculars FG, FH, and BI and DK, then we will commence on the base AB which is 12*Ch* 8*Li*, and the base FC 10*Ch* 86*Li*, the base ED 8*Ch* 68*Li*, the perpendiculars FG 6*Ch* 49*Li*, FH 4*Ch* 95*Li*, BI 6*Ch* 80*Li*, and DK 5*Ch* 90*Li*.

Perp FG=6.49	Base AB=12.08
Perp FH=4.95	Base FC=10.86
Perp BI =6.80	Base ED= 8.68
Perp DK=5.90	31.62
Divide by 2)24.14	½ Perp=12.07
12.07	22134
	63240
	3162
	acres=38.16534
	4
	roods=.66136
	40
	rods=26.45440
	272
	90880
	318080
	90880
	feet=123.59680

Art. X. To find the area of a Trapezoid.

Rule.—Add the parallel sides together, and multiply half that sum by the perpendicular breadth, and the product will be the area.

What is the area of the trapezoid ABCD (fig. 10) the side AB being 9 *f*, the side CD 12 *f* 6*i*, and the perpendicular height AE 9 *f* 8*i*.

By duodecimals.	By practice.
side AB= 9.0	10.9
side CD=12.6	9.8
21.6=sum	7.2.0
10.9 =½ half of sum	96.9
height AE= 9.8	103.11.0
96.9	
7.2.0	
ans. 103.11.0	
f i ii	

Art. XI. To find the area of any regular polygon.

Rule.—Multiply the whole perimeter or sum of the sides by half of the perpendicular, or multiply half of the perimeter by the whole perpendicular, let fall from the centre to the middle of one of the sides.

Example 1. What is the area of the pentagon ABCDE (fig. 11) the sides AB, BC &c. being 6*f.* 1*i.* and the perpendicular height from F to the centre of the figure or polygon, is 4*f*, 1*i*.

Sides = 30-0-0	or 2)30
½ Perp. FG=2-0-6	15-0
	Perp.=4-1
60-0-0	1-3-0
1-3-0	60-0
area 61-3-0	61-3-0
f. i. ii.	

To demonstrate this figure by the first method, let a line be drawn F, F in (fig. 12) and make F, F equal in length to the whole perimeter of the stretchout of the five sides in (fig. 11) of which is 30 feet, then take half of the perpendicular FG in (fig. 11) and set it up from F, F to G, G in (fig. 12) and draw the line G, G which will be parallel to FF; then take the several trapezoids from (fig. 11) and transfer them to (fig. 12) commencing at FB the half side, then the sides BC, CD, DE, EA, and AF &c., and place them on the line F, F in (fig. 12;) then take the five small angles *n, n, n, &c.* in (fig. 11) and transfer them to *n, n, n,* &c. in (fig. 12,) which completes the figure, and now we can see that any regular polygon can be formed into a parallelogram. Again, by the second rule; let FO or OF in (fig. 13 be drawn equal in length to half of the perimeter (which is 15 feet) and make FO and OF equal in height to the whole perpendicular, which being 4*f.* 1*i.* and draw OF parallel to FO, then will this figure contain the same quantity of space as (figures 11 and 12,) and it is made up of the five equal angles from (fig. 11,) that is four wholes and two halves; as may be seen in the diagram of the figure, therefore these parallelograms are equal to the pentagon, which proves the rules correct.

Example 2. What is the area of the hexagon ABCDEF (fig. 14) each side being 14*f.* 6*i.* and the perpendicular height from G to the centre of the polygon, is 12*f*, 8*i*.

half of the length of the side is = 43-6
the perpendicular is = 12-8
29-0-0
522-0
Ans. 551-0-0
f. i. ii.

This figure may also be demonstrated in the same way as (fig. 11) for every regular polygon is equal to the parallelogram or long square whose length is equal to half the sum of the sides, and breadth equal to the perpendicular of the polygon as has been proved heretofore ; this figure is made up of six equilateral triangles, and the parallelogram HIJK is also composed of six equilateral triangles, that is five whole ones and two halves, as may be seen in the diagram of the figure ; therefore the parallelogram is equal to the hexagon.

But for finding more readily the area of a regular polygon, and also the perpendicular and the radius, the following table is introduced ; containing the multipliers for all regular figures from the triangle to the duodecagon.

1 No. of Sides.	2 Name of the Polygon.	3 Area multipliers.	4 Perpendicular multipliers.	5 Radius multipliers.	6 No. of degrees.
3	Trigon	0.433013	0.2886751	0.5773603	120
4	Tetragon	1.000000	0.5000000	0.7071068	90
5	Pentagon	1.720477	0.6881910	0.8506508	72
6	Hexagon	2.598076	0.8660254	.1Side=rad.	60
7	Heptagon	3.633912	1.0382617	1.1523825	$51\frac{4}{10}$
8	Octagon	4.828427	1.2071068	1.3065630	45
9	Nonagon	6.181824	1.3737387	1.4619022	40
10	Decagon	7.694209	1.5388418	1.6186340	36
11	Undecagon	9.365640	1.7028437	1.7747329	$32\frac{7}{10}$
12	Duodecagon	11.196152	1.8660254	1.9318516	30

Rule 2.—Multiply the square of the side by the tabular area and the product will,' be the area of the polygon. Or multiply the side of the polygon by the tabular perpendicular, and the product will give the perpendicular from F to the centre of the polygon as in (fig. 11.) And by multiplying the side of the polygon also by the tabular radius, and the product will give the length from A B C, &c. to the centre of the polygon.

Examples.—1 If the side of a pentagon be 6 feet what is the area.

6 1.720477 tabular area.
6 36
36=square 10322862
5161431
answer 61.937172

2 What is the area of an octagon whose sides is 8 feet 3 inches.

8.25 4.828427
8.25 680
4125 386274160
1650 28970562
6600
680625 ans. 328.3330360

3 If the sides of a pentagon be 6 feet, what is the length of the perpendicular.

6881910
6
4.1291460

4 If the sides of a pentagon be 6 feet what is the length of the radius.

8506508
6
ans. 5.1039048

And in case it is required to find the tabular numbers answering for the perpendicular, where the polygon has more than twelve sides ; they may be found by Trigonometry. There may be instances where the sides of a polygon may be obtained where the perpendicular cannot, and if we can have the length of the sides and perpendicular, it is all we want to find the area.

Note.—There is no letter described at the centre of the polygon in the diagram of the figure ; therefore we will suppose that the angles at the centre of the polygon (as in fig. 11) to be designated by the letter O.

To find the tabular numbers ; thus, find the angle of the centre of the polygon as O in (fig. 11) by dividing the number of sides by 360° ; then suppose each side of the pentagon in (fig. 11) be 1 or AB the log. of 10 then AF would be 5 the half log.

Example ; divide 360° by 5 (the number of sides in the pentagon) and the quotient is 72° for the angle AOB the half of which is 36° the angle FOA whose complement to 90° is 54° the angle OAF. Then say.

As cosecant FOA 36° - - - - - - -	10.230701
Is to 5 the half side FA log - - - - -	0.698970
So is sine OAB 54° - - - - - - - -	9.907958
To the perpendicular FO 0.6881910 - -	0.837709

Art. XII.—The diameter of a circle being given to find the circumference ; or the circumference being given to find the diameter.

Rules.—As 7 is to 22, so is the diameter to the circumference nearly ; or as 22 is to 7 so is the circumference to the diameter nearly.

2 Or more exactly as 113 is to 355 so is the diameter to circumference, or as 355 is to 113 so is the circumference to the diameter,

Examples.—What is the circumference of a circle, whose diameter is 6 ft. 6 in.

as 7 : 22 : :6.5 :
65
110
132
7)1430(20.42 the circum. nearly
14
30
28
20
14

as 113 : 355: :6.5 :
65
1775
2130
113)23075(20.42
226
452
475
230
226

Art. XIII.—What is the diameter of a circle whose circumference is $20.\frac{42}{100}$ ft.

as 22 : 7. :20.42 :
7
22)14294(6.5 diam. nearly
132
109
110

as 355 : 1130: :20.42
2042
2260
4520
22600
355)2307460(6-5 dm. nearly.
2130
1774
1775

We now see the proportions of these examples are nearly correct, the latter however is the most accurate.

Circles like all other similar plain figures are in proportion to one another, as the squares of their diameters and their circumferences, are to one another as their diameters of circles, or radii.

The proportion of the diameter of a circle to its circumference has never yet been exactly determined. This problem has engaged the attention, and exercised the abilities of the greatest mathematicians for ages ; no squares or any other right lined figures, has yet been found that shall be perfectly equal to a given circle. But though the relation between the diameter and circumference has not been accurately expressed in numbers, it may be approximated to any assigned degree of exactness. Archimedes, about two thousand years ago, discovered the proportion to be nearly as 7 to 22 ; other and nearer ratios have since been successsvely assigned, viz : as 106 to 333, or 113 to 355 ; this last proportion is useful, for being turned into a decimal, it agrees with the truth to the 6th figure inclusively.

Art. XIV. To find the area of a circle.

Rules.—1. Multiply half the diameter by half the circumference, and the product will be the area.

2. Multiply the square of the diameter by .7854, and the product will be the area.

3. Multiply the square of the radius by 3.1416 and the product will be the area.

4. Multiply the square of the circumference by .07958, and the product will be the area.

If the diameter be given, find the circumference by Art. XII.

If the circumference be given, find the diameter by Art. XIII.

What is the area of a circle whose diameter is 6*f* 6, and circumference 20*f.* 4*i.*

By Rule I.
$\frac{1}{2}$ 20*f.* 4*i.* = 10-2
$\frac{1}{2}$ 6*f.* 6*i.* = 3-3
2-6 6
30-6
Ans. 33-0-6
f. i. s.

By Rule II.
diameter 6-5
6-5
325
390
4225 sq. of diameter
7854
16900
21125
33800
29575
Ans. 33,183150

To demonstrate the first rule, let ABCD (fig. 15) be the circle, and ABC half of the circumference ; and BE the semi-diameter. Let a line be drawn FG parallel to the diameter AC ; and divide the quadrants BA and BC into any like number of equal parts ; or take the stretch out of BA and BC and produce them to F and G ; and draw HF and GI parallel to BE ; and produce the diameter AC to H. and I. Then will the parallelogram HIGF contain the same area as the circle ABCD ; for every circle may be conceived to be a polygon of an infinite number of sides ; now the semi-diameter, must be equal to the perpendicular of such a polygon ; and the circumference of the circle equal to the periphery of the polygon ; therefore half of the circumference multiplied by half of the diameter will give the area of a circle.

Demonstration of Rule II. All circles are to each other as the squares of the diameters, and the area of a circle whose diameter is 1, is 7854 (by the second example ;) therefore as the square of 1 which is 1 to .7854, so is the square of the diameter of any circle to its area.

Art. XV. Having the diameter circumference or area of a circle given ; to find the side of a square equal in area to the circle and the side of a square inscrib-

ed in the circle, or having the side of a square given to find the diameter of its circumscribing circle, and also of a circle equal in area, &c.

RULES.—1. The diameter of any circle multiplied by 8862269 will give the side of a square equal in area, as in (fig. 16.)

2. The circumference of any circle multiplied by 2820948 will give the side of a square equal in area.

3. The diameter of any circle multiplied by 7071068 will give the side of a square inscribed in that circle.

4. The circumference of any circle multiplied by 2250791 will give the side of a square inscribed in that circle.

5. The area of any circle multiplied by 6366197 and the square root of the product extracted will give the side of a square inscribed in that circle.

6. The side of any square multiplied by 1.414236 will give the diameter of its circumscribing circle.

7. The side of any square multiplied by 4.4428829 will give the circumference of its circumscribing circle.

8. The side of any square multiplied by 1.1283791 will give the diameter of a circle equal in area to the square.

9. The side of any square multiplied by 3.5449076 will give the circumference of a circle equal in area to the square.

Art. XVI. To find the area of a semi-circle.

RULE.—Multiply the fourth part of the circumference of the whole (that is half the arc line) by the semi-diameter, the product is the area.

What is the area of the semi-circle ABC (fig. 17) the arc AB being 17f. $4\frac{1}{2}$ i. and the semi-diameter BD is 11f. 3i.

By Duodecimals.	By Practice.
Cir. AB = 17-4-6	17-4-6
11-3-0	11-3-0
191-1-6	4-4-1-6-0
4-4-1-6	191-1-6
Ans. 195-5-7-6	195-5-7-6-0
f. i. p. s.	

Art. XVII. To find the area of a quadrant.

RULE.—Multiply half the arc line of the quadrant (that is the eighth part of the circumference of the whole circle) by the semi-diameter, and the product is the area of the quadrant.

What is the area of the quadrant ABC (fig. 18) half of the arc BC is 8f. 9i. and the semi-diameter AC is 11f- 3 i.

By Duodecimals.	By Decimals.
11-3	11·25
8-9	8·75
90-0	5625
8-5-3	7875
	9000
Ans. 98-5-3	98,4375
f. i. p.	

Art. XVIII. The chord and height of a segment being given to find the chord of the half arc.

RULE.—To the square of the half chord add the square of the height, and the square root of the sum will be the length of the chord of half the arc.

EXAMPLE.—The chord AC (fig. 19) being 48 feet, and the height BD 18 feet, what is the length of the chord of half the arc.

```
 2)48                                        18
 ——                                          18
  24                                         ——
  24                                        144
  ——                                         18
  96                                         ——
  48                                        324
  ——
Add { 576  square of half the chord.
    { 324  square of the height.
      ——
      900(30, length of half the chord.
      9
      ——
      00
```

Art. XIX. To find the length of any arc of a circle, the half chord and chord of the whole arc being given.

RULE.—Subtract the chord of the whole arc from double the chord of the half arc; add one third of the remainder to the double chord of the half, and the sum will be nearly equal to the length of the arc.

EXAMPLES.—What is the length of the arc ABC (fig. 19) whose chord AC is 48 and the half chord AB is 30.

```
  2×30=60  the double chord of the half arc.
       48  the chord of the whole arc.
      ——
     3)12
        4
       60  the double chord of the half arc.
      ——
       64  the length of the arc required.
```

Art. XX. The chord and height of a segment being given to find the radius of the circle.

RULE.—To the square of the half chord, add the square of the height, and divide the sum by twice the height of the segment, and the quotient will be the radius of the circle, when it is less than a semi-circle.

The chord AC (fig. 19) of a segment being 48 feet, and the height BD 18 feet, what is the radius of the circle.

```
 ÷2)48
    ——                                   18  the height.
    24  half the chord.                  18
    24                                   ——
    ——                                  144
    96                                   18
    48                                   ——
    ——                                  324
Add { 576  square of the half chord.
    { 324  the square of the height.
      ——
   36)900(25  the radius required.       18
      72                                  2
      ——                                 ——
      180                                36  twice the height.
      180
```

Art. XXI. Given any two parallel chords in a circle, and their distance, to find the distance of the greater chord from the centre.

RULE.—To the square of the distance between the chords, add the square of half the lesser chord. The difference between this sum, and the square of half the greater chord divide by twice the distance of the chords, will give the distance of the centre from the greatest chord.

EXAMPLE.—Suppose the greater chord CD (fig. 20) is 48 feet, and the lesser AB 30, their distance EG 13 feet, what is the distance EF from the centre to the greater chord CD.

```
 13     ½ 30 = 15                     ½ 48 = 24
 13            15                            24
 ——            ——                            ——
 39            75                            96
 13            15                            48
 ——            ——                            ——
169           225 sq. of the lesser ch.     576 sq. of the greater ch.
              169 sq. of the distance.      394
              ——                            ——
              394          2 × 13 = 26)182(7 = EF dist. req'd.
                                       182
                                       ——
```

Art. XXII. Given a chord of a circle and its distance from the centre to find the radius of the circle.

RULE.—To the square of the half chord, add the square of the distance from the centre, and the square root of the sum will be the radius required.

EXAMPLE.—Given the chord CD (fig. 20) 48 feet, and its distance EF from the centre 7 feet, required the radius of the circle.

```
 ½ 48 = 24 and 24 × 24 = 576
             7 × 7 =  49
                     ——
                     625(25 the radius.
                     4
                     ——
                  45)225
                     225
```

Art. XXIII. Given any two parallel chords in a circle, and the distance between them, to find the perpendicular height from the middle of either chord to the circumference.

RULE.—Find the nearest distance of the greater chord from the centre, by Art. 21, and find the radius of the circle by Art. 22, add the distance between the two parallel chords, and the distance between the greater chord and the centre of the circle together; this sum being taken from the radius, will give the perpendicular height from the middle of the lesser chord, to the circumference or height of the lesser segment; to the lesser segment add the distance between the parallel chords and the sum will be the height of the greater segment.

EXAMPLES.—Given the greater chord CD (fig. 20) 48 feet, and the lesser chord AB 30 feet, their distance EG 13 feet, required the distance GH perpendicular from the middle of AB to the circumference. The distance from the centre to the greater chord will be found to be 7 feet by Art. 21, and the radius 25 feet by Art. 22.

Thus 13 + 7 = 20 and 25 — 20 = 5 feet height of the lesser segment.
Then 13 + to 5 = 18 the height of the greater segment.

Art. XXIV. To find the area of a secter of a circle.

RULE.—Multiply the radius or half the diameter by half the length of the arc of the secter, and the product will be the area.

EXAMPLE 1.—To find the area of the sector ABC (fig. 21) first find the half arc BD by Art. 19, which is 3f. 6i. and the radius or semi-diameter AB being 12f. 4i.

```
     Radius AB is= 12-4
The half arc BD is=  3-6
                   ——
                   6-2-0
                  37-0
                   ——
         Ans.     43-2-0
                  f. i. ii.
```

Example 2.—What is the area of the sector ABC (fig. 22) which is greater than a semi-circle, half of the arc AB being 72f. 6i. and the radius AD is 56f. 3i.

72-50
56-25
———
36250
14500
43500
36250
———
Area 4078,1250

Art. XXV. To find the area of a segment of a circle, the chord and height of the are being given.

Rule 1.—Find the length of the arc ABC (fig. 23) by Art. 19, and the radius of the circle by Art. 20, the area of the sector ABCE by Art. 24. Subtract the area of the triangle AEC as found by Art. 6, from the area of the sector, and the remainder will be the area of the segment.

Example 1.—What is the area of the segment of a circle ABC (fig. 23) the chord AC being 48 feet and the height BD 18 feet. The length of the arc will be found to be 64 feet and the radius 25 feet, then

2)64
—
32
×25
———
160
64
———
800 area of the sector.
168
———
632 area of the segment.

25
—18
—
7 perpendicular DE.
48
———
2)336
168 area of the triangle ACE

Rule 2.—To two thirds of the product of the base multiplied by the height, add the cube of the height divided by twice the length of the segment and the sum will be nearly the area.

Example 2.—What is the area of a circular segment, the chord being 48 feet, and the height 18 feet.

48
× 18
———
384
48
———
3)864
———
288
×2
———
576
+60-75
———
636-75 the area required.

18
×18
———
144
18
———
324
×18
———
2592
324
———
2×48 = 96)5832(60-75
576
———
720
672
———
480
480

To find the area of a segment of a circle which is greater than a semi-circle as ABC (fig. 24.)

Rule 3.—Find the length of the arc ABC by Art. 19, and the radius AD by Art. 20, thence find the whole area of the sector ABCD by Art. 24, Example 2; and then find the area of the triangle ACD by Art. 6, and add the area of the triangle to the area of the sector, and it will give the whole area of the segment.

Example 3.—What is the area of a segment of a circle ABC (fig. 24.) Suppose the half arc AB to be 86 feet, the radius AD or CD 40 feet, and the chord AC 68 feet.

½ arc AB = 86
radius AD = 40
———
3440=area of the sector ABC
716=area of triangle ACD
———
4156=whole area of seg. ABC

chord AC = 68
side AD = 40 | 74 | 74 | 74
side GD = 40 | 68 | 40 | 40
———
2)148 | 6 | 34 | 34
———
74
6 | 513264(716 area.
——— | 49
444 | ———
34 | 141)233
——— | 141
1776 | ———
1332 | 1426)9164
——— | 8556
15096 | ———
34 | 608
———
60384
45288
———
513264

Art. XXVI. To find the area of a circular zone as in (fig. 25,) which is that part of a circle laying between two parallel chords, and the parts of the circle intercepted by the chords.

Rule.—First find the area of the whole circle by Art. 14, then find the area of the two segments of the circle by Art. 25, and subtract the areas of the two segments from the area of the whole circle; and the remainder will be the area of the zone.

Art. XXVII. To find the area of a Lune or Crescent. A Lune is a figure made by two circular arcs which intersect each other, as ABCD in (fig. 26.)

Rule.—If it be a semi-circle, first find the whole area of the semi-circle ABC by Art. 16, then find the area of the segments ADC by Art. 25, and subtract the area of the segment from the area of the semi-circle, and the remainder will be the area of the Lune ABCD. But if it be a segment of a circle, first find the area of the whole segment ABC by Art. 25, then find the area of the lesser segment ADC (by the same Art.) and subtract the area of the lesser segment from the area of the greater, and the remainder will be the area of the Lune ABCD, &c.

Art. XXVIII. To find the area of Compound Figures. Mixed or compound figures are such as are composed of rectilineal and curvilineal figures together, as (fig. 27.)

Rule.—First find the area of the trapezium ABCD by Art 8, then find the area of the segments AED and BFC by Art. 25, and add the areas all together and you will have the area of the whole figure.

Art. XXIX. To find the circumference of an ellipsis, the transverse and conjugate axis being given.

Rule.—Multiply half the sum of the two axis by 3; to the product add $\frac{1}{7}$ part of the sum of the two axis, and this sum will give the circumference near enough for most practical purposes.

What is the circumference of an ellipsis whose transverse axis AB (fig. 28,) is 24 feet, and the conjugate CD 18 feet.

24
×18
———
2)42
———
21
×$3\frac{1}{7}$
———
63
×3
———
66 feet the circumference.

Art. XXX. To find the area of an ellipsis, the transverse and conjugate axes being given.

Rule.—Multiply the transverse axis by the conjugate, and the product by 7854, will give the area required.

What is the area of an ellipsis whose transverse axis AB (fig. 28.) is 30 feet, and the conjugate CD 20 feet.

30
×20
———
600

7854
×600
———
Area 471,2400

Note.—Ellipsis' of a large size, are frequently laid out in gardens of which they can be accurately drawn, by driving two pins into the ground at the foci of the ellipsis, for the chord to revolve around, (as described in Geometry Problem 38.)

Art. XXXI. To find the area of an Elliptical Ring, or the space included between the circumference of two concentric and similar Ellipsis.*

Rule.—First find the area of the greater ellipsis ABCD (fig. 29.) by Art. 30; then find the area of the lesser EFGH (by the same Art.) and subtract the area of the lesser from the area of the greater, and the remainder will be the area of the ring; or from the product of the two diameters of the greater ellipsis, subtract the product of the two diameters of the lesser; the remainder multiplied by 7854 will be the area of the ring.

Note.—This rule will also serve for a circular ring; for when the diameters of each ellipsis become equal to the square of the diameter of the greater circle diminished by the square of the diameter of the less, and the remainder multiplied by .7854 is the area of the circular ring, or, multiply the sum of the diameters by their difference, and that product into .7854 for the area of a circular ring.

Art. XXXII. To find the area of a parabola, the base or double ordinate being given, and the axis or height.

Rule.—Multiply the base by the height, and two thirds of this product will be the area required.

What is the area of the parabola ABC (fig. 30) the axis CD being 12f. 3i. and the double ordinate AB 18f. 6i.

12-25
×18-50
———
61250
9800
1225
———
3)2266250
———
755416
×2
———
Area 151,0832

* It is here supposed that the difference between the conjugate diameters is equal to the difference between the transverse diameters, but it is well known, that in this case the elliptic arc will not be every where equi-distant; the difference between the semi-transverse or semi-conjugate diameters being the least distance between the arc.

Art. XXXIII. To find the area AB, CD (fig. 31) of the frustum of a parabola whose parallel ends AB and CD are given, also their distance EF.

Rule.—To the square of the greatest end, add the square of the lesser end, to the product of the ends; divide the sum by the sum of the ends, and the quotient multiplied by the distance of the ends, two thirds of the product will be the answer.

Suppose the end AB 24, the end CD 20, and their distance EF 5; required the area ABCD.

```
 24              20       33            24
×24             ×20        5            20
 —              ———      ———           ———
 96             400     3)165           480
 48             576      ———
———             480       55
576            ———         2
     24+20=44)1456(33    ———
                132      110 Answer nearly.
               ———
                136
                132
               ———
                  4
```

Art. XXXIV. To find the area of a cylinder.

Rule.—Multiply the circumference by the length of the cylinder and the product will be the area.

What is the area of the cylinder AB, CD, (fig. 32,) whose diameter AB being 24 inches, and the perpendicular height or length EF is 128 inches.

First I find the circumference by Art. 12, of which is $75\frac{42}{100}$ inches, this multiplied by 128 inches, the length, and divided by 144, gives $67\frac{04}{100}$ feet the area.

```
as 7 : 22 : : 24                 75.42
              22                   128
            ———                 ———
              48                 60336
              48                 15084
            ———                  7542
          7)528(75,42 circumference ———
            49               144)965376(67,04 Area.
            ———                  864
             38                 ———
             35                  1013
            ———                  1008
              30                ———
              28                   576
            ———                    576
              20
              14
```

There is to be a circular stair-case built around this cylinder containing 15 treads and 16 risers; I demand what the width of these treads and risers will be and also the length of the hand rail.

First I divide the 15 treads by $75\frac{42}{100}$ inches, which gives $5\frac{028}{1000}$ inches for the width of the treads, being a trifle over 5 inches, then I divide the 16 risers by 128 inches, and it gives 8 inches for the height of the risers, and by squaring the circumference and perpendicular height, then extract the square root of the two sums, and it gives the length of the rail as required.

```
15)75,42(5.028     16)128(8        128
   75                 128          128
  ———                ———          ———
    42                            1024
    30                             256
   ———                            128
    120                          ———
    120                          16384
                                  5688 in parts.

          75,42                  2'20'72(148,56 length of rail.
          75,42                  1
         ———                    ———
          15084               24)120
         30168                    96
        37710                    ———
       52794                 288)2472
       ———                        2304
       5688,1764                 ———
                            2965)16800
                                 14825
                                ———
                           29706)197500
                                 178236
```

MENSURATION OF SOLIDS.

Section II.

Solid measure is the finding the number of cubic inches, feet, yards, &c. contained in any thing that consists of length, breadth and thickness. The least solid measure is a cubic inch, and all solids are measured by cubes whose sides are inches, feet, yards, &c., and the solidity of a body is said to be so many cubic inches, feet, yards, &c., as will fill the same space as the solid, or as the solid will contain; that is, 12×12×12=1728 cubic inches, which makes one cubic or solid foot.

A Table of Cubic Measure.

1728	cubic or solid	inches,	make	1	solid foot.
27	"	" feet,	"	1	" yard.
$166\frac{3}{4}$	"	" yards,	"	1	" pole.
64000	"	" poles,	"	1	" furlong.
512	"	" furlongs,	"	1	" mile.

Art. XXXV. Of a Cube. A Cube is a solid of six equal sides, each of which is an exact square.

To find the solidity.

Rule.—Multiply the side of the cubic into itself, and that product again by the side; and the last product will be the solidity.

What is the solidity of a cube as ABCD (fig. 33, pl. 34, of Mensuration,) whose sides are 12 inches, or 12 feet.

```
  12                      12
  12                      12
 ———                     ———
 144                     144
  12                      12
 ———                     ———
 1728 inches.            1728 feet.
```

Art. XXXVI. To find the solidity of a parallelopipedon. A parallelopipedon, is a solid, having six rectangular sides, every opposite pair of which are equal and parallel.

Rule.—Multiply the breadth by the depth, and that product by the length and it will give the solid contents.

Example 1.—How many cubic feet is there in a rectangular box as ABCDE (fig. 34.) whose breadth AB, is 2f. 4i., depth AC, is 3f. 8i. and length DE, 12f. 3i.

```
   AC=3-8                      Or thus AC=44 inches.
   AB=2-4                              AB=28 inches.
     1-2-8                                352
     7-4                                  88
     8-6-8 area of base.                1232 area of base.
  DE=12-3-0                         DE=147 inches.
    2-1-6-0-0                          8624
  102-8-0                              4928
Ans. 104-9-6-0-0                      1232
     f.  i. p.                  1728)181104(104,80 Ans.
                                     1728
                                     8304
                                     6912
                                    13920
                                    13824
                                       96
```

Example 2.—How many feet of wood is there in a load whose breadth is 3f. 10 in., and depth 4f. 3 in., and 8 feet long.

```
           3-10
           4- 3
          11- 6
       15- 4
       16- 3- 6 area of the end.
        8
  Ans. 130- 4- 0
        f.  i. p.
```

Art. XXXVII. To find the solidity of a prism; a prism is a body with two equal or parallel ends, either square, triangular or polygonal, and three or more sides which meet in parallel lines, running from the several angles at one end to those of the other.

Rule.—Prisms of all kinds, whether square, triangular or polygonal are measured by one general rule, viz. First find the area of the end or base, by Art. 11, and then multiply the area of the end by the perpendicular height or length of the prism, it will give the solid content.

Example 1.—How many feet of timber is there in a stick of which is hewn three square, as ABC (fig. 35) the sides being 16 inches, and the perpendicular EF is 7 inches, and the length CD is 14 feet.

```
   AB=16 inches              or thus, area of base=112 inches
   EF= 7 inches                     length CD=168 inches
   ———                                      ———
    112 area of base                         896
   CD=14 feet                               672
    448                                    112
   112                                     ———
   ———                              1728)18816(10.80 ans.
144)1568(10.80 ans.                      1728
    144                                  ———
   ———                                   15360
    1280                                 13824
    1152                                 ———
   ———                                    1536
     128
```

Example 2, How many feet is there in in the octagon, or eight squared stick, as ABCDEFGH (fig. 36) the sides being 6 inches and $\frac{5}{8}$ wide, and half of the perpendicular height is 4 inches the length 18 feet 6 inches.

the sides is=53 inches
$\frac{1}{2}$ perp. = 4 inches

212 area of the end
length EK=185 inches

1060
1696
212

144)39220(27.23 ans.
288

1042
1008

340
288

520
432

88

Art. XXXVIII. To find the solidity of a cylinder.

Rule.—The diameter of a base being given, find the area of the end by Art. 14, then multiplying the area of the base by the length, that product will be the contents of the cylinder.

What is the solidity of a cylinder as (fig. 37) whose height CD is 12 feet, and the diameter AB of the base $2f\ 6i$.

2.5
2.5

12.5
50.

62.5

7854
625

39270
15708
47124

4908750 area of the base.
12

ans. 58.905000

Note.—If the circumference of a cylinder be given, multiply the square of the circumference by 07958 and the product will be the area of the base, then multiply the area of the base by the length, and it will give you the solid content.

Art. XXXIX. To find the solidity of a cylindroid.

Rule.—The diameter of the base being given, find the area of the end by Art. 30, then multiplying the area of the end by the length, that product will be the content of the cylindroid.

What is the solidity of a cylindroid as ABCDE (fig. 38) the transverse axis AB is $3f\ 9in$, and the conjugate axis CD is $2f\ 3in$ the length DE $10f$.

AB=375
CD=225

1875
750
750

84375

84375
7854

337500
421875
675000
590625

662681250 area of base
10

ans. 66.26812500

Art. XL. To find the solidity of a pyramid.

Rule. Find the area of the base, whether triangular, square, polygonal, or circular, by the rules already mentioned in superficial measure; then multiply this area by one third of the height, and the product will be the solid content of the pyramid.

Example 1. In a triangular pyramid ABC (fig. 39) the height DF being $15f\ 9i$ and each side of the base is $6f\ 6i$ the perpendicular DE $2f\ 10i$.

side AB=6.6
perp DE=2.10

5.5.0
13.0

18.5.0
$\frac{1}{3}$ of DF= 5.3.

4.7.3
92,1

ans. 96.8.3

or thus AB=6.5
6.5

325
390

4225
tab. area=4330

126750
12675
16900

18.29.4250
$\frac{1}{3}$ of DF=5.25

9145
3658
9145

Ans. 96.0225

Example 2.—What is the solidity of a quadrangular pyramid, the height EF (fig. 40) being 30f. 6i., and each side of the base is 12f. 7i.

Sides=12-7
12-7

7-4-1
151-0

158-4-1 area of base,
$\frac{1}{3}$ EF= 10-2

26-4-8-2
1583-4-10

Ans. 1609-9-6-2

Example 3.—What is the solidity of a hexagon pyramid as (fig. 41,) the height GH being 39 feet, the sides ABC &c. is 14 f. 6i. and the perpendicular IG 12f. 6i.

$\frac{1}{2}$ of sides=43-6
perp. IG=12-6

21- 9-0
522- 0

543- 9-0 area of base,
$\frac{1}{3}$ of height=13

1638- 9
543

Ans. 7068- 9

Example 4. What is the solidity of a cone, the diameter AB (fig. 42) being 9 feet, the height CD 27 feet.

9
9

81 square of the diameter.
7854 tab. multiplier.

7854
62832

63,6174 area of base.
9=$\frac{1}{3}$ of height.

572,5568=content.

Art. XLI. To find the solidity of the frustum of a pyramid. A frustum of a pyramid is what remains after the top is cut off by a plane parallel to the base.

Rule.—If it be a frustum of a square pyramid, multiply the side of the greater base by the side of the less; to this product add one third of the square of the difference of the sides, and the sum will be the mean area between the bases; but if the base be a triangular, polygon or any regular figure, multiply this sum by the proper multiplier of its figure in the table (Art. 11) and the product will be the mean area between the bases; lastly multiply this by the height, and it will give the content of the frustum.

Example 1.—What is the solidity of the triangular frustum of a pyramid, the sides of the greater base AB (fig. 43) being 9 feet, the side of the less CD 6 feet, and the height EF is 10 feet.

AB=9
CD=6

3 difference of the sides.
×3

3)9=square of the difference.

3=$\frac{1}{3}$ of the square.

9
6

54
Add 3

57
433 tab. multiplier.

171
171
228

24,681=mean area.
10=height.

246,810=content.

Example 2.—What is the solidity of a frustum of a square pyramid, the side of the greater base AB (fig. 44) being 9 feet, the side of the less CD 6 feet, and the height ED is 10 feet.

9
6

3=difference.
×3

3)9=square of the difference.

3=$\frac{1}{3}$ of the square.

AB=9
CD=6

54
add 3

57 mean area.
height ED=10

570=content.

Example 3.—What is the solidity of a frustum of an octagonal pyramid, whose sides AB (fig. 45) of the greater base is 9 feet (as before,) the lesser CD 6 feet, and the height ED 10 feet.

AB=9
CD=6

3=difference of the sides.
×3

3)9=square of the difference.

3=⅓ of the square.

AB=9
CD=6

54
Add 3

57
4828 tab. multiplier.

33796
24140

275196=mean area.
10=height.

2751,960 content.

In measuring square timher where the sticks run tapering, the best method is, viz: Take the girth of it in the middle; square ¼ of the girth, or multiply it by itself in inches. Then say as 144 inches is to that product, so is the length taken in feet, to the contents in feet.

Example 4.—What is the solidity of a tapering square stick of timber, whose sides of the largest end is 14 inches, the least end 10, and the length 40 feet.

One fourth of the girth in the middle=12 and 12×12=144, the area in the middle; then as 144 : 144 : :40 feet : 40 feet the content.

or thus 12×12=144
40=length.

144)5760(40 content.
576

0

There is more timber of this description measured by the following rule, than any other, viz: By adding the area of the two ends together, and one half of the sum multiplied into the length and divided by 144 for the solid content; but it is not so correct as the former, see the following example.

14×14=196
10×10=100

2)296

148
×40=length.

144)5920(41,11 content.
576

160
144

160
144

160
144

16

There is more than a foot's difference in the two examples; but in measuring small timber of not much value, this rule may apply; to find the area of a board that runs wedging, this rule is correct, as may be seen by Art. 5. And if it be a tapering three square stick of timber, you may find the area midway from the end, then as 144 is to the area, so is the length taken in feet, to the content in feet. Or by multiplying the area of the two bases together, and to the square root of the product add the two areas; that sum, multiplied by one third of the length, will give the solidity of any frustum.

Example 5. What is the solidity of a tapering square stick of timber, the greater end being 14 inches (as in example 4) the lesser end 10, and the length 40 feet.

14×14=196
10×10=100

1-96-00(140
1

24)96
96

00

Area=196
100
140

436
⅓ of 40=13,33

1308
1308
1308
436

144)581188(40,36=content.
576

518
432

868
864

4

Art. XLII. To find the solidity of a wedge as ABCDEF, (fig. 46.)

Rule.—Multiply the area of the base ABC, by the length BE, and half of the product will give the solidity.

Required the solidity of a wedge ABCDEF, the side AB being 9 inches, and AC 18 inches, the length BE 4 feet.

AC=18
AB= 9

162
BE =4

144)648(4,50
576

720
720

2)4,50

2,25=content.

Art. XLIII. To find the solidity of a frustum of a cone.

Rule.—Multiply the diameters of the two bases together, and to the product add one third of the square of the difference of the diameters; then multiplying this sum by 7854 it will be the mean area between the two bases; which being multiplied by the length of the frustum, will give the solid content.

Note.—In general way in measuring or guaging a frustum, it is hest to find the mean area in inches. Then by multiplying the inches by the height in feet, and by dividing the product by 12, the quotient will be the answer in board measure, or divide by 144, the quotient will be in square measure; or if you multiply the mean area in inches, by the height in inches, and divide the product by 1728, gives it also in square feet; and when we have the solid content in cubic inches, we can easily find the number of gallons and bushels, &c., by dividing by 231 for wine gallons, 282 for ale gallons, and by 2150.4 for bushels; of guaging, this part will be fully explained hereafter.

Example 1.—What is the solidity of a frustum of a cone; whose greater base AB (fig. 47) being 60 inches, the lesser CD 51 inches, and the perpendicular height EF 6 feet 6 inches.

AB=60
CD=51

60
300

3060
Add 27

3087
7854

12348
15435
24696
21609

2424,5298=mean area.
6,5½feet=length.

12120
14544

157560=content.

60
51

9=difference.
×9

3)81 square of the difference.

27=⅓ of the square.

144)157560(109,41 feet content.
144

1356
1296

600
576

240
144

96

Example 2. What is the solidity of a mast or spar, whose diameter is 30 inches at one end, and 18 inches at the other, and 80 feet long.

30
18

240
30

540
Add 48

588

7854 tab. multiplier.
×588

62832
62832
39270

46181,52=mean area.
80=length.

144)3694480(256,56 feet content.
288

814
720

944
864

808
720

880
864

16

30
18

12=difference.
×12

3)144 square of the difference.

48=⅓ of the square.

Art. XLIV. To find the solidity of an ellipsis frustum of a cone.

Rule.—Find the area of the two bases by Art. 30, and thence multiply the areas of the two bases together, and to the square root of the product add the two areas; that sum multiplied by one third of the height. This rule will give the solidity of any frustum; for it is plain when figures run uniformly taper but not to a point, they being considered as portions of the cone or pyramid; we may find the solidity by supplying what is wanting to complete the figure, and then deducting the part cut off.

A general rule for completing every straight sided solid whose ends are parallel and similar: As the difference of the top and bottom diameters is to the perpendicular height, so is the longest diameter to the altitude of the whole cone or pyramid.

Example.—What is the solidity of a frustum of a cone, as ABCD, EFGH, (fig. 48) whose transverse axis AB of the greater base is 60 inches, the conjugate axis CD 42 inches, and the transverse axis EF of the lesser base 40 inches, and the conjugate GH 28 inches, the height DH 6 feet.

The area of the greater base ABCD=1979 inches.
The area of the lesser base EFGH= 879 inches.

```
              17811          1979=area.
              13853           879=area.
              15832          1318=sq. root of prod
                             ----
 1·73·95·41(1318             4176
 1                              2=⅓ of height.
 --
 23)73                   144)8352(58 feet content.
    69                        720
 --                           ----
 261)495                      1152
     261                      1152
 2628)23441
      21024
      -----
       2417
```

Art. XLV. To find the solidity of a parabolic conoid; the diameter AB (fig. 49) of the base being given, and the perpendicular height CD.

Rule 1. Multiply the square of the diameter of the base by .3927, and the product by the height will give the solidity.

Example 1. What is the solidity of a parabolic conoid, whose diameter AB is 30 feet, and the height 50 feet.

```
               3927 multiplier.
 AB 30×30 =     900
              --------
              3534,300
                    50=height.
            ----------
            17671,700 the solidity required.
```

Rule 2.—Multiply the area of the base by the height, and half the product will be the solid content.

Example 2. AB 30×30=900 (as before.

```
        7854
         900
     -------
     7068600=area.
          50=height.
    ----------
   2)353430000
    ----------
     17671.5=content.
```

Art. XLVI. To find the solidity of the frustum of a parabolic conoid; the greater diameter AB, (fig. 50) the lesser CD, and the perpendicular height EF, being given.

Rule.—To the square of the diameter of the greater end AB, add the square of the diameter of the lesser end CD; multiply the sum by 3927, (being one half of 7854,) and the product by the height EF will give the solidity required.

What is the solidity of a parabolic frustum, the diameter of the greater end AB being 5 feet, the lesser end CD 4 feet, and the distance of the ends EF 4 feet.

```
 AB 5×5=25           3927
 CD 4×4=16             41
        --            ----
        41 sum.       3927
                     15708
                    ------
                    161007
                         4=the distance EF.
                   64,4028=content.
```

Art. XLVII. To find the solidity of a parabolic spindle.

Rule.—Multiply the square of the middle diameter by .41888 (being $\frac{8}{15}$ of .7854) and that product by its length; the last product is the solid content.

What is the solidity of a parabolic spindle whose middle diameter AB (fig. 51) is 30 inches, and its length CD 5 feet.

```
 AB=30              41888
    30                900
    --           --------
   900=square.   37699200
                        5=length.
                 --------      carried up.

 144)1884960(13,09 content.
     144
     ---
      444
      432
      ----
       1296
       1296
```

Art. XLVIII. To find the solidity of a sphere or globe.

Rule.—Multiply the cube of the diameter by 5236 and the product is the solidity.

What is the solidity of a globe whose diameter AB (fig. 52) is 4 feet.

```
 AB=4            5236
    4              64
    --          -----
   16           20944
    4          31416
   --          ------
   64 cube.    33,5104 content.
```

Note.—If it be a sphere of a semi-circle as in (fig. 53,) first find the whole solidity as in (fig. 52,) and divide the product by 2, and the quotient will be the answer.

Art. XLIX. To find the solidity of a segment of a globe.

Rule.—To three times the square of the semi-diameter of the base, add the square of the height; then multiplying that sum by the height, and then the product multiplied by 5236 will give the solidity.

What is the solidity of a spherical segment; the diameter of the base AB (fig. 54) being 18 feet, and the height of the segment CD is 4 feet.

```
 ½ of 18 is=9 the semi-diameter.     5236
            9                        1036
           --                       -----
           81=square.               31416
           ×3                       15708
           --                      52360
          243                      --------
      4×4=16                       542,4496=solid content.
          ---
          259
           ×4=height.
          ----
         1036
```

Art. L. To find the solidity of a spherical zone, the radius AB (fig. 55) and CD of the two parallel circles at the end being given and their distance BC.

Rule.—To the square of the two radii add one third of the square of the height; multiply the sum by the height, and the product by 1.5708, will give the solidity.

What is the solidity of a spherical zone, whose greater radius AB is 10 feet, the lesser CD 8 feet, and the height or distance of the ends BC is 6 feet.

```
 AB 10×10=100           6                1,5708
 CD  8× 8= 64           6                  1056
      Add  12          --                ------
          ---       3)36=square.          94248
          176          --                 78540
           ×6=height.  12=⅓of the sq.   157080
          ----                          ---------
          1056                         1658,7648=content.
```

Art. LI. To find the solidity of a spheroid, the fixed axis and the revolving axis being given.

Rule.—Multiply the square of the revolving axis by the fixed axis, and that product by .5236 for the solidity.

What is the solidity of a prolate spheroid whose transverse axis CD (fig. 56) is 10 feet, and the conjugate AB 6 feet.

```
    AB=6             5236
        6             360
       --            -----
       36           314160
    CD=10           15708
       ---         ---------
       360         188,4960=content.
```

Art. LII. To find the solidity of an annulus or a cylindric ring, whose thickness and inner diameter are known.

Rule 1.—To the thickness of the annulus, add the inner diameter; multiply the sum by the square of the thickness, and the product by 2.4674 will give the solidity sought.

What is the solidity of an annulus, whose inner diameter AB (fig. 57) is 8 inches, and the thickness of the annulus BC is 3 inches.

```
       8             2,4674
       3                 99
      --             ------
      11             222066
  3×3=9              222066
      --            ---------
      99            244,2726=content.
```

Rule 2.—Multiply the circumference round the middle of the annulus, or that circle generated by the centre of the generating circle, by the area of the generating circle, and the product will give the solidity.

Note.—This last method will give the solidity of any part of an annulus or ring comprehended between any two planes passing through the fixed axis.

OF THE FIVE REGULAR SOLIDS.

Definitions 1. A regular solid, is a body that either may be inscribed or circumscribed by a sphere, in such a manner as to be contained under equal and similar planes; alike posited, and equally distant from the centre of the sphere.

II. The *Tetraedron*, is contained under four equilateral triangles.

III. The *Hexaedron*, is contained under six equal squares.

IV. The *Octaedron*, is contained under eight equilateral triangles.

V. The *Dodecaedron*, is contained under twelve equilateral and equiangular pentagons.

VI. The *Icosaedron*, is contained under twenty equilateral triangles.

TO FIND THE SUPERFICES, AND SOLIDITY, OF ANY OF THE FIVE REGULAR BODIES.

To find the Superfices.—Multiply the area (taken from the following Table) by the square of the linear edge of the solid, for the superficies.

To find the Solidity.—Multiply the tabular solidity by the cube of the linear edge, for the solid content.

A Table of the Surfaces and Solidities of the five regular Solids.

No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octraedron	3.46410	0.47140
12	Dodecaedron	20.64573	7.66312
20	Icosaedron	8.66025	2.18169

Example 1.—If the linear edge or side of a tetraedron be 3, required it superficial and solid content. Thus $1.73205 \times 9 = 15.58845$ superficies.
And $0.11785 \times 27 = 3.18195$ solidity.

Example 2.—What is the surface and solidity of the hexaedron, whose linear side is 2? Answer { superficies =24 / solidity = 8 }

Example 3.—Required the superficies and solidity of the octaedron, whose linear side is 2. Answer. { superficies = 13.8564 / solidity = 3.7712 }

Example 4.—What is the superficies and solidity of the dodecaedron, whose linear side is 2? Answer { superficies = 82.58292 / solidity = 61.30496 }

Example 5.—What is the superficies and solidity of an icosaedron, whose linear side is 2? Answer { superficies = 34.641 / solidity = 17.45352 }

For finding Convex Surfaces of Solids.

Art. LIII. To find the convex surfaces of a cube, or a square box as in (fig. 33, Pl. 34, of Mensuration.)

Rule.—Find the surface or area of one of the sides as in section first, and multiply it by 6, and the product will be the whole surface.

What is the convex superficies of a cube or a box whose sides is 2 feet 4 inches.

```
 2-4
 2-4
 ---
 9-4
4-8
-----
6-5-4
6
-----
32-8-0= the whole surface.
```

Art. LIV. To find the convex surfaces of a parallelopipedon as in (fig. 34.)

Rule.—Find the surfaces of the depth, breadth, and one of the ends, and double it, or multiply it by 2, which is the same thing.

What is the convex superficies of a parallelopipedon whose depth is 2f. 8i. and breadth 3f. 2i., and 4f. 7i. in length.

```
  2-8                              2- 8
  3-2                              3- 2
  ---                              -----
  5 4                              5-10=depth and breadth.
8-0                                4- 7=length.
-----                              -----
8-5-4=the surface of one end.     3-4-10
                                 23-4
                             add 8-5-4
                                 -----
                                 35-2-2=whole surface.
```

Art. LV. To find the convex surfaces of any prism, as in (figures 35, and 36.)

Rule.—Multiply the circumference of the base by the length of the prism, the product will be the upright surface, to which add the area of the bases; the sum will be the whole surface.

Art. LVI. To find the convex surfaces of a cylinder.

Rule.—Multiply the circumference by the length of the cylinder (as in figs. 37, 38,) the product will be the surface of the length, thence add the area, or the surfaces of the two ends, and the sum will give the whole surface.

Art. LVII. To find the convex superficies of a pyramid or a right cone, (as in figures 39, 40, 41, and 42,) the circumference and slant side being given.

Rule.—Multiply the circumference of the base by the slant side of the cone and half the product will be the area; or multiply the slant height by half the circumference of the base, and the product will be the upright surface. To which the area of the base may be added, for the whole surface.

Art. LVIII. To find the convex surface of a frustum of a right cone as in (figs. 43, 44, 45, 47, and 48) the circumferences of both ends being given, and the slant side of the cone.

Rule.—Multiply the sum of the circumferences by the slant side of the cone, and half the product will be the area.

Or multiply half the sum of the perimeters of the two bases by the slant height and to the product add the areas of the two bases for the whole surface.

Art. LIX. To find the superficies of a sphere or globe as in (fig. 52.) the greatest circumference being given.

Rule.—Multiply the square of the circumference by .3183, and the product will be the superficies.

Example. What is the superficies of a globe the greatest circumference being 10.6 feet. Thus $10.6 \times 10.6 \times .3183 = 35.764188$ the superficies required.

Art. LX. To find the convex superficies of the segment, sphere, or globe, as in (fig. 54.) the diameter of the base of the segment, and its height, being given.

Rule.—To the square of the diameter of the base add the square of twice the height, and the sum multiplied by .7854 will give the superficies.

Example. What is the convex surface of the segment of a globe, the diameter of the base being 17.25 feet, and the height 4.5 feet.

$2 \times 4.5 = 9$ twice the height.
$9 \times .9 = 81$ square of twice the height.
——.2
$17.25 = 297.5825$ square of the diameter of the base.

Then $297.5625 + 81 \times .7854 = 297.3229975$ the superficies required.

Art. LXI. To find the convex surfaces of a spherical zone, the diameters of the ends and their distance being given, as in (fig. 55.)

Rule.—Find the diameter of the sphere by Arts. 21, and 22, in section first of superficies; then multiply the diameter of the sphere, and the distance of the parallel ends of the zone together, and the product by 3.1416, will be the superficies required.

Example. In a spherical zone the distance of the parallel ends being 4 inches the diameter of the greater end 24 inches, and that of the lesser end 20 inches, what is the convex superficies, when the centre of the sphere is without the zone.

The distance of the greater chord from the centre, will be found to be 3.5 inches by Art. 21.

The radius will be found to be 25 inches, by Art. 22 or the diameter 50 inches. Then $50 \times 4 \times 3.1416 = 628.92$ the answer.

Note.—If the diameter is given, find the circumference, and procced as before.

Art. LXII. To find the convex superficies of an annulus, or ring as in (fig.57.) whose thickness and inner diameters are known.

Rule 1.—To the thickness of the ring add the inner diameter; multiply the sum by the thickness, and the product hy 9.869 will give the superficies required.

Rule 2.—Multiply the circumference of the generating circle by the circumference round the middle of the ring, or that line generated by the centre of the generating circles and the product will be the area.

Note.—This last method will give the convex superficies of any part of an annulus or ring, comprehended between two planes passing through the fixed axis.

Example. What is the convex superficies of an annulus or ring, whose inner diameter is 8 inches, and the thickness 3 inches.

Thus $3 + 8 \times 3 \times 9.869 = 325.677$ the superfices required.

Of measuring irregular surfaces and solids.

Definition. An irregular surface or solid, is such a surface or solid as have their bounds by lines or surfaces in any manner whatever, of no particular kind of form or shape but merely accidental, according as they are to be found or given.

Art LXIII. To measure any irregular surface whatever by means of equidistaordinates.

Rule 1.—To the half sum of the two outside ordinates, add the sum of all the other remaining ordinates; multiply the whole sum by the distance between any two ordinates, and the product will be the superficial content.

Example 1. Let (fig. 58, pl. 34, of mensuration) be the curve proposed, whose equidistant ordinates, AB, CD, EF, GH, IK, LM, and NO, are respectively 5f. 5f. 6i. 6f. 7f. 9f. 10f. and 8f. and the distance of AC, CE, EG, &c. is 3 feet, required the area of the curve.

```
 AB=5
 NO=8
   --
 2)13
   --
   6.6=half the sum of the outside ordinates.
 CD=5.6
 EF=6.0
 GH=7.0
 IK=9.0
LM=10.0
   ----
   44.0
   3
   ----
   132=superficies.
```

Example 2, Let ABCD (fig. 59) be a circle whose diameter AC or BD is 10 feet, it is required to find the area by means of equidistant ordinates, marked 3f. 4f. 4.5f. 4.9f. and 5f. being at the distance of 1 foot from each other.

0
5
—
2)5
—
2.5 halfsum of the outside ordinates.
3
4
4.5
4.9
——
18.9 area of one quarter.
4
——
75.6 feet, area of the whole.

If the diameter, which is 10 feet, be multiplied by .7854, the product, 78.54, will be the area. From hence it appears that the mode of operation by means of equidistant ordinates, is very near the truth in measuring irregular planes; for it will produce the area of a circle, which is one of the most oblique curves possible as the ends raise quite perpendicular to the axis, from only 10 equidistant spaces within the $\frac{1}{25}$ part of the truth; and would be still nearer when applied to measuring any plane surface, where it is bounded partly by concave and partly by convex curves; because, if wholly bounded by a convex curve, or curves, the area will be something less than the truth, but if bounded by a concave curve, or curves, the area will be something greater than the truth; and if the extremities of the ordinates are joined by straight lines, the area so found will be exactly true; but the following is a method of approximation still nearer the truth, whether the curve be convex or concave to the axis.

Rule 2.—Divide the given curve, by ordinates, into any even number of equal parts, then add into one sum four times the sum of all the even ordinates; twice the sum of all the odd ordinates except the first and last, and also the first and last ordinates; and if one third of that sum be multiplied by the common distance between any two ordinates, the product will be the answer.

Example 1. Let (fig. 58) be a curve of any kind, whose equidistant ordinates AB, CD, EF, GH, IK, LM, and NO, are respectively 5f. 5f. 6i. 6f. 7f. 9f. 10f. and 8f., and the distance between the ordinates is 3f. required the area of the curve.

CD, GH, and LM, will be the even ordinates; that is the second, fourth, and sixth; EF and IK, the odd ordinates, that is, the third and fifth; AB and NO, the first and last.

f. i.
CD=5.6
GH=7.0
LM=10.0
——
22.6
4
——

f.
EF=6
IK=9
—
15
2
—
30

90.0 four times the sum of the even ordinates.
30.0 twice the sum of the odd ordinates.
5.0 first ordinate.
8.0 last ordinate.
——
3)133.0 sum
——
44.4
3
——
133.0 the area or superficial content.

Now by comparing this area, viz: 133 feet, with the area found in rule 1, example 1, viz: 132 feet, there appears to be a difference of 1 foot; but the last method is the most correct.

Example 2. Let ACEGILN (fig. 60) be a concave curve, whose equidistant ordinates A, BC, DE, FG, HI, KL, and MN, are respectively 0, 1, 3, 6, 10, 15, 21, and the common distance 2, required the area.

By example 1. 0
21
—
2)21
—
10.5
1
3
6
10
15
—
45.5
2
——
91.0 the area greater than the truth.

By Example 2. 3
10
—
13
2
—
26

1
6
15
—
22
4
—

88 four times the sum of the even ordinates.
26 twice the sum of the odd ordinates.
0 first ordinate.
21 last ordinate. carried up.
——
3)135
—
45
2 common distance.
—
90 the area very near the truth

Example 3. Let AHI (fig. 61) be a parabola, whose ordinates, A, BC, DE, FG, and HI, are respectively 0, 7, 12, 15, and 16, and their common distance 6, required the area of the curve.

7
15
—
22
4
—

12
2
—
24

88 four times the sum of the even ordinates.
24 twice the sum of the odd ordinates.
16 sum of the end.
—
3)128
$42\frac{2}{3}$
6
—
256 the true area.

Art. LXIV. To find the superficial content of a mixed figure, partly a curve and partly right lined.

Rule.—Find the area of the curve part of the figure by the last example, by dividing it into equidistant ordinates; divide the right lined parts of the figure by ordinates drawn through every angle, which will divide the right lined parts of the figure into trapezoids and triangles; find the area of each part, according to their respective rules add the areas of all the parts together, and the sum will give the area of the whole figure.

Example. Let ABKNORS (fig. 62) be the figure proposed to find its area.

As the end AB turns round nearly perpendicular to the base AS, draw the ordinate CB in such a manner, as it may cut off the most perpendicular part of the curve AB at the end, and divide it by ordinates, which are respectively 1, 2, $1\frac{1}{2}$, 1, 0, at the distance of 3 from each other; the part CBKL of the equidistant space is also divided into four equal parts, between the first and last ordinates BC, KL by the ordinates ED, GF, HI, and LK, which are respectively 12, 13, 12, 10, 9, and their common distance 4; the other parts of the figure are divided into 3 trapezoids, KLMN, MNOP, OPQR, and the triangle QRS, by ordinates from the angles at K, N, O, and R; the whole figure being thus prepared, by dividing it into curvilinear parts, trapezoids, and triangles, each part will be measured according to their respective rules. The measures or dimensions are marked on their respective places on the figure; the contents of each part is computed separately, as is shown in the following operation.

$1\frac{1}{2}$
2
—
3

2
1
—
3
4
—
12
3
1
—
3)16
—
$5\frac{1}{3}$
3
—
16 area of the part ABC.

13
10
—
23
4
—
92
24
12
9
——
3)137
45.6
4
——
182.4 area of the part CBKL.

12
2
—
24

9
16
—
2)25
—
$12\frac{1}{2}$
4
—
50 area of the trapezoids KLMN.

16
13
—
2)29
14.5
3
——
43.5 area of MNOP

13
14
—
2)27
$13\frac{1}{2}$
2
—
27 area of OPQR

2)14
7
—
7
—
49 area of the triangle QRS

16.0
182.4
50.0
43.5
27.0
49.0
——
367.9 sum of the areas, or contents of the whole figure.

Art LXV. To find the superficies of a groin.

Rule 1.—When the sides of the groin are semi-circles, to the area of the base add $\frac{1}{7}$th part of itself and the sum will give the superficies required.

Example 1. What is the curve superficies of a circular groin, each side of the square, base being 14 feet, as ABCD (fig. 65.)

14
14
—
56
14
---
196 area of the base.
add 28
-----
224 area of the groin.

7)196(28=to $\frac{1}{7}$
14

56
56

Rule 2.—When the groin stands upon a rectangular plan, the sides being either segments of circles or segments of an ellipsis.

The area of each two opposite parts of the surfaces of the cylinders, or cylindroids may be computed in the following manner, viz: let ABCD (fig. 63) be the plan of the groin; AC and BD are the intersection of the planes of the diagonals; MNQO is one of the cylindrical, or cylindroid surfaces stretched out on a plane; HVI is one of the side arches. Then to find the area of any two opposite quarters of the cylindrical, or cylindroidal surfaces to the arch line HVI standing over BC on the plan; that is, NM when stretched out on the plane, add four times the arch standing over FG on the plan taken in the middle between the end BC, and the vertex at E, that is, QR when stretched out on the plane; multiply one third of the sum by ES, or PO, which is equal to it, and the product will be double the area of the two opposite cylindrical or cylindroidal surfaces, standing over AED and BEC.

Example 2.—Let ABCD be the plan of a groin, the sides AB and BC are each equal to 8 feet; let MN, the length of the arch HVI, standing over BC on the plan, be 10 feet, and PO, equal to ES be 4 feet; that is the distance measured along from the vertex at E, to either of the ends at S, and QR the length of the arch over FG, be 4 feet; required the superficies of the groin.

```
   4
   4
   —
  16 four times QR.
  10 the end.
   —
3)26
  8-8 i.
   4
  ———
  34-8 area of the two opposite parts AED, and BEC.
   2
  ———
  69-4 area of the whole groin standing over ABCD on the plan.
```

Art. LXVI.—To find the Tonnage of a Ship.

"By a law of the Congress of the United States of America, the tonnage of a ship is to be found in the following manner.

If the vessel be double-decked, take the length thereof from the fore part of the main stem to the after part of the stern post above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel; and deduct from the length three fifths of the breadth, multiply the remainder by the breadth, and the product by the depth; divide this last product by ninety-five, and the quotient will be the true content or tonnage of such vessel.

If the vessel be single-decked, take the length and breadth as above directed, in respect to a double-decked vessel, and deduct from the length three fifths of the breadth, and taking the deapth from the under side of the deck plank to the cieling in the hold; multiply and divide as aforesaid, the quotient will be the true content or tonnage of such vessel."

Example.—Suppose a double-decked vessel is 98 feet, and the breadth 30 feet; what is her tonnage.

```
  30               Length=98          95)36000(378-94 tonnage.
 X3                   18=3/5             285
 ——                  ———                 ———
5)90                  80                 750
 ——                   X30=breadth.       665
  18=3/5 of breadth.  ———                ———
                     2400                 850
            1/2 of 30=15                  760
                    —————                ———
                    12000                 900
                     2400                 855
                    —————                ———
                    36000                 450
                                          380
                                          ———
                                           70
```

Carpenters, in finding the tonnage, multiply the length of the keel by the breadth of the main beam and the depth of the hold in feet, and divide the product by 95; the quotient is the number of tons. In double decked vessels, half the breadth is taken for the depth.

OF GAUGING.

Gauging is the art of measuring and finding the contents of all kinds of vessels, in gallons or cubic inches; such as casks, brewers vessels, &c.

Having found the number of cubic inches in any body by the rules given in (section 2 of solids.) You may thence determine the contents in gallons, bushels, &c. by dividing that number of cubic inches in a gallon, hushel, &c. respectively.

A *wine gallon*, by which most liquors are measured, contains 231 cubic inches. A *beer gallon* by which beer, ale, and a few other liquors are measured, contains 282 cubic inches. A bushel of corn, malt, &c. contains 2150.4 cubic inches.

Note.—In all the following rules, it will be supposed that the dimensions of the body are given in inches and decimal parts of an inch.

Art. LXVII.—To find the number of gallons or bushels in a body of a cubic form, (see fig. 33, pl. 34 of mensuration.)

Rule.—Divide the cube of the side by 231, the quotient will be the answer in wine gallons, or by 282, and the quotient will be the answer in beer gallons; or by 2150.4 and the quotient will be the number of bushels.

Example.—Required the number of wine gallons contained in a cubic cistern, the length of whose side is 60 inches. Multiplying 60 by itself, and the product again by 60 gives the solidity 216000; which divided by 231, gives the content $935.\frac{6}{100}$ wine gallons.

In the like manner the content of any other figure may be found, by finding the number of cubic inches in the body by the rules already taught in section second of mensuration; then bring it into gallons, bushels, &c. by dividing the number of cubic inches found in the body by 231 for wine gallons, and by 282 for ale gallons; and hy 2150.4 for bushels; or you may multiply the number of cubic inches found in the body by 004329 for wine gallons, and the product will be the number of gallons; and by 003546 for ale gallons; though the shortest and best method of gauging a frustum of a cone, and a body in a cylindrical form will be found by the following examples.

Art. LXVIII.—To find the number of gallons or bushels contained in a body of a cylindrical form (see fig. 37, pl. 34, of mensuration.)

Rule.—Multiply the square of the diameter by the height of the cylinder, and divide the product by 294, the quotient will be the number of wine gallons; if you divide by 359, the quotient will be the number of ale gallons; and if you divide by 2738, the quotient will be the number of hushels.

Note.—These divisors are found by dividing 231,282 and 2150.4 by 7854.

Example.—How many wine gallons is there in a cylinder, whose diameter of its base is 30 inches, and the length 50 inches.

```
     30=diameter.
    X30
    ———
    900 square of the diameter.
    X50=height.
294)45000(153.06 gallons.
    294
    ————
    1560
    1470
    ————
     900
     882
    ————
    1800
    1764
    ————
      36
```

Art. LXIX.—To find the number of gallons or bushels contained in a body of the form of a frustum of a cone, (see fig. 47, pl. 34 of mensuration.)

Rule.—Multiply the top and bottom diameters together, and to the product add one third of the square of the difference of the same diameters: multiply this sum by the perpendicular height, and divide the product by 294 for wine gallons, by 359 for ale gallons, and by 2738 for bushels.

Example.—How many wine gallons is there in a frustum of a cone whose greatest diameter is 70 inches, the lesser 61, and the height 72.

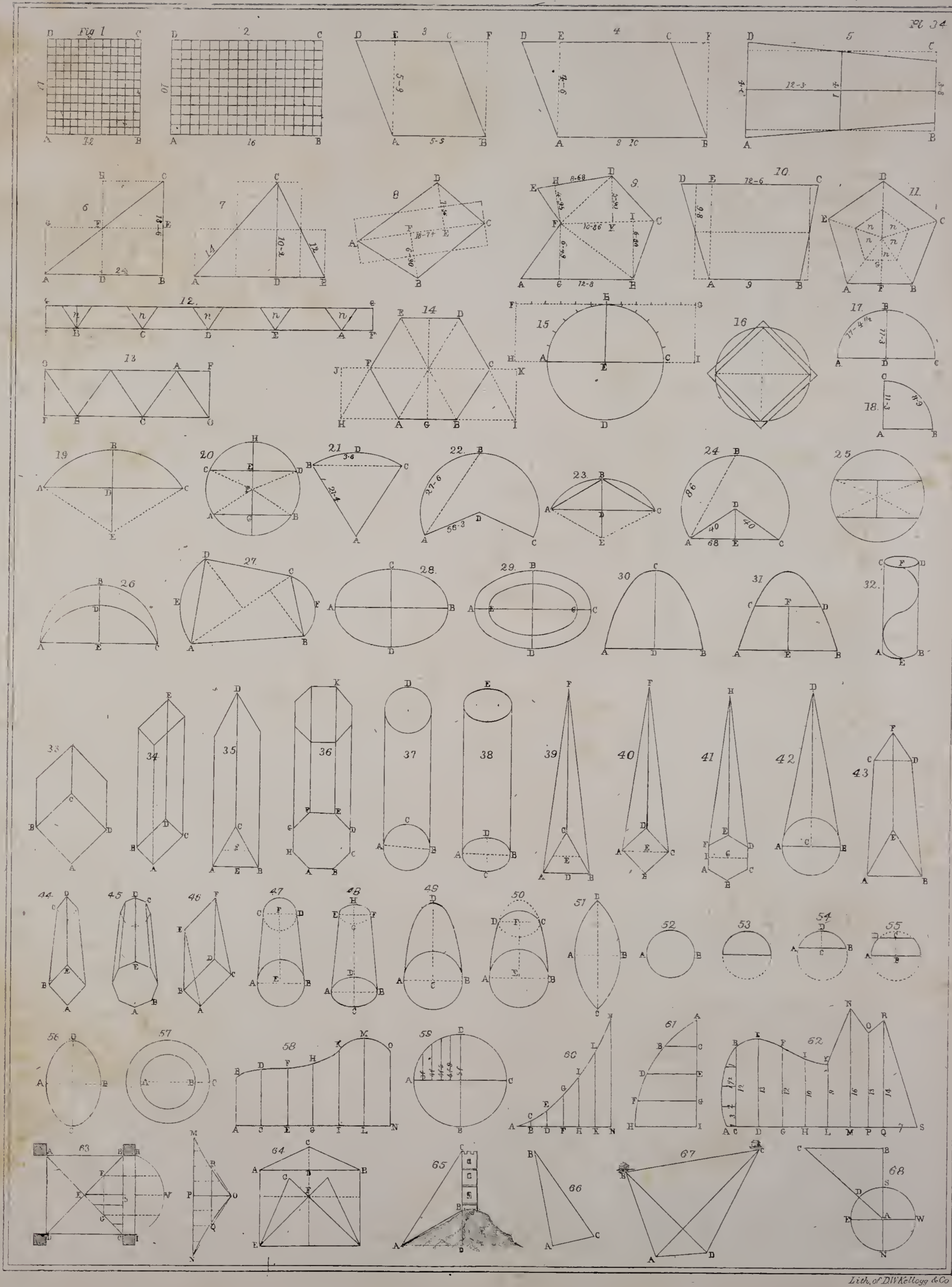
```
 70                 61=the lesser diameter.     294)309384(1052.32=galls.
 61                X70=the greater diameter.        294
 —                 ———                              ————
  9 difference.    4270                             1538
 X9                add 27                           1470
 —                 ————                             ————
3)81(27=1/3 of square. 4297                          684
  6                X72=height.                       588
 —                 ————                             ————
 21                8594                              960
 21               30079                              882
                  ——————                            ————
                  309384                             780
                                                     588
                                                    ————
                                                     192
```

Note.—It may be proper here to remark, that cisterns, frustums, &c. built either of wood, brick or stone, are generally made by the hogshead; and in gauging the above, we bring it first into wine gallons, then allow one hundred gallons for a hogshead.

Cisterns that are built of brick, are mostly built in the form of a cylinder, and a cylindroid, (see figs. 37, and 38, pl. 34 of mensuration,) and are arched over. And in gauging the above, if it be in the form of a cylinder, find the contents of the straight part by Art. 38, or by Art. 67; and if the arch be a semicircle find the contents by Art. 48; but if it be a segment of circle find the contents by Art. 49, and add them together which willgive the whole content.

But if it be in the form of a cylindroid, find the contents of the straight part by Art. 39; and the contents of the arch by Art. 52, then add them together which will give the whole content; and when you have obtained the content in cubic inches, thence you may determine the contents in gallons, bushels, &c. (by the rules given at Art. 64,) by dividing the number of cubic inches by 231, the quotient will be the answer in wine gallons, or by 232, the quotient will be the answer in beer gallons, or by 2150.4, and the quotient will be the number of bushels, &c.

Pl. 34



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LOGARITHMS.

The learner, who for the first time becomes acquainted with the wonderful properties of Logarithms, may be not a little surprised to find himself introduced to a system of numbers, so new in their nature, and which, surpassing all his former knowledge of figures, afford so many facilities for shortening the labor and lessening the difficulty of arithmetical calculations.

He will admire to find, that by help of these, the labor of hours, and, in some calculations even, the labor of days, may be reduced to as many minutes! The invention of Logarithms was justly regarded as "a favor from heaven;" because, in many departments of science, essential to the happiness of man, they have saved him ages of toil.

Although it does not come appropriately into the design of this volume to enter minutely into the history of their invention, nor the yet more difficult process by which they were originally constructed, yet a familiar explanation of their properties and uses, adapted to the apprehension and wants of the learner, is necessary, in order to his making a proper application of their great advantages in practice.

Logarithms, then, we may first observe, never stand for the numbers themselves, of which they are composed, but invariably for *other numbers*, of which they are only the representative exponents, or indices. Their great utility in arithmetical operations consists, chiefly, in this—that addition takes the place of multiplication, and subtraction, that of division. That is, to multiply numbers we have only to add their logarithms; to divide, we have only to subtract the logarithm of the divisor from that of the dividend; to raise a number to any power, we multiply its logarithm by the exponent of that power; and to extract the root of any number, we merely divide its logarithm by the number expressing the root to be found.

The constant number upon which the tables in common use are constructed, and which is called the base of the tables, is 10; and every conceivable number, large or small, integral, mixed, or decimal, is considered as some ascertained power or root of 10.

10^1	the first power of ten is	10,	whose exponent is	1.
10^2	the second power of ten is	100,	whose exponent is	2.
10^3	the third power of ten is	1,000,	whose exponent is	3.
10^4	the fourth power of ten is	10,000,	whose exponent is	4.
10^5	the fifth power of ten is	100,000,	whose exponent is	5.
10^6	the sixth power of ten is	1,000,000,	whose exponent is	6.

It may be remarked, that the first power of any number, is that number *once* repeated; or it is the number itself: The second power of any number, is the product of that number multiplied once by itself: The third power of a number, is the product of the number multiplied twice by itself: The fourth power of a number, is the product multiplied three times by itself, &c. The index denoting the power, is called, in common arithmetic the *exponent* of that power; and is, in other words, the *logarithm* of the power.

Logarithms, then, are the Exponents of a series of powers and roots.

In the above series, the *logarithms* indicate how many *cyphers* belong to their corresponding numbers. Thus, the logarithm 1 stands for 10, or 1 and one cypher; the logarithm 2 stands for 100, or 1 and two cyphers; the logarithm 3, for 1000, or 1 and three cyphers, &c. Now if we multiply 10,000 by 100, the product will be 1,000,000, whose logarithm is 6: but to obtain this, we need only add the logarithms 2 and 4, which stand opposite the numbers to be multiplied. On the contrary, if we divide 1,000,000 by 100, the quotient will be 10.000, whose logarithm is 4: but to obtain this, we need only subtract 2, the logarithm of the divisor, from 6, the logarithm of the dividend.

Again, the square of 1000, that is, the product of 1000 multiplied by itself, is 1,000,000, whose logarithm is 6: but to obtain the square of 1000, we need only double its logarithm 3. On the other hand, the cube root of 1000,000 is 100, whose logarithm is 2; but this is obtained by dividing 6, the logarithm of the given number, by 3, the index of the root. Hence it is manifest, that the protracted labor of multiplying or dividing one large number by another, the tedious evolution of roots, and the various mistakes incident to long operations, may be almost entirely obviated by the use of logarithms.

As the logarithm of 1 is always 0, and that of 10 is but 1, the logarithms of all numbers *below* 10, will be decimals: and as the logarithms in the common system increase regularly by 1, according to the integral powers of 10, it follows that the logarithms of all numbers between 10 and 100, will be more than 1, but less than 2—that is, they will be 1 and a decimal: the logarithms of all numbers between 100 and 1000, will be between 2 and 3—that is, they will be two and a decimal: and the logarithms of all numbers between 1000 and 10,000, will be between 3 and 4—that is, 3 and a decimal.

A logarithm generally consists of two parts; a *whole number*, and a *decimal*. This whole number or integer is called the *characteristic*, or *index*, of the logarithm, and is always *one less* than the number of *integral figures* in the natural number whose logarithm is sought. As the index of the logarithm is *omitted* in the tables, it is important to recollect the principle, or rule, by which it is to be supplied, whenever it is wanted in calculation. Thus, the logarithm of 8 is 0.903090. Here, the number (8) consists of but one figure, and the index of its logarithm, being *one less*, must be 0. Again, the logarithm of 16 is 1.204120. Here, the given number (16) consists of two figures, and the index of its logarithm, being *one less*, must be 1.—Again, the logarithm of 640 is 2.806180. Here, the given number (640) consists of 3 figures, and the index of its logarithm, being *one less*, must be 2, &c. The rule holds universally true, that *the index of a logarithm is always one less, than the number of integral figures in the natural number whose logarithm is sought.*

The same rule holds in *mixed* numbers. The logarithm of 6.40 is 0.80618, the same as for 640 (see the last example) differing only in the index. Here, the integral part (6) of the given number, consists of but one figure, and the index of its logarithm, being *one less*, must be 0. And, generally—having obtained the logarithm of any number, large or small, we have only to change the index, agreeably to the above rule, in order to obtain the logarithm of every other number, consisting of the *same significant figures*, whether they be integral, fractional, or mixed. Thus:—

The logarithm of	7596	is 3.880585
	759.6	2.880585
	75.96	1.880585
	7.596	0.880585
	.7596	— 1.880585
	.07596	— 2.880585
	.007596	— 3.880585

When the natural number is less than 1, the index of its logarithm becomes less than 0, or *negative*; and is indicated by placing the sign — just before or above it. Suppose it were required to affix the proper index to the logarithm of .000007596. Here, the number of cyphers on the left *including the decimal point*, is 6, which, being fitted with the negative sign —, becomes the proper index of the logarithm. And universally. *The negative index is always equal to the number of cyphers on the left, including the decimal point.*

Before any one can avail himself of the great advantages of logarithms in expediting the operations of Arithmetic and Trigonometry, he must become so familiar with the tables, that he can readily find the logarithm of any number; and, on the other hand, the number to which any logarithm belongs.

DIRECTIONS FOR TAKING LOGARITHMS AND THEIR NUMBERS FROM THE TABLE.

In the common tables. the *Indices* to the logarithms of the first 100 numbers are inserted. But, for all other numbers, the *decimal part* only of the logarithms is given; while the index is left to be supplied, according to the principles already laid down.

PROBLEM I.

To find the Logarithm of any number between 1 and 100:

Rule.—Look for the proposed number on the left; and against it, in the next column, will be the logarithm, with its index.

Example. The logarithm of 50 is 1.698970. The logarithm of 89 is 1.949390.

PROBLEM II.

To find the Logarithm of any number between 1 and 1000: or of any number consisting of not more than three significant figures, with cyphers annexed.

Rule.—Find the given number in the left hand column of the table, and directly opposite, in the next column, is the decimal part of its logarithm, to which apply the index as already taught.

Example. The logarithm of 140 is 2.146128. The logarithm of 781 is 2.892651
of 358 2.553883. of 974 2.988559

The decimal part only of these logarithms are found in the table; the index 2, was affixed to each, because the given numbers consisted, each of *three integral* figures. If there had been cyphers annexed to the significant figures of the given numbers, as 14000, 358000, &c. their logarithms would have been precisely the same, with the exception of the *index* only; and, consequently, would be found in the same place in the table. Thus—

The log. of 1400 is 3.146128. The log. of 781000 is 5.892651.
of 35800 4.553883. of 9740000 6.988559.

Here the *decimal part* of the logarithm is the same as before; while the index has been increased *as many units, as there are cyphers annexed to the given numbers.* This rule will hold good in all similar cases.

PROBLEM III.

To find the Logarithm of any number consisting of four figures, either with, or without, cyphers annexed.

Rule. Look for the three first figures, on the left hand, and for the fourth figure, at the *top* of one of the columns; the logarithm will be found opposite the three first figures, and in the column which, at the head, is marked with the fourth figure.

By reference to the table, it will be seen, that each page contains 10 columns of logarithms, which are severally numbered from 0 to 9. The first column, alone, contains six figures; while every other column has only four figures: but it is to be always remembered that *the two first figures, of the left hand column, are common to each of the other columns*, and were *omitted* only to avoid re-

petition. These two initial figures, therefore, are to be prefixed to each of the other four, since every logarithm, in our table, consists of six figures, besides the index.

EXAMPLE. The log. of 3657	is 3.563125	The log. of 3657	is	3.826704.
of 5696	3.755570	8512		3.930032.

In the last example, as it will frequently happen, the *two initial* figures (93) of the logarithm, are not found, *in the same line*, with the given number (851,) but in the next below it:—And, universally, whenever the third figure of the logarithm changes, from 9 to 10, the cypher only is retained in the column, while the 1 is carried down to the *next lower initial*, on the left. To guard against a mistake here, *points* have been substituted in place of cyphers; and whenever these points are found, the cyphers are to be reinstated, and the two initials taken from the line below.

PROBLEM IV.

To find the Logarithm of a number consisting of five or six figures.

RULE.—Find the logarithm of the *first four figures* of the given number, as taught in the last problem. Take the remaining figures and multiply them into the number standing opposite, in the outside column, headed D; from the right of the product, reject as many figures as you multiplied by, and add what is left to the logarithm previously found. This sum, being fitted with a proper index, will be the logarithm required.

EXAMPLE. Required the logarithm of 45263. Thus—

The logarithm of 45260 is	4.655715
The difference D is 96, which being multiplied by 3, gives	28.8
Logarithm of 45263 required.	4.655743

EXAMPLE 2. Required the logarithm of 758936. Thus—

The logarithm of 758900 is	5.880185
The difference D is 57, which being multiplied by 36, gives	20.52
Logarithm of 758936 required.	5.880205

This process of finding the logarithms of large numbers supposes that they increase in the same ratio as their numbers, which is not strictly true, though sufficiently near the truth for general practice. It may be remarked, however, that these ratios approach that of equality, the larger the numbers, and the less they differ from each other.

The column marked D, contains the average mean differences of the ten logarithms against which they stand, and, consequently, do not always correspond exactly to each of the differences, taken seperately; wherefore, when great accuracy is required, it may be necessary, particularly in the first part of the table, to work by

PROBLEM V.

To find the logarithm of a number consisting of six or seven figures.

RULE.—Find the logarithm of the first four figures, as before, and take the difference between this logarithm and the next greater in the table; multiply this difference by the remaining figures of the given number; reject, from the right of the product, as many figures as you multiplied by, and add what is left to the logarithm before found; this sum being fitted with a proper index; will be the logarithm required.

EXAMPLE. Required the Logarithm of 4526375.

To the first four figures, add as many cyphers as there are *other* figures in the proposed number, find the logarithm, which subtract from the next greater. Thus—

The next greater number is	4527000	and its log. is	6.655810
given number	4526000	"	6.655715
diff. of number.	1000	diff. of logs.	95
multiply by the other figures of the given number			375
gives the proportional part to be added			35.625
to the log. of the first part			6.655715
Gives the log. of 4526375, as required.			6.655750

PROBLEM VI.

To find the Logarithm of a *Fraction*.

RULE.—Subtract the logarithm of the denominator from that of the numerator; or, reduce the fraction to a *decimal*, and take out the logarithm as for a whole number, fitting it with a proper index.

EXAMPLE. Required the logarithms of $\frac{3}{4}$ or .75.

The log of the numerator (3) is		0.477121
of the denominator (4) is		0.602060
Log. of $\frac{3}{4}$ or 75.	Answer.	—1.875061

PROBLEM VII.

To find the Natural Number, belonging to any Logarithm.

RULE 1.—If the logarithm be found within the limits of the table; that is, if its index do not exceed 3; then neglecting the index, look down in the column of logarithms, under 0, for the two or three first figures of your given logarithm, and if you exactly find *all* the figures of the given logarithm in that column, you will have the corresponding number in the adjoining column, on the left.

RULE 2.—If the logarithm be not found, exactly, in the column under 0, look through the other columns, on the right, till you find it exactly, or very nearly, and in the column of numbers directly against it, you will have the first *three* figures of the number sought, to which join the figure at the top of the column in which the logarithm was found, and you will have the number required.

NOTE.—When the number is found, you must point off decimals from it, or annex cyphers to it, if necessary, to make it correspond with the *index* of your logarithm, as already taught.

EXAMPLES.

Logarithms.	Numbers.
3.880585	7596
2.402089	252.4
1.514946	32.73
0.629919	4.265
1.811508	.6479
2.907680	.08085
3.962464	.009172

When great accuracy is required, and the given logarithm is not exactly, or very nearly, found in the table, it will be necessary to reverse the rule, under problem 4, or 5. Thus—

RULE 3.—From the given logarithm, subtract the next less, in the table; annex to the difference as many cyphers, as you wish the number of figures in the answer to exceed four. Then divide this difference by the common difference in the side column D, and annex the quotient to the natural number belonging to the less logarithm, and you will have the number required.

EXAMPLE 1. Required the natural number, belonging to the logarithm

	3.441049		
next less log.	3.440909	corresponding number	2760
divide by com. D=	157)14000(quotes		.89
Number required.			2760.89

EXAMPLE 2. Required the number corresponding to the logarithm —3.441049

The operation in this example, is the same as in the first. Having found the corresponding number, as before, prefix the number of cyphers indicated by the negative index, thus, 0.00276089.

PROBLEM VIII.

DIRECTIONS FOR TAKING THE LOGARITHMIC SINES, TANGENTS, AND SECANTS, FROM THE TABLE.

RULE 1.—If the given angle is less than 45°, look for the degrees at the *top* of the table, and the minutes on the *left*; then, opposite to the minutes, and under the word *sine*, at the head of the column, will be found the sine; under the word *tangent*, will be found the tangent, and under the word *secant*, will be found the secant of the angle required. Thus:—

Sine of 12° 35′ is	9.338176	Cotangent of 9° 55′ is	10.757390
Cosine of 21° 40′ is	9.968179	Secant of 11° 30′ is	10.008807
Tangent of 38° 5′ is	9.894111	Cosecant of 39° 5′	10.200349

The first figure is the index, and the other figures are the decimal part of the logarithm.

2.—If the given angle is between 45 and 90°, look for the degrees at the *bottom* of the table, and the minutes on the *right*; then, opposite to the minutes, and *over* the word *sine* at the foot of the column, will be found the sine; over the word tangent, will be found the tangent; and over the word secant, will be found the tabular secant &c. of the angle sought. Thus:—

Sine of 81° 20′ is	9.995013	Cosine of 89° 15′ is	8·116926
Tangent of 73° 25′ is	10.526081	Cotangent of 54° 5′ is	9.859932
Secant of 64° 45′ is	10.386455	Cosecant of 45° 55′ is	10.143677

3.—If the given angle is between 90 and 180 degrees; subtract it from 180 degrees, and the tabular sine, tangent, or secant, of the *remainder*, will be the tabular sine, tangent, or secant, of the required angle. Thus:—

Sine of 99° is the sine of	81°	Cosine of 111° 55′=Cosine	68° 5′
Tang. of 102° is the tang. of	78°	Cotang. of 127° 8′=Cotang.	52° 52′
Sec. of 158° 10′=sec. of	21° 50′	Cosec. of 131° 40=Cosec.	48° 20′

Having thus found the *supplement* of the angle, when it is obtuse, that is, when it exceeds 90 degrees, the tabular sine, tangent, or secant, is found by rule 1st or 2d.

It must be observed, that the table of sines, tangents, and secants, in this work, are calculated only to every five minutes.

If, however, greater accuracy is required, work as follows:

4.—Find the difference between the tabular sine, tangent, or secant, of the angle which is next *less*, and of that which is next *greater*, than the given one; multiply this difference by double the odd minutes, and add the product, rejecting the last figure of it, to the tabular sine, tangent, or secant, of the lesser angle. Thus:

EXAMPLE. Required the Logarithmic sine of 53° 8′

Next greater, in the table, is	53	10	=	9.903298
Next less, do.	53	5		9.902824
Tabular difference for		5		474
Mult. by twice the odd min.		3×	=	6
Add to the lesser angle	53	5		284.4
Gives log. sine of	53	8	=	9.903108

PROBLEM IX.

To find the degrees and minutes corresponding to any given tabular sine, tangent, or secant within the compass of the table.

RULE 1.—Look in the column of the same name, for the sine, tangent, or secant, which is *nearest* to the given one; and if the title be at the *head* of the column, take the degrees at the *top* of the table, and the minutes on the *left*; but if the title be at the *foot* of the column, take the degrees at the *bottom*, and the minutes on the *right*.

EXAMPLE. Find the number of degrees and minutes corresponding to the logarithmic sine 9.673971.

The nearest sine in the tables is 9.673977. The title of sine is at the head of the column in which the numbers are found. The degrees at the top of the page

are 28, and the minutes on the left are 10. The angle required, is, therefore, 28° 10′.

2.—When the given logarithmic sine, tangent, or secant, is not found exactly, or very nearly, in the table.

Find the difference between the next less and the next greater tabular sine, tangent, or secant, as in the last example; multiply this difference by five, and divide the product by the difference between the given logarithm and the next less, and add the quotient to the degrees and minutes belonging to the less. Thus:

Example. Required the degrees and minutes corresponding to the logarithmic sine 9.903108.

Given log.	9.903108	Next greater log.	9.903298
Next less.	9.902824=53° 5′	Next less	9.902824
	284		474
	5		

Divide by 474)1420(3 minutes, added to the less
Gives 53°.8′

MULTIPLICATION BY LOGARITHMS.

Rule.—Take from the table the logarithms of all the numbers to be multiplied, add them together, and their sum will be the logarithm of the product. Then, by means of the table, take out the natural number, corresponding to this sum, for the product sought.

Observe, that whatever is to be carried from the decimal part of the logarithm, *is always positive*, and must be *added to the positive*, or subtracted from the negative index or indices.

Examples.

	Numbers.	Logarithms.		Numbers.	Logarithms.
1. Mult.	326	2.513218	2. Mult.	8.25	0.916454
by	85	1.929419	by	112	2.049218
Prod.	27710	4.442637	Prod.	924.00	2.965672

3. Multiply the following numbers together: 3.902, 597.16, and .0314728.

	Numbers.	Logarithms.
	3.902	0.591287
	597.160	2.776091
	.0314728	$\bar{2}$.497935
Prod.	73.3333	1.865313

Here, the $\bar{2}$ cancels the 2, and the 1 to carry, from the decimal part of the logarithms is set down.

4. Multiply 3.586, and 2 1046, and 0.8372, and 0.0294 all together.

	Numbers.	Logarithms.
	3.586	0.554610
	2.1046	0.323170
	0.8372	$\bar{1}$.922829
	0.0294	$\bar{2}$.468347
Product	0.105762	$\bar{1}$.268956

Here the 2 to carry cancels the $\bar{2}$, and there remains the $\bar{1}$ to set down, for the index of the product.

In practice, however, it is usual to make *all the indices positive*. This is done by adding 10 to each negative index; observing, to reject an equal number from the final result.

Thus, for negative —1, we may put down positive 9.
for negative —2, we may put down positive 8.
for negative —3, we may put down positive 7. &c.

Because, minus 1, plus 10, equals 9.
minus 2, plus 10, equals 8.
minus 3, plus 10, equals 7. &c.

5. Repeating the 3d example we have

	Numbers.	Logarithms.		Logarithms.
	3.902	0.591287 =		0.591287
	597.160	2.776091 =		2.776091
	.0314728	$\bar{2}$.497935	or	8.497935
Prod.	73.3333	1.865313		1.865313

Here the sum of the indices is 11: from which reject 10, and the result is the same as before.

DIVISION BY LOGARITHMS.

Rule.—From the Logarithm of the Dividend, subtract the logarithm of the divisor; the number answering to the remainder will be the quotient required. The decimal part of the logarithm may be subtracted as in common arithmetic. But observe, always to *change the index of the divisor*, or suppose it to be changed from positive to negative, or from negative to positive; then *add* the indices of the Dividend and Divisor together where their signs are *alike*, but *subtract* the less from the larger, if *unlike*, and prefix to the difference the sign of the greater index.

The sum, or difference, thus found, will be the index to the logarithm of the quotient.

Observe also, that where 1 is borrowed, in the left hand place of the decimal part of the logarithm, *add* it to the index of the divisor when that index is *positive*, but *subtract* it, from the index if it is *negative*, and then proceed, as above stated.

Examples.

	Numbers.	Logarithms.		Numbers.	Logarithms.
1. Divide	24163	4.383151	2. Divide	37.149	1.569947
by	4567	3.659631	by	523.760	2.719132
Quot.	5.2908	0.723520	Quot.	.070927	$\bar{2}$.850815

Here, in the 2d example, the 1 which is borrowed for the 7, being added to the index 2. because it is *positive*, makes it 3. But, before it is subtracted, the sign is changed from positive 3, to negative —3, from which, take the 1 above it and there remains —2 for the index of the quotient.

3. Divide	.06314	—2.800305	or	8.800305
by	.007243	—3.859799	or	7.859799
	8.7198	0.940506		0.940506

Here, 1 carried from the decimal, to the —3, reduces it to —2, which, taken from the other —2, leaves 0 for the index of the quotient. To the learner, who is unacquainted with algebra, much of the perplexity arising from the employment of the negative indices, may be avoided, by making them all positive.

INVOLUTION BY LOGARITHMS.

A number is involved, by multiplying it one, or more times, into itself. But a number is multiplied by itself, by adding its logarithm to itself; that is, a number is raised to the second power, by multiplying its logarithm by 2. In like manner a number is raised to the third power by multiplying its logarithm by 3, and so on. Wherefore, we have this general

Rule.—Multiply the logarithm of the number to be involved, by the index of the power. The number, corresponding to this product, will be the power required.

Example 1. What is the square of 291?

Number 291, its logarithm is	2.463893
Index of the power	2
Power required 84681	4.927786

2. Required the cube of 3.07146

Number	3.07146 logarithm	0.487345
	Index of the power	3
	Power required 28.9758	1.462035

3. Required the 10th power of 2.

Number	2 logarithm	0.301030
	Index of the power	10
Power required 1024		3.010300

EVOLUTION BY LOGARITHMS.

Evolution is the opposite of Involution. Therefore to extract the roots of numbers, by Logarithms, we have this

Rule.—Divide the Logarithm of the given number, by the number expressing the root to be found.

Note.—When the index of the logarithm is negative, and does not exactly contain the divisor, without some remainder, increase the index by such a number as will make it exactly divisible, and carry the units thus borrowed, as so many tens, to the first figure of the decimal, and then divide as in whole numbers.

Example. Required the square root of 84681, and the cube root of 19683000.

1.	Numbers.	Logarithms.	2.	Numbers.	Logarithms.
Power	84681	2)4.927786	Power	19683000	3)7.294091
Root	291	2.463893	Root	270	2.431364
	Numbers.	Logarithms.		Numbers.	Logarithms.
3. Power	0.0932	2)—2.969416	4. Power	0 00048	3)—4.681241
Root	0.30496	—1.484708	Root	0 078297	—2.893747

Here, in the 3d example, the divisor 2 is contained *exactly once*, in the negative index —2; the index of the quotient is therefore —1. But in the 4th example, the divisor 3 not being exactly contained in —4, the latter is augmented by 2 to make up 6. in which the divisor is contained just twice. The 2 thus borrowed, being carried to the decimal figure 6, makes 26, which divide by 3 &c.

But when to avoid the perplexity of negative signs, the index of the logarithm is made positive, it will be needful to add as many tens to the negative index as there are units in the number expressing the root to be found.

5. What is the 5th root of 0.008926?

Power	0 008926	5)—3.950657	or	5)47.950657
	0.38916	—1.590131		9.590131

A *power of a root*, that is, a fractional root of any given number may be found by the following

Rule.—Multiply the logarithm of the given number by the index of the power, and then divide the product by the number, expressing the root. In other words, multiply the logarithm by the numerator, and divide by the denominator.

Example. What is the $(654)^{\frac{8}{9}}$ that is, the 8th power of the 9th root of 654.

Given number 654	log.	2.815578
	Multiply by	8
	Divide by	9)22.524624
Power required 318-23		2.502936

COMPOUND INTEREST BY LOGARITHMS.

Rule.—Find the amount of 1 dollar for 1 year; multiply its logarithm by the number of years; and to the product, add the logarithm of the principal. The sum will be the logarithm of the *amount* for the given time.

From the amount subtract the principal, and the remainder will be the interest.

Example, The last example in Dabolls arithmetic, under Compound Interest. is as follows:—"What will fifty dollars amount to, in 20 years, at 6 per cent?"

This question, wrought out in the most expeditious manner in common arithmetic, would take the student scarcely less than 3 hours, and the final result would be, if done correctly, $160.3567756106422365941496492288974572748800. But by logarithms this sum may be done in as many minutes, and the correct answer, in dollars and cents, obtained with a very few figures. Thus—

Amount of 1 dollar for 1 year is 1.06	logarithm	0.0253059
	Multiply by the time	20
		0.5061180
	Add log. of principal 50	1.6989700
Amount as above=$160.35,7		2.2050880

From the foregoing general principles of the nature and application of logarithms, are derived an infinite number of specific rules, adapted to particular cases; some of which will be more fully shown under the article Trigonometry. We shall in this place, therefore, give only one more illustration of the great utility of logarithms in facilitating some of the most laborious operations in arithmetic, viz:

Between two numbers given, to find any number of mean proportionals required.

Rule 1.—From the logarithm of the greater number subtract the logarithm of the less, and divide the remainder by the number of means, increased by 1.

2.—Add the quotient to the logarithm of the less number, and the sum will be the logarithm of the 1st mean proportional.

3.—To the logarithm last found, add the said quotient, and the sum will be the sum of the 2d mean proportional; and thus proceed always adding the same quotient to the logarithm of the last proportional found, as far as the question requires.

Example. Required to find between 16 and 64, five mean proportionals.

Log. of 64		1.806180
of 16		1.204120
Divide by 5 means, +1	=6)	0.602060
Com. ratio, 1.26 nearly		0.100343⅓
Add log of 19		1.204120
1st proportional 20.158=		1.304463⅓
To which add said quotient		0.100343⅓
2nd proportional 25.398	=	1.404806⅔
Add quotient		0.100343⅓
3d proportional 32	=	1.505150
Add quotient		0.100343⅓
4th proportional 40.317	=	1.605493⅓
Add quotient		0.100343⅓
5th proportional 50.796		1.705836⅔

TRIGONOMETRY.

Trigonometry is that branch of the general science of Geometry which treats of the properties and relations of certain straight lines drawn in and about a circle, and also teaches to compute the sides and angles of triangles. It is divided into two parts, plane, and spherical.

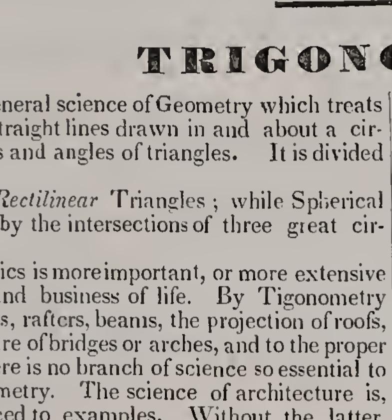
Plane Trigonometry treats only of *Rectilinear* Triangles; while Spherical Trigonometry treats of triangles, formed by the intersections of three great circles upon the surface of a sphere.

Scarcely any department of mathematics is more important, or more extensive in its application to the useful purposes and business of life. By Trigonometry the builder determines the length of braces, rafters, beams, the projection of roofs, and whatever appertains to the architecture of bridges or arches, and to the proper delineation of plans and drawings. There is no branch of science so essential to the practical architect, as that of trigonometry. The science of architecture is, indeed, the science of trigonometry reduced to examples. Without the latter, the former would have remained, at best, but as an art without rules, exactness, or order.

The *terms* of this science, such as *Sines, Tangents, Secants,* &c., have already been explained, among the definitions of Geometry. It remains now to show their application, and the manner of computing them. Each of these terms, under different circumstances, is but another name for the perpendicular, the base, and the hypothenuse of a triangle. In one triangle for instance, the perpendicular becomes the *Sine*, and in another perfectly similar to the former, it is as often the *Tangent*, of an angle. The same thing is true of the base, also. It becomes first of all, necessary, therefore, to know, under what cirumstances, the respective sides of a triangle become sines, tangents, and secants, or cosines, cotangents and cosecants.

This depends on which side of the triangle the circle is described: for if any side whatever be made radius, each of the other sides will be the sine, tangent or secant, of the arc described by this radius.

Fig. 1.

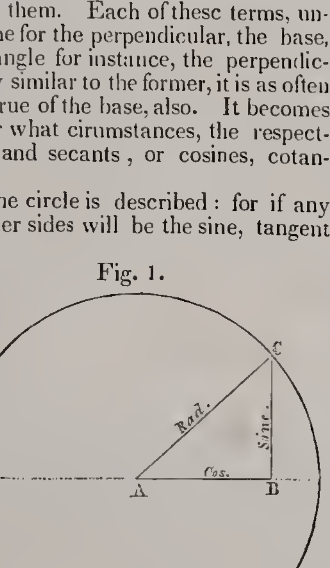


Case 1.—In every right angled triangle, if the Hypothenuse be made radius, one of the sides will be a *sine* of its opposite angle, and the other side a *cosine* of the same angle. Thus:

The triangle ABC being constructed and a circle described around the centre A, with the radius AC, then the perpendicular BC will be the *sine*, and the base AB the *cosine* of the angle at A. But as the sine of either of the acute angles of a right angled triangle is the cosine of the other, and the contrary; therefore the perpendicular BC, being the *sine* of the angle at A, is also *cosine* of the angle at C.

Case 2.—If either the base or perpendicular be made radius, the other will be a *tangent* of its opposite angle, and the Hypothenuse will be a *secant* of the same angle; that is, of the angle between the secant and the radius. Thus:

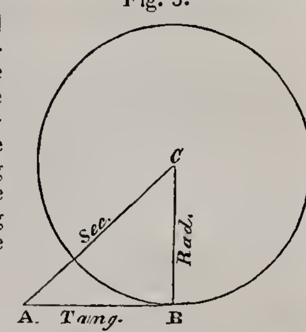
Fig. 2.



Let an arc or circle, be described around A, with the base AB, as radius, then the perpendicular BC will be the *tangent*, and the hypothenuse AC the *secant* of the angle at A. And because the side which is the sine, tangent or secant, of one of the acute angles of a right angled triangle, is the *cosine, cotangent,* or *cosecant* of the other angle, therefore, the perpendicular BC, being tangent of the angle at A, is also *cotangent* of the angle at C. And the hypothenuse AC, being secant of the angle at A, is, also, *cosecant* of the angle at C.

Case 3.—If the perpendicular BC be made radius, with the centre at C, then the base AB will be the *tangent*, and the hypothenuse AC (as before) the *secant* of the angle at C. And, because every side of a right angled triangle, which is the sine, tangent, or secant, of one of the acute angles, is the *cosine, cotangent,* or *cosecant,* of the *other* acute angle, therefore, the base AB being *tangent* of the angle at C, is also *cotangent* of the angle at A. And the hypothenuse AC, being *secant* of the angle at C, is also cosecant of the angle at A.

Fig. 3.



The solution of all the cases in right angled trigonometry, depends upon the principle that the radius of one circle, is to the radius of any other, as the sine tangent or secant, in one, is to the sine, tangent, or secant, of the same number of degrees, in the other. It is also plain, that if the side of any triangle, which is made radius should remain unchanged, the other two sides would vary according to the size of the angle; that is, they would be *longer* or *shorter* according as they subtended a greater or less number of degrees. Upon this principle the table of sines, tangents, or secants, was computed, to radius 1, for every 5' of a degree.

In every triangle, there are six parts, three sides, and three angles. Of these parts *three* must be given, including a side, to enable us to find the the rest. To find a *side*, we begin the statement of the problem with an *angle*. To find an *angle*, we must begin with a *side*, as in the following rules.

1. *To find a Side*.—Call any one of the sides of a triangle radius, and write upon it the word radius; observe whether the other sides become sine, tangent, or secants, and write these words on them, or suppose them to be written, as in some one of the three last figures. Consider the word thus written upon each side, as the *tabular name*, of that side. Then institute the following proportion:

As the *tabular name** of the given side,
Is to the *length* of the side,
As is the tabular name* of the required side,
To the length of the required side,

2. *To find an Angle*.—One of the given sides must be made radius, then institute the following proportion:

As the length of the given side made radius,
Is to its tabular name,* that is, radius;
So is the length of the other given side,
To its tabular name*.

Having thus arranged the terms of the proportion, look for the corresponding logarithms, in the logarithms of numbers, for the length of the sides; and in the table of sines, tangents and secants, for the logarithmic sine, tangent, or secant. Then proceed by this general

Rule.—ADD TOGETHER THE LOGARITHMS OF THE SECOND AND THIRD TERMS, AND FROM THEIR SUM, SUBTRACT THE LOGARITHM OF THE FIRST TERM. THE REMAINDER WILL BE THE LOGARITHM OF THE ANSWER.

NOTE.—From the property of a plane triangle, that the three angles are together equal to two right angles, or 180 degrees, the following useful corollaries, arise.

1. When two angles of a triangle are given, the third is also said to be given; for it is the supplement of the other two, and may be found by subtracting their sum from 180 degrees.

2. When one angle of a triangle is given, the sum of the other two may be found, by subtracting the given angles from the two right angles, or 180 degrees.

3. If one angle of a triangle be right, the other two are acute, and together make another right angle; and if one of the acute angles be given, the other is also given, being the complement of the other given one, or what it wants of 90 degrees.

4. The sine or tangent of any angle is to the side opposite to it, as the sine, or tangent, of any other angle is to its opposite side, and the contrary.

* Which will be either Sine, Tangent, or Secant, &c.

PROBLEM I.

GIVEN THE ANGLES AND THE HYPOTHENUSE OF A RIGHT ANGLED TRIANGLE, TO FIND THE BASE AND PERPENDICULAR.

EXAMPLE 1. In the triangle ABC, right angled at B, suppose the angle at A=50 degrees 30 minutes, and the hypothenuse, AC=125 feet, or yards; required the sides AB and BC.

Here, the hypothenuse being the *given* side must be made radius; and BC will then become the sine of the angle at A, and AB the cosine of the same angle, *see fig* 1.

To find the perp. BC		To find the base AB	
As rad. or sine of 90°	10.000000	As radius, sine of 90°	10.000000
Is to hypothenuse 125	3.096910	Is to hyp. AC 125	2.096910
So is the sine of ∠ A 50° 30′	9.827406	So is Cos. of ∠ A 50° 30	9.803511
	10.984316		11.900421
(Subtract 1st term)	10.000000	(Subtract 1st term)	10.000000
To perp. BC 96.4	1.984316	To base AB 79.51	1.900421

NOTE.—Where the first term is radius, it may be subtracted from the sum of the other two, by merely rejecting 10 in the index, without the trouble of setting it down a second time.

To find the angle at C, we have only to subtract the angle at A from 90°. Thus 90°—50° 30′=39° 30′ which is the angle at C.

2. *Making the base radius*, the perpendicular BC becomes the tangent of the angle A, and AC becomes the secant of A. *see fig.* 2. Thence.

To find BC		To find AB	
As secant of it 50° 30′ =	10.196489	As secant of A 50° 30′ =	10.196489
Is to hyp. AC 125	2.096910	Is to hyp. AC 125	2.096910
So is tangent of A 50° 30′	10.083896	So is radius 90°	10.000000
	12.180806		12.096910
(Subtract 1st term)	10.196489	(Subtract 1st term.)	10.196489
To perp. BC 96.45	1.984317	To base AB 79.51	1.900421

3. *Making the Perpendicular Radius*, the base AB will be the tangent of the angle at C, or cotangent of A; and the hypothenuse AC, will be the secant of C, or the cosecant of A. *See fig.* 3. Hence.

To find BC		To find AB.	
As cosecant A 50° 30′ =	10.112594	As cosecant of A 50° 30′=	10.112594
Is to hyp. AC 125	2.096910	Is to hyp. AC 125	2.096910
So is radius 90°	10.000000	So is cotang. of A 50° 30	9.916104
	12.096910		12.013014
(Subtract 1st term)	10.112594	(Subtract 1st term)	10.112594
To perp. BC 96.45	1.984316	To base AB 79.51	1.900420

PROBLEM II.

BY GUNTER'S SCALE.

1st. Extend the compasses, from 36° 52′ the complement of A to 90, on the line of sines; that extent will reach from 288 to 480= the hypothenuse AC.

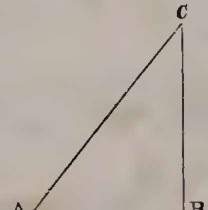
2d. Extend the compasses from 36° 52′ to 53° 8, on the line of sines; that extent will reach from 288 to BC 384 on the line of numbers.

In working the several cases of right angled trigonomety by Gunters scale, we shall always suppose the hypothenuse radius, (where it can be done,) because it is the most simple of the three.

GIVEN THE ANGLES AND THE BASE, TO FIND THE HYPOTHENUSE AND THE PERPENDICULAR.

In the triangle ABC, right angled at B, given the angle at A, 53° 8′, and the base AB=288, to find the hypothenuse AC, and the perpendicular BC.

Fig. 4.



1. *Making the hypothenuse radius*, the perpendicular BC, will be the sine of the angle at A, and the base AB the cosine. (See fig. 1,) Hence.

To find AC.		To find BC.	
As cosine A 53° 8=	9.778119	As cos. A 53° 8′ =	9.778119
Is to base AB 288	2.459392	Is to base AB 288	2.459392
So is radius 90°	10.000000	So is sine A 53° 8′	9.903108
	12.459392		12.362500
(Subtract 1st. term)	9.778119	(Sub. 1st. term)	9.778119
To hyp. AC 480	2.681273	To perp. BC 384	2.584381

2. *Making the base radius*, the perpendicular BC will be the tangent of the angle at A, and the hypothenuse will be secant of the same angle. See fig. 2. Hence.

To find AC		To find BC	
As radius 90° =	10.000000	As radius 90° =	10.000000
Is to the base AB 288	2.459392	Is to the base AB 288	2.454392
So is secant of A 53° 8′	10.221881	So is tang. of A 53° 8′	10.124990
To hyp. AC 480	2.681273	To perp. BC 384	2.584382

3. *Making the perpendicular radius*, the base AB will be the tangent of C, or the cotangent of A; and the hypothenuse AC will be the secant of C, or cosecant of A. See fig. 3. Hence.

To find AC.		To find BC.	
As cotangent A 53° 8′=	9.875010	As cotangent A 53° 8′=	9.875010
Is to the base AB 288	2.459392	Is to the base AB 288	2.459392
So is cosecant A 53° 8′	10.096892	So is radius 90°	10.000000
	12.556284		12.459392
(Subtract 1st term)	9.875010	(Subtract 1st term)	9.875010
To hyp. AC 480	2.681274	To hyp. AC 384.	2.584382

BY GUNTER'S SCALE.

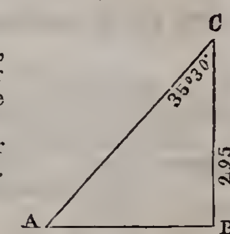
1. Extend the compasses from 90° to 50° 30′ on the line of sines, and then that extent will reach from 125 to 96.45=BC on the line of numbers.

2. Extend the compasses from 90° to 39° 30′ the complement or cosine of the angle A, and that extent will reach from the hypothenuse AC=125, to the base AB=79.51, on the line of numbers.

PROBLEM III.

GIVEN THE ANGLES AND THE PERPENDICULAR TO FIND THE HYPOTHENUSE AND THE BASE.

Fig. 5.



EXAMPLE 1. In the triangle ABC, right angled at B, let the angle at C be 35° 30′, and the perpendicular BC 295 feet; required the base AB and the hypothenuse AC.

1. *Making the hypothenuse radius*, the perpendicular BC becomes the sine of A, and the base AB the cosine. *See fig.* 1. Hence.

To find AB		To find AC.	
As cosine of C 35° 30′=	9.910686	As cosine of C 35° 30′	9.910686
Is to perp. BC 295	2.469822	Is to perp. BC 295	2.469822
So is sine C 35° 30′	9.763954	So is rad. 90	10.000000
	12.233776		12.469822
(Sub. 1st. term)	9.910686	(Sub. 1st. term)	9.910686
To base AB 210.4	2.323090	To hyp. AC 362.3	2.559136

NOTE.—It is customary to subtract the 1st term from the sum of the other two, *where it stands*, without writing it down the second time, as, for the sake of plainness, we have done in the foregoing examples.

2. *Making the base radius*, BC will be the tangent of A, and AC the secant thereof. *See fig* 2. Hence.

To find AB.		To find AC.	
As cotang. of C 35° 30′=	10.146732	As cotangent C 35° 30′=	10.146732
Is to perp. BC 295	2.469822	Is to perp. BC 295	2.469822
So is radius 90	10.000000	So is cosecant C 35° 30′	10.236046
	12.469822		12.705868
To base AB 210.4	2.323090	To hyp. AC 362.3	2.559136

3. *Making the perpendicular radius*, the base will be the tangent of C, or the cotangent of A; and the hypothenuse will be the secant of C, or cosecant of A. *See fig.* 3. Hence.

To find AB.		To find A C	
As radius or sine 90°=	10.000000	As rad. or sine 90°=	10.000000
Is to perp. BC 295	2.469822	Is to perp. BC 295	2.469822
So is tangent C 35° 30′	9.853268	So is secant C 35° 30′	10.089314
	12.323090		12.559136
To base AB 210.4	2.323090	To hyp. AC 362.3	2.559136

BY GUNTER'S SCALE.

1. Extend the compasses from 35° 30′ to its cosine 54° 30′ on the line of sines; and that extent will reach from the perpendicular 295 to the base 210.4 on the line of numbers.

2. Extend the compasses, from 54° 30 the complement of C to 90°, on the line of sines; and that extent will reach from the perpendicular 295 to the hyponuse 362.3 on the line of numbers.

PROBLEM IV.

GIVEN THE HYPOTHENUSE AND THE BASE TO FIND THE ANGLES AND THE PERPENDICULAR.

In the right angled plane triangle ABC, fig. 4. Given the hypothenuse AC = 480, and the base AB=288, to find the angles A and C, and the perpendicular BC.

1. *Making the hypothenuse radius*, BC will be the sine of the angle A, and AB the cosine of the same angle. *See fig.* 1. Hence.

To find the angle A.		To find the perpendicular BC.	
As hyp. AC 480 =	2.681241	As radius 90°= .	10 000000
Is to radius 90	10.000000	Is to hyp. AC 480	2.681241
So is base AB 288	2.459392	So is sine A 53° 8′	9.903108
	12.459392		12.584349
To cosine A 53° 8′	9.778151	To perp. BC 384	2.584349

2. *Making the base radius.* BC will be the tangent of A, and AC the secant of A. *See fig.* 2. Hence.

To find the angle A.		To find the perpendicular BC.	
As the base AB=288	2.459392	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to the base AB 288	2.459392
So is hyp. AC 480	2.681241	So is tang. A 53° 8′	10.124990
	12.681241		12.584312
To secant A 53° 8′	10.221849	To perp. BC=384	2.584312

BY GUNTER'S SCALE.

1st. Extend the compasses from the hypothenuse 480, to the base 288, on the line of numbers; that extent will reach from 90 to 36° 52′, on the line of sines, the complement of the angle A.

PROBLEM V.

GIVEN THE HYPOTHENUSE AND PERPENDICULAR, TO FIND THE ANGLES AND THE BASE.

In the right angled plane triangle ABC, fig. 4. Given the hypothenuse AC =480, and the perpendicular BC=384; required the angles and the base.

1. *Making the hypothenuse radius*, BC will be the sine of the angle A, and AB the cosine thereof. *See fig.* 1. Hence.

To find the angle A.		To find the base AB.	
As hyp. AC=480	2.681241	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to hyp. AC=487	2.681241
So is perp. BC 384	2.584331	So is cosine A 53° 8′	9.778119
To sine A 53° 8′	9.903090	To base AB 288	2.459360

2. *Making the perpendicular radius*, AB will be the tangent of C, or the cotangent of A; and AC will be the secant of C, or the cosecant of A. *See fig.* 3. Hence.

To find the angle A.		To find the base AB.	
As perp. BC=384	2.584331	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to perp. BC=384	2.584331
So is hyp. AC 480	2.681241	So is cotangent A 53° 8′	9.875010
To cosecant A 53° 8′	10.096910	To base AB=288.	2.459341

BY GUNTER'S SCALE.

1st. Extend the compasses from the hypothenuse 480 to the perpendicular 384. on the line of numbers, and that extent will reach from 90° to 53 8′ on the line of sines.

2d. Extend the compasses from 90° to 36° 52′ the complement of A, on the line of sines, and that extent will reach from 480 to 288, on the line of numbers.

PROBLEM VI.

GIVEN THE BASE AND PERPENDICULAR TO FIND THE ANGLES AND THE HYPOTHENUSE.

In the right angled plane triangle ABC, fig. 4, given the base AB=288, and the perpendicular BC=384, to find the angles at A and C, and the hypothenuse AC.

1. *Making the base radius*, BC will be the tangent, and AC the secant of the angle A. *See fig.* 3. Hence,

To find the angle A.		To find the hypothenuse AC.	
As base AB = 288	2.459392	As radius 90°	10.000000
Is to radius 90°	10.000000	Is to base AB=288	2.459392
So is perp. BC 384	2.584331	So is Secant A 53° 8	10.221881
To tangent A 53° 8	10.124939	To hypoth. AC 480	2.681273

2. *Making the perpendicular radius*, AB will be the tangent of C, or the cotangent of A; and AC will be the secant of C, or the cosecant of A. *See fig.* 3. Hence

To find the angle A.		To find the hypothenuse AC.	
As perp. AC = 384	2.584531	As radius 90°	10 000000
Is to radius 90°	10.000000	Is to perp. BC=384	2.584331
So is base AB 288	2.459392	So is cosec. A 53° 8′	10.096892
To cotangent A 53° 8′	10.124961	To hyp. AC 480	2.681223

BY GUNTER'S SCALE.

1st. Extend the compasses from 384 to 288 on the line of numbers, and that extent will reach from 45° to 53° 8′, on the line of tangents.

2d. Extend the compass from 53° 8′ to 90°, on the line of sines, and that extent will reach from 384 to 480, on the line of numbers.

The foregoing examples embrace all the variety of cases that can be solved in right angled trigonometry. We are now about to treat of

OBLIQUE ANGLED TRIGONOMETRY.

RULE 1.—WHEN TWO OF THE THREE GIVEN PARTS ARE A SIDE, AND ITS OPPOSITE ANGLE.

Any one side of a triangle,
Is to the sine of its opposite angle;
As any other side
Is to the sine of its opposite angle.
And, the sine of any angle,
Is to its opposite side;
As the sine of any other angle,
Is to its opposite side.

An *angle*, found by this rule is sometimes *ambiguous*; for trigonometry gives us only the sine of an angle, and not the angle itself, and the sine of every angle is also the sine of its supplement. When the *given* side' opposite to the *given* angle, is greater than the other *given* side; then the angle opposite to that other given side is always acute. But when the *given* side opposite to the *given* angle is less than the other given side, then the angle, opposite that other given side may be either acute or obtuse, and consequently it is *ambiguous*.

RULE 2.—WHEN TWO SIDES AND THEIR INCLUDED ANGLE ARE GIVEN.

As the sum of the two given sides,
Is to their difference;
So is the tangent of half the sum of the opposite angles,
To the tangent of half their difference.

This half difference between the two required angles being *added* to half their sum, gives the *greater* angle, and *subtracted* from half their sum, gives the *less*.

The remaining side of the triangle is then found by rule 1.

RULE 3.—WHEN THE THREE SIOES ARE GIVEN TO FIND THE AGLES.

Assume the longest of the three sides as base, upon which suppose a perpendicular to be let fall from the opposite angle. Then.
As the base or longest side,
Is to the sum of the two other sides.
So is the difference of those sides,
To the difference of the segments of the base.

Then *half* the base, or longest side, added to the said difference, gives the greater segment of the base, and subtracted, gives the less.

The triangle being thus divided into two right angled triangles, each of which contains two given sides, the remaining angles may be found by rule 1. It is to be observed, that the greater segment is always adjacent to the greater side.

To enable us, therefore, to find the sides and angles of an oblique angled triangle, *three* of them must be given. These may be either

1. Two angles and a side, or
2. Two sides and an angle *opposite* to one of them, or
3. Two sides and the *included* angle, or
4. The three sides.

PROBLEM 1.

GIVEN THE ANGLES AND ONE SIDE TO FIND THE OTHER ANGLE AND REMAINING SIDES.

In the plane triangle ABC, given the angle at A=32° 15′, the angle at B=114° 25′ and the side AB=98, to find the angle C, and the sides AC and BC.

Fig. 6.

To the angle A=32° 15′ add the angle B=114° 25′ and the sum will be 146° 40′; which subtract from 180° and it leaves the angle C=33° 20′. Then

BY RULE 1.

To find the side AC.		To find the side BC.	
Sine of C=33° 20′	9.739975	Sine C=33° 20′	9.739975
Is to side AB=98	1.991226	Is to side AB 98	1.991226
As sine B 114° 25′, or 65° 35′	9.959310	As sine A 32° 15′	9.727228
	11.950536		11.718454
To side AC=162.39	2.210561	To side BC, 95.17	1-978479

NOTE.—In all cases where the angle is *obtuse*, or greater than 90°, subtract it from 180, and with the remainder, take out the sine, tangent, or secant, from the tables. Thus, if we subtract 114° 25′ (as in the above example,) from 180, it leaves 65° 35′, the tabular sine of which is 9,959310, and is exactly the same for 114° 25′. And, generally—The tabular sine, tangent, or secant of an obtuse angle, is *the same* as that of its supplement, and the contrary.

BY GUNTER'S SCALE.

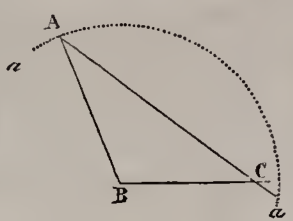
1. Extend the compasses from 33° 20′ to 65° 35′ the supplement of B, on the line of sines, and that extent will reach from 98 to 162, on the line of numbers.
2. Extend the compasses from 33° 20′ to 32° 15′, on the line of sines, and that extent will reach from 98 to 95, on the line of numbers.

PROBLEM II.

GIVEN TWO SIDES AND AN ANGLE OPPOSITE TO ONE OF THEM, TO FIND THE REMAINING SIDE AND ANGLES.

In the triangle ABC, (fig. 7.) given the angle C=33° 20′ the side AB=98, and the side BC=95, 17, to find the angles at A and B, and the side AC.

Fig. 7.



Here, agreeably to the observation under Rule I. the angle A is acute, and not ambiguous; but had the side AB been less than the side BC, the arc *aa* would evidently have cut the side AC in two points on the same side of BC.

BY RULE I.

To find the angles A and B.		To find the side AC.	
AB=98	1.991226	Sine C=33° 20′	9.739975
: Sine C 33° 20′	9.739975	: Side AB=98	1.991226
:: BC 95.17	1.978479	:: Sine B 114° 25′ or 65° 35′	9.959310
	11.718454		11.950536
: Sine A 32° 15′	9.727228	: Side AC=162.39	2.210561

The sum of the angles A and C, subtracted from 180°, leaves the obtuse angle B=114° 25′.

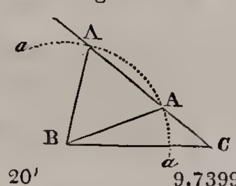
BY GUNTER'S SCALE.

1. Extend the compasses from 98 to 95, on the line of numbers, and that extent will reach from 33° 20′ to 32° 15′, on the line of sines.
2. Add the angles A=32° 15′ and C = 33° 20′, together, and the sum will be 65° 35; then extend the compasses from 33° 20′ to 65° 35′ on the line of sines, and that extent will reach from 98 to 162.4 on the line of numbers.

2. Given. { The angle C=33° 20′; The side BC=95. 17; The side AB=60 } Required the angles A and B, and the side AC.

The geometrical construction of this triangle, (fig. 8.) is exactly the same as in the preceeding example; only AB, being shorter than BC, cuts AC in two points on the same side of BC, wherefore the angle A may be either acute or obtuse. We may work

Fig. 9.



BY RULE I.

To find the angles A and B.		To find the side AC.	
Side AB=60	1.778151	Sine C=33° 20′	9.739975
: Sine C 33° 20′	9.739975	: Side AB 60	1.778151
:: Side BC 95.17	1.978479	:: Sine B 86° 1′	9.998950
	11.718454		11.777101
: Sine A { 60° 39′ acute; 119° 21′ obtuse }	9.940303	: Side AC 108.92	2.037127
		OR	
		Sine C=33° 20′	9.739975
		: Side AB 60	1.778151
		:: Sine B 27° 19′	9.661726
			11.439877
		: Side AC=50.11	1.699902

The sum of the angles C and A subtracted from 180°, leaves the angle B =86° 1′, if it be acute; or 27° 1′ if it be acute. It it is evident that the two sides BA, and BA, and exactly equal, because they are radii of the same arc *aa*.

PROBLEM III.

GIVEN TWO SIDES AND THEIR INCLUDED ANGLE, TO FIND THE OTHER SIDE AND REMAINING ANGLES.

In the plain triangle ABC, (fig. 6.) given the side AB=98, the side BC=95.17, and the included angle B=114° 25′, to find the rest.

First, find the sum and difference of the two given sides.

The side AB	=	98	The side AB	=	98
BC		95.17	BC		95.17
Sum of the sides,		193.17	Difference of the sides,		2.83

Next, subtract the given angle B = 114° 25′, from 180, and it leaves 65° 35′ for the *sum of the other two angles*. Half of 65° 35′ is 32° 47′ 30″, Then,

BY RULE 2.—As the sum of the two sides	193.17	2.285940
Is to their difference	2.83	0.451786
So is tangent of half the unknown angles	32° 47′ 30	9.809055
		10.260841
To tangent of half their difference	0° 32′ 30″	7.974901
Add this to ½ the sum of the unknown angles	32 47 30	
Gives the greater angle at C =	33 20 00	
Subtracted, gives the less, at A =	32 15 00	

Or—

As the sum of the two sides	193.17	2.285940
Is to their difference	2.83	0.451786
So is cotangent of ½ the given angle 57° 12′ 30″		9.809055
		10.260841
To tangent of ½ the difference of unknown angles.	0° 32′ 30″	7.974901

To find the side AC.

As sine of C =33° 20′		9.739975
Is to the side AB 98		1.991226
So is sine B=114° 25′, or 65° 35′		9.959310
		11.950536
To side AC.	162.39	2.210561

BY GUNTER'S SCALE.

1. Extend the compasses from 193.17 to 2.83 on the line of numbers, and that extent will reach from 32° 47½′ to 0° 32½′ on the line of tangents. This is the method of working such examples as this; but so small an angle as 0° 32½′ cannot be taken from the scale.
2. Extend from 33° 20′ to 65° 35′ on the line of sines, and that extent will reach from 98 to 162.4 on the line of numbers.

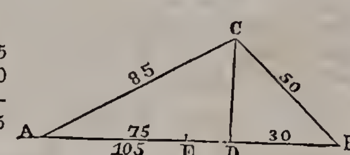
PROBLEM IV.

GIVEN THE THREE SIDES TO FIND THE ANGLES.

First, find the sum and difference of the two shorter sides.

Fig. 9.

Side AC	85	Side AC.	85
BC	50	BC.	50
Sum of the two sides	135	Difference	35



By RULE 3.—As the longest side AB =	105	2.021189
Is to the sum of the other two	135	2.130334
So is their difference	35	1.544068
		3.674402
To the difference of the segments ED	45	1.653213
Half of the said difference, is	22.5	
Added to half the base	52.5	
Gives the greater segment AD	75	
Subtracted, gives the less	30	

Thus, the triangle is divided into two right angled triangles, ADC and BDC; in each of which the hypothenuse and one side are given, to find the angles, as already taught in right-angled trigonometry.

To find the angle DCA.		To find the angle DCB.	
As hyp. AC 95	1.929419	As hyp. BC 50	1.698970
Is to rad. 90°	10.000000	Is to rad.	10.000000
So is seg. AD 75	1.875061	So is seg. BD. 30	1.477121
	11.875061		11.477121
To sine DCA 61° 56′	9.945642	To sine DCB, 36° 52′	9.778151

The angle DCA 61° 56′ subtracted from 90°, leaves the angle at A 28° 4′.
The angle DCB 36° 52′ subtracted from 90, leaves the angle at B 53° 8′.
The angle DCA 61° 56′ added to the angle DCB 36° 52′ gives 98° 48′ for the obtuse angle at C, which was required.

The preceding solutions are all effected by means of the tabular sines, tangents, and secants. But when any two sides of a right angled triangle are given, the third side may be found without the aid of trigonometrical tables, by the proposition, that the square of the hypothenuse is equal to the sum of the squares of the two perpendicular sides; (See Geometry, Theorem 17.)

If the legs be given, extracting the square root of the sum of their squares, will give the hypothenuse. Or if the hypothenuse and one leg be given, extracting the square root of the difference of their squares, will give the other leg. It is generally most convenient to find the difference of the squares by *logarithms*. But this is not to be done by *subtraction*. For subtraction, in logarithms, performs the office of *division*. If we subtract the logarithm of the square of the base, from the logarithm of the square of the hypothenuse, we shall have the logarithm, not of the *difference* of the squares, but of their quotient.

To obtain the difference of the squares of two quantities, add the logarithm of the sum of the quantities, to the logarithm of their difference. After the logarithm of the difference of the squares is found; the *square root* of this difference is obtanied, by dividing the logarithm by 2.

EXAMPLE 1. If the base be 60 inches, and the perpendicular 45, what is the length of the hypothenuse.

1 By extracting the square root.	2. By Logarithms.	Logarithm.
60×60 = 3600		
45×45 = 2025	Sq. of the given sides=5625	2)3.750123
56·25(75 inches.	Side required = 75	1.875061
49		
145)725		
725		

Example 2.—If the hypothenuse be 75 inches, and the base 60, what is the length of the perpendicular.

1. By extracting the square root.	2. By Logarithms.	Logarithm.
75×75=5625		
60×60=3600	Sum of the given sides=135	2.130334
20·25(45 inches.	Difference of do. = 15	1 176091
16		2)3.306425
85)425	Side required 45	1.653212
425		

Example 3. If the hypothenuse be 75 inches, and the perpendicular 45, what is the length of the base.

1. By extracting the square root.	2. By Logarithms.	Logarithm.
75×75=5625		
45×45=2025	Sum of the given sides=120	2.079181
36 00(60 inches.	Difference of do. = 30	1.477121
36		2)3.556302
00	Side required 60	1.778151

There is a roof whose span or width is 40 feet, and height 10 feet, standing upon a rectangle plan, hiped at each end. What is the length of the common rafters and likewise the hip rafters (See Fig. 64, Pl. 34 of Mensuration.)

Example.—One half of the span AD is 20 feet; and the perpendicular height CD is 10 feet, to find the length of the common rafters AC or BC. The square of 20 and 10 is 400, the square root of which is 22.36 feet, for the length of the common rafters AC or BC.

Now the hip rafters are the hypothenuses of right-angled triangles; having a common rafter for one of the legs and the other leg being equal to half the width of the roof. Now we will suppose AC or BC to be the length of the common rafter, which is 22.36 feet, and AD half the width of roof, (which is 20 feet,) to find the length of the hip rafter EG. The square of 22.36 and 20 is 899.9696, the square root of which is 29.99 feet, being a trifle less than 30 feet.

HEIGHTS AND DISTANCES.

PROBLEM I.

TO FIND THE PERPENDICULAR HEIGHT OF AN ACCESSIBLE OBJECT STANDING ON A HORIZONTAL PLANE.

Rule 1.—Measure from the object to a convenient station, on a base line, and there take the angle of the elevation, subtended by the object.

2. Then say: As radius is to the base :: so is the tangent of the angle of elevation, to the perpendicular height.

3. Or more briefly, thus—From the logarithm of the distance of the station from the object, increased by 10 in the index, subtract the tangent of the elevation; the remainder will be the logarithm of the perpendicular height, in the same denomination of measure, as the *distance* was taken in.

Example 1.—Wanting to know if a particular tree was of sufficient height to make a sill of a required length. I measured off 40 feet from the foot of the tree, and there found the angle, subtended by the tree, to be 56° 30′. Required the height of the tree? (See Fig. 14, Plate 1.)

In this example, we have a plane triangle, right-angled at B, with the base and other angles given, to find the perpendicular. Making the base AB *radius*, because it is the side which is *given* (See case 2, Trig.) the perpendicular becomes the *tangent* of the angle at A, and the proportion is stated thus:

As radius, or sine	90°	10.000000
Is to the base AB = 40		1.602060
So is tangent A = 56° 30		10.179217
To the perp. BC 60.43		1.781277

Example 2. Wanting to ascertain the evelation of a church and steeple, (See Fig. 13, Pl. 1) similar to one I had undertaken to build, and not being able, otherwise, to obtain the measurements, I measured off 275 feet from the base of the porch and by means of a hemistant (fig. 11) took the following angles:

1. To the top of the ridge, the angle ADE was 6° 14′.
2. To the top of the belfrey, the angle ADD was 10° 18′.
3. To the commencement of the spire ADC was 13° 18′.
4. To the top of the spire ADB was 20° 0′.

Required the height of each part.

Here the same things are given, and required as in the first example, and the operations being precisely the same, may be abridged by simply adding together the first and second terms, and rejecting 10 from the sum of their indices: thus:

1. Base DA=275	2.439333	3. Base DA 275 =	2.439333
Tang. ADE=6° 14′	9.038216	Tang. ADC 13° 18′ =	9.373629
Perp. AE 30.03 =	1.477649	Perp. AC = 65	1.812961
2. Base DA=275	2.439333	4. Base DA 275 =	2.439333
Tang. ADD 10° 18′	9.259428	Tang. ADB 20° =	9.561066
Perp. AD 49.97	1.698761	Perp. AB 100.1 =	2.000399

Example 3. Three places, A, B, and C, (See Fig. 66, Plate 34 of Mensuration) are so situated, that A is directly south of B, and C directly east of A, at the distance of four miles. The bearing of C from B is 25° east. Required the distance of B, from A and C.

Here, making the hypothenuse BC radius, the proportion will be

As sine B =	25°	9.625948
Is to perp. AC	4	0.681241
So is rad.	90°	10.000000
To dist. BC = 11.36		1.055293

In like manner, the distance AB will be found to be 10.29.

PROBLEM II.

TO FIND THE PERPENDICULAR HEIGHT OF AN OBJECT STANDING ON AN EMINENCE.

Rule 1. Measure from the foot of the object to a convenient station on the plane beneath, and there take the angle of elevation, both of the top and bottom of the object.

2. Then say—As the cosine of the larger angle is to the difference of the two angles; so is the distance of the station from the foot of the object, to the height of the object.

Example.—Wanting to know the height of a tower standing on a hill (See Fig. 65.) I measured the distance from the base of the object at B, to the foot of the hill at A, and found it 136 feet. At A, I took the angle to the bottom of the tower, 48° 30 and to the top, 67°. Required the height of the building, and the elevation of the hill. Thus:

As cosine 67°	9.591878
Is to the diff. of the ∠s. 18° 30′	9.501476 sine.
So is dist. AB = 136	2.133339
	11635015
To height BC = 110.44	2.043137

2. To find the height of the hill, say

As radius	10.000000
Is to 110.44	2.043137
So is sine 48° 30′	9 874456
To height of hill = 82.71	1.917593

PROBLEM III.

TO FIND THE DISTANCE BETWEEN TWO INACCESSIBLE OBJECTS.

Rule.—Measure a base line between two stations, and the angles between this base and lines drawn from each of the stations to each of the objects.

Example. Let B and C, (fig. 67) be two buildings on the opposite banks of a river, and A, D two stations, lying in the same plane, 113 rods apart. Let the angle BAD made by the first building and the second station, be 100°; the angle CAD made by the second building and the second station, 36° 30′; let the angle CDA made by the second building and the first station, be 121°, and the angle BDA made by the first building and the first station, 49°. Required the distance between the two buildings B and C,

180° less the sum of BDA and BAD=ABD=31°
180 less the sum of CAD and CDA=ACD=22° 30′.

1. In the oblique angled triangle ABD, find DB, thus:

As the sine of ABD=31°	9.711839
Is to the distance AD=113	2.053078
So is the sine of BAD=100°, (or 80°)	9.993352
	12.046430
To the distance DB 216.	2.334591

2. In the the triangle ADC, find DC thus:

As the sine of ACD=22° 30′	9.582840
Is to the distance AD=113	2.053078
So is the sine of CAD=36° 30	9.774388
	11.827466
To the distance DC=175.64	2.244626

3. In the triangle BDC, we find the angle BDC, by subtracting the angle ADB=49° from ADC=121°: which gives 72°, with which, and the sides DB and DC, (as already found) we determine the length of the required side BC, by case 2 of oblique angled trigonometry, wherein two sides, and their contained angle are given to find the remaining sides and angles. Thus:

	DB+DC		
As the sum of	216+175.64=391.64		2.592887
Is to their diff.	216—175.64	40.36	1.605951
So is tang. of ½ supp, of BDC=54°			10.138759
			11.744690
To tang. of half the diff. of rem. angles.=8° 4′			9.151803

4. Having thus found half the difference of the required, angles, the required side is soonest found by the following proportion. viz.

As the sine of half the diff.=8° 4′	9.147136
Is to the sine of half their sum=54°	9.907958
So is the diff. of the given sides 40.36	1.605951
	11.513909
To the distance BC=232.6	2.366773

The principles of trigonometry are no less essential to the measurement of lines and angles, on water, than they are on land: Hence their application to

NAVIGATION.

In applying the principles of trigonometry to the art of navigation, the *distance* which a ship sails, is represented by the hypothenuse of a right angled triangle; the *course* or direction of the ship, is the angle at the perpendicular. The base represents the *departure*, which is always opposite to the course; and the perpendicular, which is always opposite to the compliment of the course, represents the *difference of latitude*. To illustrate these observations by an example: (see fig. 68.)

Suppose a ship sails from the point A, on a course of 45°. S.E. 84 miles to C. Required her departure AB, and diff. of latitude CB.

Making the distance, that is, the hypothenuse, radius, the perpendicular will be,

As radius or 90°	10.000000
Is to the distance 84	1.924279
So is the sine of the course 45°	9.849485
To the departure AB 59.40	1.773764

To find the difference of latitude, use the *cosine* of the course, and proceed as above, exactly.

Note.—The line NS, that is drawn in the diagram of the circle, represents the meridian. (See fig. 15. pl. 1.)

MATHEMATICAL INSTRUMENTS, SCALES, &C.

The Mathematical Instruments, commonly included in a case for the purposes of drawing, are *compasses* with their appendages, viz: *steel drawing pen, pencil holder*, with black lead pencil, *protractor*, or graduated semicircle, *plane scales*, and parallel ruler; to which, are often added other scales and implements, adapted to particular purposes, as land surveying, &c.

The use of the compasses is too well known to require particular explanation There are in the case, generally, two kinds, one with fixed steel points, and the other with one point fixed. When the unfixed point is taken off, there may be put in its place, a steel drawing pen point. The steel point is put in on the compass, when it is intended therewith to describe circles, or arcs, with ink, which are intended to remain. Occult arcs or such as are to be rubbed out again are most conveniently described with a pencil holder. The other steel pen is used for drawing right lines from any given points in any direction. An explanation of the use of the compasses has been fully given under the head of Practical Geometry, pages 8 and 9, &c,

The Scale is so called from a Greek word, which signifies a wooden measure of length It is a thin broad rule of wood, ivory, or brass, divided into different lines of various names and use. The best and most useful scales for architectural puposes,, are represented in (figs. 1, 2, and 3, pl. 1); and are of the exact size in which they are usually made. The graduations of these scales have been made with such care, that I believe they may be relied on, for practice, by them who have not the instruments at hand.

(Fig. 1). The breadth is divided into seven parts, and is numbered, (on the left of the scale) viz. 55, 45, 40, &c. to 20. These numbers are decimals of an inch, as may be seen in the first divisions at the right, that is on the lower part of each division; which is divided into ten parts, or the tenth part of a foot, and the uppper part of which is divided into twelve parts, or the twelfth part of a foot. At the right through these several divisions, they are numbered from 1, 2, 3, &c. to 10: again 1, 2, 3, &c. to 20, and so on. And in the upper division on the right, is a line of *chords*, which is numbered from C, viz: 10, 20, 30, &c. to 90 The construction and use of the line of chords is given on the *plane scale*.

In plotting and making architectural drawings, it is most convenient to work upon a scale of one quarter of an inch to a foot; or half an inch. The scale or division which is numbered 40 at the left, is just one quarter of an inch; and that which is numbered 20 in the lower division, is half an inch.

To take of feet and inches from the scale.—Suppose for example it was required to take the distance of ten feet and six inches from the scale, (say the quarter inch) set one foot of the compasses on 10, and open the other leg to the centre of the first division on the left (which is divided into twelve parts). The extent will be the distance required. Again, suppose it is required to take the distance of ten feet and five tenths of a foot, (say from the same scale): set one foot of the compasses on 10, and open the other leg to the centre of the first divison on the lower part of the scale, (which is divided into ten parts). The extent will be the distance required.

(Fig. 2) Exhibits the back of the same scale, with inches, eighths, and tenths of inches, &c. And it contains also a *Decimal Diagonal Scale*, for plotting or planning.

The *diagonal scale* is subdivided to hundredth parts of one half and one quarter of an inch; its principle and application will be obvious on inspection; as it may be seen that the perpendiculars are divided into ten equal parts, and through the divisions parallel lines are drawn, the whole length of the scale. Again, the length of the first division is divided both at top and bottom, into ten equal parts, and the points are connected by diagonal lines, so as to take off dimensions or numbers of two or more figures.

Examples. If the largest divisions be taken as units, the *exterior* smaller divisions will be tenth parts, and the divisions in the heighth will be *hundredth* parts. If the larger divisions be taken as tens, the next smaller will be hundredths, and the smallest thousandths, &c. Each set of divisions being tenth parts of the former ones.

To take the distance representing one and four tenths from the scale, (say the half inch), set one foot of the compasses on the upper line, to the larger division 1, and open the other leg to 4 in the subdivision on the right. The extent will be the distance required.

To take a distance equal to 25, set, in like manner, one foot of the compasses on the larger division 2, and extend the other to the subdivision 5, which will be the distance.

For 346, the larger division being in this case taken as hundredths, set one leg in 3, upon the line marked 6 at the end, and extend the other to the diagonal 4, which will be the extent required.

(Fig. 3) Represents another set of plotting scales for half an inch, one quarter of an inch three eighths, and one eighth of an inch to the foot, &c. and subdivided diagonally for greater exactness. The uses of these are too clear to require farther explanation.

(Fig. 4.) Represents another which is called the *Plane Scale*. The construction of this scale is as follows:

1. With the radius you intend for your scale, describe a semicircle ADB (as in fig 6) and from the centre C draw CD perpendicular to AB, which will divide the semicircle into two quadrants, AD, BD: continue the line CD to S: and draw the tangent BT perpendicular to AB.
2. Divide the quadrant BD, into nine equal parts, then will each of these be ten degrees; subdivide each of these parts into single degrees, and if your radius will admit of it, into minutes, or some aliquot parts of a degree greater than minutes.
3. Set one foot of the compasses in B, and transfer each of the divisions of the quadrant BD to the right line BD, then will BD be a line of *Chords*.
4. From the points 10, 20, 30, &c. in the quadrant BD draw right lines parellel to CD, to cut the radius CB, and they will divide that line into a line of *sines*, which must be numbered from C towards B.
5. If the same line of *sines* be numbered from B towards C, it will become a line of *versed sines*, which my be continued to 180°, if the same divisions be transferred on the same line on the other side of the centre C.
6. From the centre C, through the several divisions of the quadrant BD, draw right lines till they cut the tangent BT, then will the line BT, become a line of *Tangents*.
7. Setting one foot of the compasses in C, extend the other to the several divisions 10, 20, 30, &c. in the tangent line BT, and transfer these extents severally to the right line CS, then will that line be a line of *Secants*.
8. Right lines drawn from A to the several divisions 10, 20, 30, &c. in the quadrant BD, will divide the radius CD into a line of *Semi-tangents*.
9. Divide the quadrant AD into eight equal parts, and from A as a centre, transfer these divisions severally to the right line AD, then will AD be a line of *Rhumbs* each division answering to 11° 15′ upon the line of chords. The use of this line is for protracting and measuring angles, according to the common division of the mariners compass. There is another line also on the plane scale, that is not described in this figure, and is Marked ML, which is joined to a line of chords; and shows how many miles of easting or westing correspond to a degree of longitude in every latitude. To describe this line, let the quadrant ABC (fig. 7) be described with the same radius as in fig, 6; thence divide the line AB into sixty equal parts, and through each point draw lines parallel to AC to intersect the arc BC: on B as a centre, transfer the several points of intersection to the right line BC, and then number it from C towards B from 0 to 60, and it will be the line of longitude. And then take these several lines of chords sines, tangents, secants, &c., and place them upon a ruler, as represented in fig. 4, they will form the instrument called the *Plane Scale*.

(Fig. 5.) Represents an *Instrument*, or *Scale*, called the *Sector*; and consist of two rules or legs, moveable round an axis or joint as a centre, having several scales drawn on the faces. some single, others double; the single scales are like those upon a common *Gunter's Scale*, the double scales are those which proceed from the centre, each being laid twice on the same face of the instrument, viz: once on each leg. From these scales dimensions or distances are to be taken when the legs of the instrument are placed in an angular position.

The single scales are used exactly like those upon a common *Gunter Scale*.

And those of the double scales, the number of which is seven, viz: the scale of lines marked Lin. or L. the scale of chords marked Cho. or C., the scale of sines marked Sin. or S., the scale of tangents to 45°, and another scale of tangents from 45° to about 76°, both of which are marked Tan. or T., the scale of secants marked Sec. or S., and a scale of Polygons marked Pol.

The scale of lines, chords, sines and tangents under 45°, are all of the same radius beginning at the centre of the instrument, and terminating near the other extremity of each leg, viz: the lines at the division 10, the chords at 60° the sines at 90°, and the tangents at 45°; the remainder of the tangents or those above 45°, arc on the scales beginning at a quarter of the length of the former, counted from the centre, where they are marked with 45°, and extend to 76°. The secants also begin at the same distance from the centre, where they are marked with 0, and are from thence continued to 75°. The scales of polygons are set near the inner edge of the legs, and where these scales begin, they are marked with four and from thence are numbered backward, or towards the centre, to 12.

In describing the use of a sector the terms *lateral distance* and *transverse distance* often occur. By the former is meant the distance taken by the compasses on one of the scales only, beginning at the centre of the sector; and by the latter, the distance taken between any two corresponding divisions of the scales of the same name, the legs of the sector being in an angular position.

The use of the sector depends upon the proportions of the corresponding sides of similar triangles. Suppose for example the triangle ABC (fig 8) be the form of the *sector* when set in an angular position; then if we take AB equal to AC, and AD equal to AE, and join BC and DE it is evident that BC and DE will be parallel, therefore by the above mentioned proposition AB:BC::AD:DE; so whatever part AD is of AB, the same part DE will be of BC: hence if DE be the *chord sine* or *tangent* of any arc to the radius AD, BC will be the same to the radius AB.

The most useful scales or lines on the *sector* for *Architectural* purposes is the *Line of Lines*, marked L, the *Line of chords* marked C, and the line of *Polygons* marked Pol., as represented in fig 5.

Uses of the Line of Lines.—The line of lines is useful to divide a given line into any number of equal parts, or in any proportion, or to find third and fourth proportionals, or to increase a given line in any proportion.

Example 1. To divide a given line into any number of equal parts, (suppose 8) make the length of the given line a transverse distance to 8 and 8, the number of parts proposed; then will the transverse distance of 1 and 1 be one of the eight parts, or the eighth part of the whole: and the transverse distance of 3 and 3 will be 3 of the equal parts or $\frac{3}{8}$ of the whole line, &c.

Example 2. If a ship sails 59 miles in 9 hours, how far would she sail in 3 hours at the same rate.

Take 59 in your compasses as a transverse distance and set it off from 9 to 9, then the transverse distance 3 and 3 being measured laterally, will be equal to $19\frac{2}{3}$ miles.

Example 3. Having a plan or chart constructed upon a scale of 5 feet to an inch, or 5 miles to an inch, it is required to open the sector so that a corresponding scale may be taken from the line of lines.

Make the transverse distance 5 and 5 equal to 1 inch, and this position of the sector will produce the given scale.

Example 4. It is required to reduce a scale of 6 inches to a degree, another of 3 inches to a degree.

Make the transverse distance 6 and 6, equal to the lateral distance 3 and 3; then set off any distance from the chart laterally, and the corresponding transverse distance will be the reduced distance required.

Example 5. One side, of any triangle being given of any length to measure the other two sides on the same scale.

Suppose for example the triangle ABC (fig. 7, pl. 34 of Mens.) being constructed, and the side AB is 15 feet, to find the side AC and BC.

Take AB in your compasses, and apply it transversely to 15 and 15; to this opening of the sector apply the distance AC in your compasses to the same number on both sides of the rule transversely; and where the two points fall will be the measure on the line of lines of the distance required; the distance AC will fall against 14, 14, and BC against 13, 13, on the line of lines.

Uses of the line of Chords on the Sector.—The use of the line of chords upon the sector, is convenient for protracting any angle, when the paper is so small that an arc cannot be drawn upon it with the radius of a common line of chords.

Suppose for example that the triangle ABD (fig. 9, pl. 1), it was required to set off 20° from the point B on the arc BC. Take the radius AB in your compasses, and set it off transversely from 60° to 60° on the chords, then take the transverse extent from 20° to 20° on the chords; and place one foot of the compasses in B, the other will reach to C, and BC will be the arc required. And by the converse operation any angle or arc may be measured, viz: with any radius describe an arc, and then set that radius transversely from 60° to 60°; thence take the distance of the arc intercepted between the two legs, and apply it transversely to the chords, which will show the degrees of the given angle.

Note.—When the angle to be protracted exceeds 60° you must lay off 60°, and the remaining part; or if it be above 120° lay off 60° twice, and then the remaining part. And in a similar manner any arc above 60° may be measured.

Use of the line of Polygons on the Sector.—This line is also very useful to inscribe a regular polygon in a circle, in plotting on Paper. For example, let it be required to inscribe a regular octagon in a circle. Open the sector till the transverse distance 6 and 6 be equal to the radius of a circle; then will the transverse distance 8 and 8 be the side of the inscribed polygon.

To inscribe any regular polygon in a circle.—Suppose, for example, it is reuired to inscribe a hexagon as represented in the circle ABCDA (fig. 10, pl. 1.) First find the angle of centre E, by dividing 361° by the number of sides of which the proposed figure is to consist: then from the centre E draw the radiating lines EA, EF, EG, &c. in the angle given by the quotient, and the chord of the angle included between them, will be the side of the hexagon required. Thus, may any regular polygon be described, the radius being always 60°. In the like manner, any regular polygon may be described in a circle with the radius of a common line of chords as represented on the *plane scale*.

Use of the line of chords on the plane scale.—The definition of a chord is given among those under the head of Geometry, and the lines of this name are divided for the purpose of laying off and measuring angles, on the established principle, that the radius or semidiameter of a circle is equal to the side of a hexagon inscribed in the same circle; or, in other words to the chord of 60°. Hence by taking the extent of the chord of 60° in the compasses, applying one foot to an angular point, and sweeping an arc with the other from leg to leg, (*produced if required*) the exact measure of the angle may be found.

Example 1. In the triangle ABD (fig. 9, pl. 1) it is required to construct an angle at the point A of the line AB of any number of degrees, suppose 23.

From the line of chords take in your compasses the extent of 60; and setting one foot in A, describe the arc BC; then take 23, the number proposed, from the same line of chords, in your compasses, and set it off from B to C. Join AC, and the angle BAC will contain 23 degrees as required.

Example 2. Suppose it is required, to find the pitch of a roof, the angle of which is 25 degrees.

Take, as above, in your compasses the extent of 60, and set one foot in A, and describe the arc BC, then take 25, the number proposed, in your compasses and set it off from B to C, and join AC. Then will the angle BAC contain 25° as required.

This method of constructing a roof will be found the simplest and best. For if the span of the roof is given, we have only to take one half for the base. Then (by Problem 2, Trigonometry) you may ascertain the length of the rafter, and also the perpendicular heighth. Though *Carpenters* generally divide the breadth of the building, or span, into some number of equal parts, and then give one of these parts for the heighth. Suppose it is required to make the heigth of the roof equal to $\frac{1}{6}$ of the span. The angle of $\frac{1}{6}$ will be found to contain 18°. If it be equal to $\frac{1}{5}$ the angle will be 21° 30′. If $\frac{1}{4}$, the angle is 26°. If $\frac{1}{3}$, 33° 30′, &c.

(Fig. 11, pl. 1) Exhibits an *Instrument* or *Hemistant*, suitably constructed, to take levels and angles of elevation or depression in measuring heights, distances &c. This instrument should be made of hard wood. The diameter AB of the semicircle may be from 2 to 4 feet in length; the larger, the more accurate; and the straight part which extends out from AB, may be from 4 to 5 inches wide; this straight part, and the graduated semicircle, should be made from one piece of board; and $\frac{3}{4}$ of an inch will be sufficient for the thickness. To construct the graduated semicircle, let the line AB be drawn, and from D with the radius AD sweep the semicircle ACB; and from the centre D, draw DC at right angles with AB, which will divide the semicircle into two quadrants. To divide these quadrants into degrees; first divide them into 9 equal parts each; then will each of these be 10 degrees; thence subdivide each of these parts into single degrees, and if the radius will admit of it, into minutes, or some aliquot part of a degree greater than minutes, and number them from 0° to 90° each quadrant from C.

In taking angles of elevation, you can use a plumet as represented at H, or there may be a line drawn through the centre of the staff GH, and then let the degrees be drawn across the edge of the board, across the graduated semicircle, which will answer all purposes of a plumet, or even better. E represents the level. and is of the same with that of the straight part AB, and the thickness may be from one and a quarter, to one and a half inches. F represents the spirit level as let into the upper edge.

(Fig. 12.) Shows the plan and section of the same. A represents that part of the level; B the graduated semicircle; C the mortice which is harved out of that part of the level for the staff to slide through; E represents the screw to hold it fast when the level is placed in its right position.

A, at the upper part of this plate, shows a representation of a *Draught board*, commonly called the *Trustle board*, to which the paper used in drawing is to be fixed. This board is composed of a frame of mahogany or other hard wood; the edges of the frame should be made perfectly straight and square, with a pannel about half the thickness of the frame, which is to be let in from the back, and to lie in a rabbit, in the frame, and there to be secured by small buttons; B shows a section of the board, and the buttons by which the pannel is kept in its place. Eight in number will be sufficient for a common sized board. It would not be amiss before making the board, to ascertain the size of the paper to be used, and make the pannel about two inches less than the sheet. In applying this board to use, lay the paper on a table, and moisten one side of it with a sponge; then place the board upside down near it; take out the pannel and lay it on the paper, (one inch of which will extend beyond the pannel all around) take hold of the edges of the paper and lift them both into the frame; then fasten the buttons, and when it becomes dry it will be perfectly smooth.

C and D represents the T square for drawing right lines; the blade as represented at D, should not exceed $\frac{3}{16}$ of an inch in thickness.

E represents Pool's Geometrical Protractor. It is an instrument highly useful for the practical illustrations of elementary principles. I have used it for the last three years, and would recommend it to all Architects on account of its simplicity, and the ease with which it can be applied. Those wishing to purchase, will find them in all of our principal cities.

A TABLE
OF
LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
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7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
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16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
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22	1.342423	47	1.672098	72	1.857333	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
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113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	.320	379
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149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
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159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
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161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	..51	.319	.586	.853	1121	1388	1654	1921	267
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
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166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
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190	278754	8982	9211	9439	9667	9895	.123	.351	.578	.806	228
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
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193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
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233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
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237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	..30	181
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255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
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261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
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287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
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292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
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303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
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307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	9958	..99	.239	.380	.520	.661	.801	.941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	9687	9824	9962	..99	.236	.374	.511	.648	.785	.922	137
317	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
322	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
323	9203	9337	9471	9606	9740	9874	...9	.143	.277	.411	134
324	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4414	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	9828	9959	..90	.221	.353	.484	.615	.745	.876	1007	131
332	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	8917	9045	9174	9302	9430	9559	9687	9815	9943	..72	128
339	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	9076	9202	9327	9452	9578	9703	9829	9954	..79	.204	125
347	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	.106	123
355	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907	..26	.146	.265	.385	.504	.624	.743	.863	.982	119
364	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959	..76	.193	.309	.426	117
372	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	9898	..12	.126	.241	.355	.469	.583	.697	.811	114
381	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	9950	..61	.173	.284	.396	.507	.619	.730	.842	.953	112
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	.101	.210	.319	.428	.537	.646	.755	.864	109
399	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	602060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	9594	9701	9808	9914	..21	.128	.234	.341	.447	.554	107
408	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	..32	104
417	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
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420	623249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	9410	9512	9613	9715	9817	9919	.. 21	.123	.224	.326	102
427	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	3569	3670	3771	3872	3973	4074	4175	4276	4376	100
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	100
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	99
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	99
436	9486	9586	9686	9785	9885	9984	.. 84	.183	.283	.382	99
437	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	99
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	3551	3650	3749	3847	3946	4044	4143	4242	4340	98
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8162	98
445	8360	8458	8555	8653	8750	8848	8845	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	.. 16	.113	.210	97
447	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	4177	4273	4369	4465	4592	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	6821	95
457	9916	.. 11	.106	.201	.296	.391	.486	.581	.676	.771	95
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	94
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5106	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	.. 60	.153	93
468	670241	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	.. 63	.154	.245	91
479	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	681241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	2116	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	2767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	1756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9012	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	.. 19	.107	89
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2756	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7405	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	9057	9144	9231	9317	9404	9191	9578	9664	9751	87
501	9838	9924	.. 11	.. 98	.184	.271	.358	.444	.531	.617	87
502	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1641	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3895	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	2229	7315	7400	7485	85
510	707570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	.. 33	85
513	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	85
514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
215	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3650	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
520	716003	6087	6170	6254	6337	6421	6504	6588	6671	6754	83
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	.. 77	83
525	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	.2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	.. 55	.136	.217	.298	.378	.459	.540	.621	.702	81
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
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541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
513	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	.. 47	.126	.205	.284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2646	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	.. 45	.123	.200	.277	.354	.431	77
563	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2133	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8158	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	9668	9743	9819	9894	9970	.. 45	.121	.196	.272	.347	75
576	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
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578	1928	2003	2078	2153	2228	2303	2378	2153	2529	2604	75
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580	763428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8116	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	.. 42	74
589	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	770852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957	.. 29	.101	.173	.245	72
603	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8151	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	... 4	.. 74	.144	.215	70
617	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
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620	792392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8316	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	9560	9627	9694	9762	9829	9896	9964	.. 31	.. 98	.165	67
616	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2415	2512	2579	2646	2713	2780	2847	67
650	812913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
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661	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	9947	.. 11	.. 75	.139	.204	.268	.332	.396	.460	.525	64
677	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	.. 43	63
692	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3120	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
700	845098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	9739	9799	9859	9918	9978	.. 38	.. 98	.158	.218	.278	60
725	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	9818	9877	9935	9994	.. 53	.111	.170	.228	.287	.345	59
742	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
714	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7917	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9410	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	.. 13	.. 70	.127	.185	57
759	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
760	880814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
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785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
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789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
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798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
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839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
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843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
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875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
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878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	944483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975	..24	..73	.121	.170	.219	.267	.316	49
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
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907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
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922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
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937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
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941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
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947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
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955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
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964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
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975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
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981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
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985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
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990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

A TABLE OF LOGARITHMIC SINES, TANGENTS, AND SECANTS,

FOR EVERY DEGREE AND FIVE MINUTES OF THE QUADRANT.

0 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	0.000000	10.000000	0.000000	Infinite.	10.000000	Infinite.	60
5	7.162696	10.000000	7.162696	12.837304	10.000001	12.738304	55
10	7.463726	9.999998	7.463727	12.536273	10.000002	12.536274	50
15	7.639816	9.999996	7.639820	12.360180	10.000004	12.360184	45
20	7.764754	9.999993	7.764761	12.235239	10.000007	12.235246	40
25	7.861662	9.999989	7.861674	12.138326	10.000011	12.138338	35
30	7.940812	9.999983	7.940858	12.059142	10.000017	12.059158	30
35	8.007787	9.999977	8.007809	11.992191	10.000023	11.992213	25
40	8.065776	9.999971	8.065806	11.934194	10.000029	11.934224	20
45	8.116926	9.999963	8.116963	11.883037	10.000037	11.883074	15
50	8.162681	9.999954	8.162727	11.837273	10.000046	11.837319	10
55	8.204070	9.999944	8.204126	11.795874	10.000056	11.795930	5
60	8.241855	9.999934	8.241922	11.758078	10.000066	11.758145	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

89 Degrees.

1 Degree.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	8.241855	9.999934	8.241921	11.758079	10.000066	11.758145	60
5	8 276614	9 999922	8 276691	11 723309	10 000078	11 723386	55
10	8 308794	9 999910	8 308884	11 691116	10 000093	11 691206	50
15	8 338753	9 999897	8 338856	11 661144	10 000103	11 661247	45
20	8 366777	9 999882	8 366895	11 633105	10 000118	11 633223	40
25	8 393101	9 999867	8 393234	11 606766	10 000133	11 606899	35
30	8 417910	9 999851	8 418068	11 581932	10 000149	11 582081	30
35	8 441394	9 999834	8 441560	11 558440	10 090166	11 558606	25
40	8 463665	9 999816	8 463849	11 536151	10 000184	11 536335	20
45	8 484848	9 999797	8 485050	11 514950	10 000203	11 515152	15
50	8 505045	9 999778	8 505267	11 494733	10 000222	11 494955	10
55	8 524343	9 999657	8 524586	11 475414	10 000243	11 475657	5
60	8 542819	9 999735	8 543084	1 456916	10 000265	11 457181	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

88 Degrees.

2 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	8 542819	9 999735	8 543084	11 456916	10 000265	11 457181	60
5	8 560540	9 999713	8 560828	11 439172	10 000287	11 439460	55
10	8 577566	9 999689	8 577877	11 422123	10 000311	11 422434	50
15	8 593948	9 999665	8 594283	11 405717	10 000335	11 406052	45
20	8 609734	9 999640	8 610094	11 389906	10 000360	11 390266	40
25	8 624965	9 999614	8 625352	11 374648	10 000386	11 375035	35
30	8 639680	9 999586	8 640093	11 359907	10 000414	11 360320	30
35	8 653911	9 999558	8 654352	11 345648	10 000442	11 346089	25
40	8 667689	9 999529	8 668160	11 331840	10 000471	11 332311	20
45	8 681043	9 999500	8 681544	11 318456	10 000500	11 318957	15
50	8 693998	9 999469	8 694529	11 305471	10 000531	11 306002	10
55	8 706577	9 999437	8 707140	11 292860	10 000563	11 293423	5
60	8 718800	9 999404	8 719396	11 280604	10 000596	11 281200	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

87 Degrees.

3 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	8 718800	9 999404	8 719396	11 280604	10 000596	11 281200	60
5	8 730688	9 999371	8 731317	11 268683	10 000629	11 269312	55
10	8 742259	9 999336	8 742922	11 257078	10 000664	11 257741	50
15	8 753528	9 999301	8 754227	11 245773	10 000699	11 246472	45
20	8 764511	9 999265	8 765246	11 234754	10 000735	11 235489	40
25	8 775223	9 999227	8 775995	11 224005	10 000773	11 224777	35
30	8 785675	9 999189	8 786486	11 213514	10 000811	11 214325	30
35	8 795881	9 999150	8 796731	11 203269	10 000850	11 204119	25
40	8 805852	9 999110	8 806742	11 193258	10 000890	11 194148	20
45	8 815599	9 999069	8 816529	11 183471	10 000931	11 184401	15
50	8 825130	9 999027	8 826103	11 173897	10 000973	11 174870	10
55	8 834456	9 998984	8 835471	11 164529	10 001016	11 165544	5
60	8 843585	9 998941	8 844644	11 155356	10 001059	11 156415	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

86 Degrees.

4 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	8 843585	9 998941	8 844644	11 155356	10 001059	11 156415	60
5	8 852525	9 998896	8 853628	11 146372	10 001104	11 147475	55
10	8 861283	9 998851	8 862433	11 137567	10 001149	11 138717	50
15	8 869868	9 998804	8 871064	11 128936	10 001196	11 130132	45
20	8 878285	9 998757	8 879529	11 120471	10 001243	11 121715	40
25	8 886542	9 998708	8 887833	11 112167	10 001292	11 113458	35
30	8 894643	9 998659	8 895984	11 104016	10 001341	11 105357	30
35	8 902596	9 998609	8 903987	11 096013	10 001391	11 097404	25
40	8 910404	9 998558	8 911846	11 088154	10 001442	11 089596	20
45	8 918073	9 998506	8 919568	11 080432	10 001494	11 081927	15
50	8 922609	9 998453	8 927156	11 072844	10 001547	11 074391	10
55	8 933015	9 998399	8 931616	11 065384	10 001601	11 066985	5
60	8 940296	9 998344	8 941952	11 058048	10 001656	11 059704	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

85 Degrees.

5 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	8 940296	9 998344	8 941952	11 058048	10 001656	11 059704	60
5	8 947456	9 998289	8 949168	11 050832	10 001711	11 052544	55
10	8 954499	9 998232	8 956267	11 043733	10 001768	11 045501	50
15	8 961429	9 998174	8 963255	11 036745	10 001826	11 038571	45
20	8 968249	9 998116	8 970133	11 029867	10 001884	11 031751	40
25	8 974962	9 998056	8 976906	11 023094	10 001944	11 025038	35
30	8 981573	9 997996	8 983577	11 016123	10 002004	11 018427	30
35	8 988083	9 997935	8 990149	11 009851	10 002065	11 011917	25
40	8 994497	9 997872	8 996624	11 003376	10 002128	11 005503	20
45	9 000816	9 997809	9 003007	10 996993	10 002191	10 999184	15
50	9 007014	9 997745	9 009298	10 990702	10 002255	10 992956	10
55	9 013182	9 997680	9 015502	10 984498	10 002320	10 986818	5
60	9 019235	9 997614	9 021620	10 978380	10 002386	10 980765	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

84 Degrees.

6 Degrees.

M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M
0	9 019235	9 997614	9 021620	10 978380	10 002386	10 980765	60
5	9 025203	9 997547	9 027655	10 972345	10 002453	10 974797	55
10	9 031089	9 997480	9 033609	10 966391	10 002520	10 968911	50
15	9 036896	9 997411	9 039485	10 960515	10 002589	10 963104	45
20	9 042625	9 997341	9 045284	10 954716	10 002659	10 957375	40
25	9 048279	9 997271	9 051008	10 948992	10 002729	10 951721	35
30	9 053859	9 997199	9 056659	10 943341	10 002801	10 946141	30
35	9 059367	9 997127	9 062240	10 937760	10 002873	10 940633	25
40	9 064806	9 997063	9 067752	10 932218	10 002947	10 935194	20
45	9 070176	9 996979	9 073197	10 926803	10 003021	10 929824	15
50	9 075480	9 996904	9 078576	10 921424	10 003096	10 924520	10
55	9 080719	9 996828	9 083891	10 916109	10 003172	10 919281	5
60	9 085894	9 995751	9 089144	10 910856	10 003249	10 914106	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co sec.	Secant.	M

83 Degrees.

7 Degrees.

M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M
0	9 085894	9 996751	9 089144	10 910856	10 003249	10 914106	60
5	9 091008	9 996673	9 094336	10 905664	10 003327	10 908992	55
10	9 096062	9 996594	9 099468	10 900532	10 003406	10 903938	50
15	9 101056	9 996514	9 104542	10 895458	10 003486	10 898944	45
20	9 105992	9 996433	9 109559	10 890441	10 003567	10 894008	40
25	9 110873	9 996351	9 114521	10 885479	10 003649	10 889127	35
30	9 115698	9 996269	9 119429	10 880571	10 003731	10 884302	30
35	9 120469	9 996185	9 124284	10 875716	10 003815	10 879531	25
40	9 125187	9 996100	9 129087	10 870913	10 003900	10 874813	20
45	9 129854	9 996015	9 133839	10 866161	10 003985	10 870146	15
50	9 134470	9 995928	9 138542	10 861458	10 004071	10 865530	10
55	9 139037	9 995841	9 143196	10 856804	10 004159	10 860963	5
60	9 143555	9 995753	9 147803	10 852197	10 004247	10 856445	0
M	Co-sine	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

82 Degrees.

8 Degrees.

M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M
0	9 143555	9 995753	9 147803	10 852197	10 004247	10 856445	60
5	9 148026	9 995664	9 152363	10 847637	10 004336	10 851974	55
10	9 152451	9 995573	9 156877	10 843123	10 004427	10 847549	50
15	9 156830	9 995482	9 161347	10 838653	10 004518	10 843170	45
20	9 161164	9 995390	9 165774	10 834226	10 004610	10 838836	40
25	9 165454	9 995297	9 170157	10 829843	10 004703	10 834546	35
30	9 169702	9 995203	9 174499	10 825501	10 004797	10 830298	30
35	9 173908	9 995108	9 178799	10 821201	10 004892	10 826092	25
40	9 178072	9 995013	9 183059	10 816941	10 004987	10 821928	20
45	9 182196	9 994916	9 187280	10 812720	10 005084	10 817804	15
50	9 186280	9 994818	9 191462	10 808538	10 005182	10 813720	10
55	9 190325	9 994720	9 195606	10 804394	10 005281	10 809675	5
60	9 194332	9 994620	9 199713	10 800287	10 005380	10 805668	0
M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M

81 Degrees.

9 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	9 194332	9 994620	9 199713	10 800287	10 005380	10 805668	60
5	9 198302	9 994519	9 203782	10 796218	10 005481	10 801698	55
10	9 202234	9 994418	9 207817	10 792183	10 005582	10 797766	50
15	9 206131	9 994316	9 211815	10 788185	10 005684	10 793869	45
20	9 209992	9 994212	9 215780	10 784220	10 005788	10 790008	40
25	9 213818	9 994108	9 219710	10 780290	10 005892	10 786182	35
30	9 217609	9 994003	9 223607	10 776393	10 005997	10 782391	30
35	9 221367	9 993897	9 227471	10 772529	10 006103	10 778633	25
40	9 225092	9 993789	9 231302	10 768698	10 006211	10 774908	20
45	9 228784	9 993681	9 235103	10 764897	10 006319	10 771216	15
50	9 232444	9 993572	9 238872	10 761128	10 006428	10 767556	10
55	9 236073	9 993462	9 242610	10 757390	10 006538	10 763927	5
60	9 239670	9 993351	9 246319	10 753681	10 006649	10 760330	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

80 Degrees.

10 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 239670	9 993351	9 246319	10 753681	10 006649	10 760330	60
5	9 243237	9 993240	9 249998	10 750002	10 006760	10 756763	55
10	9 246775	9 993127	9 253648	10 746352	10 006873	10 753225	50
15	9 250282	9 993013	9 257269	10 742731	10 006987	10 749718	45
20	9 253761	9 992898	9 260863	10 739137	10 007102	10 746239	40
25	9 257211	9 992783	9 264428	10 735572	10 007217	10 742789	35
30	9 260633	9 992666	9 267967	10 732033	10 007334	10 739367	30
35	9 264027	9 992549	9 271479	10 728521	10 007451	10 735973	25
40	9 267395	9 992430	9 274964	10 725036	10 007570	10 732605	20
45	9 270735	9 992311	9 278424	10 721576	10 007689	10 729265	15
50	9 274049	9 992190	9 281858	10 718142	10 007810	10 725951	10
55	9 277337	9 992069	9 285268	10 714732	10 007931	10 722663	5
60	9 280599	9 991947	9 288652	10 711348	10 008053	10 719401	0
M	Co-sine.	Sinc.	Co-tang.	Tang.	Co-sec.	Secant.	M

7[illegible] Degrees.

11 Degrees.

M	sine.	co-sine	tangent.	co-tang.	secant.	co-sec.	M
0	9 280599	9 991947	9 288652	10 711348	10 008053	10 719401	60
5	9 283836	9 991823	9 292013	10 707987	10 008177	10 716164	55
10	9 287048	9 991699	9 295349	10 704651	10 008301	10 712952	50
15	9 290236	9 991574	9 298662	10 701338	10 008426	10 709764	45
20	9 293399	9 991448	9 301951	10 698049	10 008552	10 706601	40
25	9 296539	9 991321	9 305218	10 694782	10 008679	10 703461	35
30	9 299655	9 991193	9 308463	10 691537	10 008807	10 700345	30
35	9 302748	9 991064	9 311685	10 688315	10 008936	10 697252	25
40	9 305819	9 990934	9 314885	10 685115	10 009066	10 694181	20
45	9 308867	9 990803	9 318064	10 681936	10 009197	10 691133	15
50	9 311893	9 990671	9 321222	10 678778	10 009329	10 688107	10
55	9 314897	9 990538	9 324358	10 675642	10 009462	10 685103	5
60	9 317879	9 990404	9 327475	10 672525	10 009596	10 682121	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

78 Degrees.

12 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 317879	9 990404	9 327474	10 672526	10 009596	10 682121	60
5	9 320840	9 990270	9 230570	10 669430	10 009730	10 679160	55
10	9 323780	9 990134	9 333646	10 666354	10 009866	10 676220	50
15	9 326700	9 989997	9 336702	10 663298	10 010003	10 673300	45
20	9 329599	9 989860	9 339739	10 660261	10 010140	10 670401	40
25	9 332478	9 989721	9 342757	10 657243	10 010279	10 667522	35
30	9 335337	9 989582	9 345755	10 654245	10 010418	10 664663	30
35	9 338176	9 989441	9 348735	10 651265	10 010559	10 661824	25
40	9 340996	9 989300	9 351697	10 648303	10 010700	10 659004	20
45	9 343797	9 989157	9 354640	10 645360	10 010843	10 656203	15
50	9 346579	9 989014	9 357566	10 642434	10 010986	10 653421	10
55	9 349343	9 988869	9 360474	10 639526	10 011131	10 650657	5
60	9 352088	9 988724	9 363364	10 636636	10 011276	10 647912	0
M	co-sine.	Sine.	co-tang.	tangent.	co-sec.	secant.	M

77 Degrees.

13 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 352088	9 988724	9 363364	10 636636	10 011276	10 647912	60
5	9 354815	9 988578	9 366237	10 633763	10 011422	10 645185	55
10	9 357524	9 988430	9 369094	10 630906	10 011570	10 642476	50
15	9 360215	9 988282	9 371933	10 628067	10 011718	10 639785	45
20	9 362889	9 988133	9 374756	10 625244	10 011867	10 637111	40
25	9 365546	9 987983	9 377563	10 622437	10 012017	10 634454	35
30	9 368185	9 987832	9 380354	10 619646	10 012168	10 631815	30
35	9 370808	9 987679	9 383129	10 616871	10 012321	10 629192	25
40	9 373414	9 987526	9 385888	10 614112	10 012474	10 626586	20
45	9 376003	9 987372	9 388631	10 611369	10 012628	10 623997	15
50	9 378577	9 987217	9 391360	10 608640	10 012783	10 621423	10
55	9 381134	9 987061	9 394073	10 605927	10 012939	10 618866	5
60	9 383675	9 986904	9 396771	10 603229	10 013096	10 616325	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

76 Degrees.

14 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 383675	9 986904	9 396771	10 603229	10 013096	10 616325	60
5	9 386201	8 986746	9 399455	10 600545	10 013254	10 613799	55
10	9 388711	9 986587	9 402124	10 597876	10 013413	10 611289	50
15	9 391206	9 986427	9 404778	10 595222	10 013573	10 608794	45
20	9 393685	9 986266	9 407419	10 592581	10 013734	10 606315	40
25	9 396150	9 986104	9 410045	10 589955	10 013896	10 603850	35
30	9 398600	9 985942	9 412658	10 587342	10 014058	10 601400	30
35	9 401035	9 985778	9 415257	10 584743	10 014222	10 598965	25
40	9 403455	9 985613	9 417842	10 582158	10 014387	10 596545	20
45	9 405862	9 985447	9 420415	10 579585	10 014553	10 594138	15
50	9 408254	9 985280	9 422974	10 577026	10 014720	10 591746	10
55	9 410632	9 985113	9 425519	10 574481	10 014887	10 589368	5
60	9 412996	9 984944	9 428052	10 571948	10 015056	10 587004	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

75 Degrees.

15 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 412996	9 984944	9 428052	10 571948	10 015056	10 587004	60
5	9 415347	9 984774	9 430573	10 569427	10 015226	10 584653	55
10	9 417684	9 984603	9 433080	10 566920	10 015397	10 582316	50
15	9 420007	9 984432	9 435576	10 564424	10 015568	10 579993	45
20	9 422318	9 984259	9 438059	10 561941	10 015741	10 577682	40
25	9 424615	9 984085	9 440529	10 559471	10 015915	10 575385	35
30	9 426899	9 983911	9 442988	10 557012	10 016089	10 573101	30
35	9 429170	9 983735	9 445435	10 554565	10 016265	10 570830	25
40	9 431429	9 983558	9 447870	10 552130	10 016442	10 568571	20
45	9 433675	9 983381	9 450294	10 549706	10 016619	10 566325	15
50	9 435908	9 983202	9 452706	10 547294	10 016798	10 564092	10
55	9 438129	9 983022	9 455107	10 544893	10 016978	10 561871	5
60	9 440338	9 982842	9 457496	10 542504	10 017158	10 559662	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

74 Degrees.

16 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 440338	9 982842	9 457496	10 542504	10 017158	10 559662	60
5	9 442535	9 982660	9 459875	10 540125	10 017340	10 557465	55
10	9 444720	9 982477	9 462242	10 537758	10 017523	10 555280	50
15	9 446893	9 982294	9 464599	10 535401	10 017706	10 553107	45
20	9 449054	9 982109	9 466945	10 533055	10 017891	10 550946	40
25	9 451204	9 981924	9 469280	10 530720	10 018076	10 548796	35
30	9 453342	9 981737	9 471605	10 528395	10 018263	10 546658	30
35	9 455469	9 981549	9 473919	10 526081	10 018451	10 544531	25
40	9 457584	9 981361	9 476223	10 523777	10 018639	10 542416	20
45	9 459688	9 981171	9 478517	10 521483	10 018829	10 540312	15
50	9 461782	9 980981	9 480801	10 519199	10 019019	10 538218	10
55	9 463864	9 980789	9 483075	10 516925	10 019211	10 536136	5
60	9 465935	9 980596	9 485339	10 514661	10 019404	10 534065	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

73 Degrees.

17 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 465935	9 980596	9 485339	10 514661	10 019404	10 534065	t0
5	9 467996	9 980403	9 487593	10 512407	10 019597	10 532004	55
10	9 470046	9 980208	9 489838	10 510162	10 019792	10 529954	50
15	9 472086	9 980012	9 492073	10 507927	10 019988	10 527914	45
20	9 474115	9 979816	9 494299	10 505701	10 020184	10 525885	40
25	9 476133	9 979618	9 496515	10 503485	10 020382	10 523867	35
30	9 478142	9 979420	9 498722	10 501278	10 020580	10 521858	30
35	9 480140	9 979220	9 500920	10 499080	10 020780	10 519860	25
40	9 482128	9 979019	9 503109	10 496891	10 020981	10 517872	20
45	9 484107	9 978817	9 505289	10 494711	10 021183	10 515893	15
50	9 486075	9 978615	9 507460	10 492540	10 021385	10 513925	10
55	9 488034	9 978411	9 509622	10 490378	10 021589	10 511966	5
60	9 489982	9 978206	9 511776	10 488224	10 021794	10 510018	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

72 Degrees.

18 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 489982	9 978206	9 511776	10 488224	10 021794	10 510018	60
5	9 491922	9 978001	9 513921	10 486079	10 021999	10 508078	55
10	9 493851	9 977794	9 516057	10 483943	10 022206	10 506149	50
15	9 495772	9 977586	9 518185	10 481815	10 022414	10 504228	45
20	9 497682	9 977377	9 520305	10 479695	10 022623	10 502318	40
25	9 499584	9 977167	9 522417	10 477583	10 022833	10 500416	35
30	9 501476	9 976957	9 524520	10 475480	10 023043	10 498524	30
35	9 503360	9 976745	9 526615	10 473385	10 023255	10 496640	25
40	9 505234	9 976532	9 528702	10 471298	10 023468	10 494766	20
45	9 507099	9 976318	9 530781	10 469219	10 023682	10 492901	15
50	9 508956	9 976103	9 532853	10 467147	10 023897	10 491044	10
55	9 510803	9 975887	9 534916	10 465084	10 024113	10 489197	5
60	9 512642	9 975670	9 536972	10 463028	10 024330	10 487358	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

71 Degrees.

19 Degrees.

M	sine.	co-sinc.	tangent.	co-tang.	secant.	co-sec.	M
0	9 512642	9 975670	9 536972	10 463028	10 024330	10 487358	60
5	9 514472	9 975452	9 539020	10 460980	10 024548	10 485528	55
10	0 516294	9 975233	9 541061	10 458939	10 024767	10 483706	50
15	9 518107	9 975013	9 543094	10 456906	10 024987	10 481893	45
20	9 519911	9 974792	9 545119	10 454881	10 025208	10 480089	40
25	9 521707	9 974570	9 547138	10 452862	10 025430	10 478293	35
30	9 523495	9 974347	9 549149	10 450851	10 025653	10 476505	30
35	9 525275	9 974122	9 551152	10 448848	10 025878	10 474725	25
40	9 527046	9 973897	9 553149	10 446851	10 026103	10 472954	20
45	9 528810	9 973671	9 555139	10 444861	10 026329	10 471190	15
50	9 530565	9 973444	9 557121	10 442879	10 026556	10 469435	10
55	9 532312	9 973215	9 559097	10 440903	10 026785	10 467688	5
60	9 534052	9 972986	9 561066	10 438934	10 027014	10 465948	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

70 Degrees.

20 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 534052	9 972986	9 561066	10 438934	10 027014	10 465948	60
5	9 535783	9 972755	9 563028	10 436972	10 027245	10 464217	55
10	9 537507	9 972524	9 564983	10 435017	10 027476	10 462493	50
15	9 539223	9 972291	9 566932	10 433068	10 027709	10 460777	45
20	9 540931	9 972058	9 568873	10 431127	10 027942	10 459069	40
25	9 542632	9 971823	9 570809	10 429191	10 028177	10 457368	35
30	9 544325	9 971588	9 572738	10 427262	10 028412	10 455675	30
35	9 546011	9 971351	9 574660	10 425340	10 028649	10 453989	25
40	9 547689	9 971113	9 576576	10 423424	10 028887	10 452311	20
45	9 549360	9 970874	9 578486	10 421514	10 029126	10 450640	15
50	9 551024	9 970635	9 580389	10 419611	10 029365	10 448976	10
55	9 552680	9 970394	9 582286	10 417714	10 029606	10 447320	5
60	9 554329	9 970152	9 584177	10 415823	10 029848	10 445671	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

69 Degrees.

21 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 554329	9 970152	9 584177	10 415823	10 029848	10 445671	60
5	9 555971	9 969909	9 586062	10 413938	10 030091	10 444029	55
10	9 557606	9 969665	9 587941	10 412059	10 030335	10 442394	50
15	9 559234	9 969420	9 589814	10 410186	10 030580	10 440766	45
20	9 560855	9 969173	9 591681	10 408319	10 030827	10 439145	40
25	9 562468	9 968926	9 593542	10 406458	10 031074	10 437532	35
30	9 564075	9 968678	9 595398	10 404602	10 031322	10 435925	30
35	9 565676	9 968429	9 597247	10 402753	10 031571	10 434324	25
40	9 567269	9 968178	9 599091	10 400909	10 031822	10 432731	20
45	9 568856	9 967927	9 600929	10 399071	10 032073	10 431144	15
50	9 570435	9 967674	9 602761	10 397239	10 032326	10 429565	10
55	9 572009	9 967421	9 604588	10 395412	10 032579	10 427991	5
60	9 573575	9 967166	9 606410	10 393590	10 032834	10 426425	0
M	co-sine.	sine.	co-tang.	tangent.	co-scc.	secant.	M

68 Degrees.

22 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 573575	9 967166	9 606410	10 393590	10 032834	10 426425	60
5	9 575136	9 966910	9 608225	10 391775	10 033090	10 424864	55
10	9 576689	9 966653	6 610036	10 389964	10 033347	10 423311	50
15	9 578236	9 966395	9 611841	10 388159	10 033605	10 421764	45
20	9 579777	9 966136	9 613641	10 386359	10 033864	10 420223	40
25	9 581312	9 965876	9 615435	10 384565	10 034124	10 418688	35
30	9 582840	9 965615	9 617224	10 382776	10 034385	10 417160	30
35	9 584361	9 965353	9 619008	10 380992	10 034647	10 415639	25
40	9 585877	9 965090	9 620787	10 379213	10 034910	10 414123	20
45	9 587386	9 964826	9 622561	10 377439	10 035174	10 412614	15
50	9 588890	9 964560	9 624330	10 375670	10 035440	10 411110	10
55	9 590387	9 964294	9 626093	10 373907	10 035706	10 409613	5
60	9 591878	9 964026	9 627852	10 372148	10 035974	10 408122	0
M	co-sinc.	sine.	co-tang.	tangent.	co-sec.	sccant.	M

67 Degrees.

23 Degrees.

M	sine.	co-sine.	tangent.	[illegible]	secant.	co-sec.	M
0	9 591878	9 964026	9 627852	10 372148	10 035974	10 408122	60
5	9 593363	9 963757	9 629606	10 370394	10 036243	10 406637	55
10	9 594842	9 963488	9 631355	10 368645	10 036512	10 405158	50
15	9 596315	9 963217	9 633098	10 366902	10 036783	10 403685	45
20	9 597783	9 962945	9 634838	10 365162	10 037055	10 402217	40
25	9 599244	9 962672	9 636572	10 363428	10 037328	10 400756	35
30	9 600700	9 962398	9 638302	10 361698	10 037602	10 399300	30
35	9 602150	9 962123	9 640027	10 359973	10 037877	10 397850	25
40	9 603594	9 961846	9 641747	10 358253	10 038154	10 396406	20
45	9 605032	9 961569	9 643463	10 356537	10 038431	10 394968	15
50	9 606465	9 961290	9 645174	10 354826	10 038710	10 393535	10
55	9 607892	9 961011	9 646881	10 353119	10 038989	10 392108	5
60	9 609313	9 960730	9 648583	10 351417	10 039270	10 390687	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

6[illegible] Degrees.

24 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 609313	9 960730	9 648583	10 351417	10 039270	10 390687	60
5	9 610729	9 960448	9 650281	10 349719	10 039552	10 389271	55
10	9 612140	9 960165	9 651974	10 348026	10 039835	10 387860	50
15	9 613545	9 959882	9 653663	10 346337	10 040118	10 386455	45
20	9 614944	9 959596	9 655348	10 344652	10 040404	10 385056	40
25	9 616338	9 959310	9 657028	10 342972	10 040690	10 383662	35
30	9 617727	9 959023	9 658704	10 341296	10 040977	10 382273	30
35	9 619110	9 958734	9 660376	10 339624	10 041266	10 380890	25
40	9 620488	9 958445	9 662043	10 337957	10 041555	10 379512	20
45	9 621861	9 958154	9 663707	10 336293	10 041846	10 378139	15
50	9 623229	9 957863	9 665366	10 334634	10 042137	10 376771	10
55	9 624591	9 957570	9 667021	10 332979	10 042430	10 375409	5
60	9 625948	9 957276	9 668672	10 331328	10 042724	10 374052	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

65 Degrees.

25 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 625948	9 957276	9 668673	10 331327	10 042724	10 374052	60
5	9 627300	9 956981	9 670320	10 329680	10 043019	10 372700	55
10	9 628647	9 956684	9 671963	10 328037	10 043316	10 371353	50
15	9 629989	9 956387	9 673602	10 326398	10 043613	10 370011	45
20	9 631326	9 956089	9 675237	10 324763	10 043911	10 368674	40
25	9 632658	9 955789	9 676869	10 323131	10 044211	10 367342	35
30	9 633984	9 955488	9 678496	10 321504	10 044512	10 366016	30
35	9 635306	9 955186	9 680120	10 319880	10 044814	10 364694	25
40	9 636623	9 954883	9 681740	10 318260	10 045117	10 363377	20
45	9 637935	9 954579	9 683356	10 316644	10 045421	10 362065	15
50	9 639242	9 954274	9 684968	10 315032	10 045726	10 360758	10
55	9 640544	9 953968	9 686577	10 313423	10 046032	10 359456	5
60	9 641842	9 953660	9 688182	10 311818	10 046340	10 358158	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

64 Degrees.

26 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 641842	9 953660	9 688182	10 311818	10 046340	10 358158	60
5	9 643135	9 953352	9 689783	10 310217	10 046648	10 356865	55
10	9 644423	9 953042	9 691381	10 308619	10 046958	10 355577	50
15	9 645706	9 952731	9 692975	10 307025	10 047269	10 354294	45
20	9 646984	9 952419	9 694566	10 305434	10 047581	10 353016	40
25	9 648258	9 952106	9 696153	10 303847	10 047894	10 351742	35
30	9 649527	9 951791	9 697736	10 302264	10 048209	10 350473	30
35	9 650792	9 951476	9 699316	10 300684	10 048524	10 349208	25
40	9 652052	9 951159	9 700893	10 299107	10 048841	10 347948	20
45	9 653308	9 950841	9 702466	10 297534	10 049159	10 346692	15
50	9 654558	9 950522	9 704036	10 295964	10 049478	10 345442	10
55	9 655805	9 950202	9 705603	10 294397	10 049798	10 344195	5
60	9 657047	9 949881	9 707166	10 292834	10 050119	10 342953	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

63 Degrees.

27 Degrees.

M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M
0	9 657047	9 949881	9 707166	10 292834	10 050119	10 342953	60
5	9 658284	9 949558	9 708726	10 291274	10 050442	10 341716	55
10	9 659517	9 949235	9 710282	10 289718	10 050765	10 340483	50
15	9 660746	9 948910	9 711836	10 288164	10 051090	10 339254	45
20	9 661970	9 948584	9 713386	10 286614	10 051416	10 338030	40
25	9 663190	9 948257	9 714933	10 285067	10 051743	10 336810	35
30	9 664406	9 947929	9 716477	10 282523	10 052071	10 335594	30
35	9 665617	9 917600	9 718017	10 281983	10 052400	10 334383	25
40	9 666824	9 947269	9 719555	10 280445	10 052731	10 333176	20
45	3 668027	9 946937	9 721089	10 278911	10 053063	10 331973	15
50	9 669225	9 946604	9 722621	10 277379	10 053396	10 330775	10
55	9 670119	9 946270	9 724149	10 275851	10 053730	10 329581	5
60	9 671609	9 945935	9 725674	10 274326	10 054065	10 328391	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

62 Degrees.

28 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 671609	9 945935	9 725674	10 274326	10 054065	10 328391	60
5	9 672795	9 945598	9 727197	10 272803	10 054402	10 327205	55
10	9 673977	9 945261	9 728716	10 271284	10 054739	10 326023	50
15	9 675155	9 944922	9 730233	10 269767	10 055078	10 324845	45
20	9 676328	9 944582	9 731746	10 268254	10 055418	10 323672	40
25	9 677498	9 944241	9 733257	10 266743	10 055759	10 322502	35
30	9 678663	9 943899	9 734764	10 265236	10 056101	10 321337	30
35	9 679824	9 943555	9 736269	10 263731	10 056445	10 320176	25
40	9 680982	9 943210	9 737771	10 262229	10 056790	10 319018	20
45	9 682135	9 942864	9 739271	10 260729	10 057136	10 317865	15
50	6 683284	9 942517	9 740767	10 259233	10 057483	10 316716	10
55	9 684430	9 942169	9 742261	10 257739	10 057831	10 315570	5
60	9 685571	9 941819	9 743752	10 256248	10 058181	10 314429	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

61 Degrees.

29 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 685571	9 941819	9 743752	10 256248	10 058181	10 314429	60
5	9 686709	9 941469	9 745240	10 254760	10 058531	10 313291	55
10	9 687843	9 941117	9 746726	10 253274	10 058883	10 312157	50
15	9 688972	9 940763	9 748209	10 251791	10 059237	10 311028	45
20	9 690098	9 940409	9 749689	10 250311	10 059591	10 309902	40
25	9 691220	9 940054	9 751167	10 248833	10 059946	10 308780	35
30	9 692339	9 939697	9 752642	10 247358	10 060303	10 307661	30
35	9 693453	9 939339	9 754115	10 245885	10 060661	10 306547	25
40	9 694564	9 938980	9 755585	10 244415	10 061020	10 305436	20
45	9 695671	9 938619	9 757052	10 242948	10 061381	10 304329	15
50	9 696775	9 938258	9 758517	10 241483	10 061742	10 303225	10
55	9 697874	9 937895	9 759979	10 240021	10 062105	10 302126	5
60	9 698970	9 937531	9 761439	10 238561	10 062469	10 301030	0
M	co-sine.	sine.	co-tang	tangent.	co-sec.	secant.	M

60 Degrees.

30 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 698970	9 937531	9 761439	10 238561	10 062469	10 301030	60
5	9 700062	9 937165	9 762897	10 237103	10 062835	10 299938	55
10	9 701151	9 936799	9 764352	10 235648	10 063201	10 298849	50
15	9 702236	9 936431	9 765805	10 234195	10 063569	10 297764	45
20	9 703317	9 936062	9 767255	10 232745	10 063938	10 296683	40
25	9 704395	9 935692	9 768703	10 231297	10 064308	10 295605	35
30	9 705469	9 935320	9 770148	10 229852	10 064680	10 294531	30
35	9 706539	9 934948	9 771592	10 228408	10 065052	10 293461	25
40	9 707606	9 934574	9 773033	10 226867	10 065426	10 292394	20
45	9 708670	9 934199	9 774471	10 225529	10 065801	10 291330	15
50	9 709730	9 933822	9 775908	10 224092	10 066178	10 290270	10
55	9 710786	9 933445	9 777342	10 222658	10 066555	10 289214	5
60	9 711839	9 933066	9 778774	10 221226	10 066934	10 288161	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

59 Degrees.

31 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 711839	9 933066	9 778774	10 221226	10 066934	10 288161	60
5	9 712889	9 932685	9 780203	10 219797	10 067315	10 287111	55
10	9 713935	9 932304	9 781631	10 218369	10 067696	10 286065	50
15	9 714978	9 931921	9 783056	10 216944	10 068079	10 285022	45
20	9 716017	9 931537	9 784479	10 215521	10 068463	10 283983	40
25	9 717053	9 931152	9 785900	10 214100	10 068848	10 282947	35
30	9 718085	9 930766	9 787319	10 212681	10 069234	10 281915	30
35	9 719114	9 930378	9 788736	10 211264	10 069622	10 280886	25
40	9 720140	9 929989	9 790151	10 209849	10 070011	10 279860	20
45	9 721162	9 929599	9 791563	10 208437	10 070401	10 278838	15
50	9 722181	9 929207	9 792974	10 207026	10 070793	10 277819	10
55	9 723197	9 928815	9 794383	10 205617	10 071185	10 276803	5
60	9 724210	9 928420	9 795789	10 204211	10 071580	10 275790	0
M	co-sine.	Sine.	co-tang.	tangent.	co-sec.	secant.	M

58 Degrees.

32 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 724210	9 928420	9 795789	10 204211	10 071580	10 275790	60
5	9 725219	9 928025	9 797194	10 202806	10 071975	10 274781	55
10	9 726225	9 927629	9 798596	10 201404	10 072371	10 273775	50
15	9 727228	9 927231	9 799997	10 200003	10 072769	10 272772	45
20	9 728227	9 926831	9 801396	10 198604	10 073169	10 271773	40
25	9 729223	9 926431	9 802792	10 197208	10 073569	10 270777	35
30	9 730217	9 926029	9 804187	10 195813	10 073971	10 269783	30
35	9 731206	9 925626	9 805580	10 194420	10 074374	10 268794	25
40	9 732193	9 925222	9 806971	10 193029	10 074778	10 267807	20
45	9 733177	9 924816	9 808361	10 191639	10 075184	10 266823	15
50	9 734157	9 924409	9 809748	10 190252	10 075591	10 265843	10
55	9 735135	9 924001	9 811134	10 188866	10 075999	10 264865	5
60	9 736109	9 923591	9 812517	10 187483	10 076409	10 263891	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

57 Degrees.

33 Degrees.

M	sine.	co-sine	tangent.	co-tang.	secant.	co-sec.	M
0	9 736109	9 923591	9 812517	10 187483	10 076409	10 263891	60
5	9 737080	9 923181	9 813899	10 186101	10 076819	10 262920	55
10	9 738048	9 922768	9 815279	10 184721	10 077232	10 261952	50
15	9 739013	9 922355	9 816658	10 183342	10 077645	10 260987	45
20	9 739975	9 921940	9 818035	10 181965	10 078060	10 260025	40
25	9 740934	9 921524	9 819410	10 180590	10 078476	10 259066	35
30	9 741889	9 921107	9 820783	10 179217	10 078893	10 258111	30
35	9 742842	9 920688	9 822154	10 177846	10 079312	10 257158	25
40	9 743792	9 920268	9 823524	10 176476	10 079732	10 256208	20
45	9 744739	9 919846	9 824893	10 175107	10 080154	10 255261	15
50	9 745683	9 919424	9 826259	10 173741	10 080576	10 254317	10
55	9 746624	9 919000	9 827624	10 172376	10 081000	10 253376	5
60	9 747562	9 918574	9 828987	10 171013	10 081426	10 252438	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

56 Degrees.

34 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	9 747562	9 918574	9 828987	10 171013	10 081426	10 252438	60
5	9 748497	9 918147	9 830349	10 169651	10 081853	10 251503	55
10	9 749429	9 917719	9 831709	10 168291	10 082281	10 250571	50
15	9 750358	9 917290	9 833068	10 166932	10 082710	10 249642	45
20	9 751284	9 916859	9 834425	10 165575	10 083141	10 248716	40
25	9 752208	9 916427	9 835780	10 164220	10 083573	10 247792	35
30	9 753128	9 915994	9 837134	10 162866	10 084006	10 246872	30
35	9 754046	9 915559	9 838487	10 161513	10 084441	10 245954	25
40	9 754960	9 915123	9 839838	10 160162	10 084877	10 245040	20
45	9 755872	9 914685	9 841187	10 158813	10 085315	10 244128	15
50	9 756782	9 914246	9 842535	10 157465	10 085754	10 243218	10
55	9 757688	9 913806	9 843882	10 156118	10 086194	10 242312	5
60	9 758591	9 913365	9 845227	10 154773	10 086635	10 241409	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

55 Degrees.

35 Degrees.

M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M
0	9 758591	9 913365	9 845227	10 154773	10 086635	10 241409	60
5	9 759492	9 912922	9 846570	10 153430	10 087078	10 240508	55
10	9 760390	9 912477	9 847913	10 152087	10 087523	10 239610	50
15	9 761285	9 912031	9 849254	10 150746	10 087969	10 238715	45
20	9 762177	9 911584	9 850593	10 149407	10 088416	10 237823	40
25	9 763067	9 911136	9 851931	10 148069	10 088864	10 236933	35
30	9 763954	9 910686	9 853268	10 146732	10 089314	10 236046	30
35	9 764838	9 910235	9 854603	10 145397	10 089765	10 235162	25
40	9 765720	9 909782	9 855938	10 144062	10 090218	10 234280	20
45	9 766598	9 909328	9 857270	10 142730	10 090672	10 233402	15
50	9 767475	9 908873	9 858602	10 141398	10 091127	10 232525	10
55	9 768348	9 908416	9 859932	10 140068	10 091584	10 231652	5
60	9 769219	9 907958	9 861261	10 138739	10 092042	10 230781	0
M	Co-sine	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

54 Degrees.

36 Degrees.

M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M
0	9 769219	9 907958	9 861261	10 138739	10 092042	10 230781	60
5	9 770087	9 907498	9 862589	10 137411	10 092502	10 229913	55
10	9 770952	9 907037	9 863915	10 136085	10 092963	10 229048	50
15	9 771815	9 906575	9 865240	10 134760	10 093425	10 228185	45
20	9 772675	9 906111	9 866564	10 133436	10 093889	10 227325	40
25	9 773533	9 905645	9 867687	10 132113	10 094355	10 226467	35
30	9 774388	9 905179	9 869209	10 130791	10 094821	10 225612	30
35	9 775240	9 904711	9 870529	10 129471	10 095289	10 224760	25
40	9 776090	9 904241	9 871849	10 128151	10 095759	10 223910	20
45	9 776937	9 903770	9 873167	10 126833	10 096230	10 223063	15
50	9 777781	9 903298	9 874484	10 125516	10 096702	10 222219	10
55	9 778624	9 902824	9 875800	10 124200	10 097176	10 221376	5
60	9 779463	9 902349	9 877114	10 122886	10 097651	10 220537	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co sec.	Secant.	M

53 Degrees.

37 Degrees.

M	Sine.	co-sine.	Tang.	co-tang.	Secant.	co-secant.	M
0	9 779463	9 902349	9 877114	10 122886	10 097651	10 220537	60
5	9 780300	9 901872	9 878428	10 121572	10 098128	10 219700	55
10	9 781134	9 901394	9 879741	10 120259	10 098606	10 218866	50
15	9 781966	9 900914	9 881052	10 118948	10 099086	10 218034	45
20	9 782796	9 900433	9 882363	10 117637	10 099567	10 217204	40
25	9 783623	9 899951	9 883672	10 116328	10 100049	10 216377	35
30	9 784447	9 899467	9 884980	10 115020	10 100533	10 215553	30
35	9 785269	9 898981	9 886288	10 113712	10 101019	10 214731	25
40	9 786089	9 898494	9 887594	10 112406	10 101506	10 213911	20
45	9 786906	9 898006	9 888900	10 111100	10 101994	10 213094	15
50	9 787720	9 897516	9 890204	10 109796	10 102484	10 212280	10
55	9 788532	9 897025	9 891507	10 108493	10 102975	10 211468	5
60	9 789342	9 896532	9 892810	10 107190	10 103468	10 210650	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant	M

52 Degrees.

38 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 789342	9 896532	9 892810	10 107190	10 103468	10 210658	60
5	9 790149	9 896038	9 894111	10 105889	10 103962	10 209851	55
10	9 790954	9 895542	9 895412	10 104588	10 104458	10 209046	50
15	9 791757	9 895045	9 896712	10 103288	10 104955	10 208243	45
20	9 792557	9 894546	9 898010	10 101990	10 105454	10 207443	40
25	9 793354	9 894046	9 899308	10 100692	10 105954	10 206646	35
30	9 794150	9 893544	9 900605	10 099395	10 106456	10 205850	30
35	9 794942	9 893041	9 901901	10 098099	10 106959	10 205058	25
40	9 795733	9 892536	9 903197	10 096803	10 107464	10 204267	20
45	9 796521	9 892030	9 904491	10 095509	10 107970	10 203479	15
50	9 797307	9 891523	9 905784	10 094216	10 108477	10 202693	10
55	9 798091	9 891013	9 907077	10 092923	10 108987	10 201909	5
60	9 798872	9 890503	9 908369	10 091631	10 109497	10 201128	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

51 Degrees.

39 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	9 798872	9 890503	9 908369	10 091631	10 109497	10 201128	60
5	9 799651	9 889990	9 909660	10 090340	10 110010	10 200349	55
10	9 800427	9 889477	9 910951	10 089049	10 110523	10 199573	50
15	9 801201	9 888961	9 912240	10 087760	10 111039	10 198799	45
20	9 801973	9 888444	9 913529	10 086471	10 111556	10 198027	40
25	9 802743	9 887926	9 914817	10 085183	10 112074	10 197257	35
30	9 803511	9 887406	9 916104	10 083896	10 112594	10 196489	30
35	9 804276	9 886885	9 917391	10 082609	10 113115	10 195724	25
40	9 805039	9 886362	9 918677	10 081323	10 113638	10 194961	20
45	9 805799	9 885837	9 919962	10 080038	10 114163	10 194201	15
50	9 806557	9 885311	9 921247	10 078753	10 114689	10 193443	10
55	9 807314	9 884783	9 922530	10 077470	10 115217	10 192686	5
60	9 808067	9 884254	9 923813	10 076187	10 115746	10 191933	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

50 Degrees

40 Degrees.

M	Sine.	Co-sine.	Tang.	Co-tang.	Secant.	Co-sec.	M
0	9 808067	9 884254	9 923813	10 076187	10 115746	10 191933	60
5	9 808819	9 883723	9 925096	10 074904	10 116277	10 191181	55
10	9 809569	9 883191	9 926378	10 073622	10 116809	10 190431	50
15	9 810316	9 882657	9 927659	10 072341	10 117343	10 189684	45
20	9 811061	9 882121	9 928940	10 071060	10 117879	10 188939	40
25	9 811804	9 881584	9 930220	10 069780	10 118416	10 188196	35
30	9 812544	9 881046	9 931499	10 068501	10 118954	10 187456	30
35	9 813283	9 880505	9 932778	10 067222	10 119495	10 186717	25
40	9 814019	9 879963	9 934056	10 065944	10 120037	10 185981	20
45	9 814753	9 879420	9 935333	10 064667	10 120580	10 185247	15
50	9 815485	9 878875	9 936610	10 063390	10 121125	10 184515	10
55	9 816215	9 878328	9 937887	10 062113	10 121672	10 183785	5
60	9 816943	9 877780	9 939163	10 060837	10 122220	10 183057	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

49 Degrees.

41 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 816943	9 877780	9 939163	10 060837	10 122220	10 183057	60
5	9 817668	9 877230	9 940438	10 059562	10 122770	10 182332	55
10	9 818392	9 876678	9 941714	10 058286	10 123322	10 181608	50
15	9 819113	9 876125	9 942988	10 057012	10 123875	10 180887	45
20	9 819832	9 875571	9 944262	10 055738	10 124429	10 180168	40
25	9 820550	9 875014	9 945535	10 054465	10 124986	10 179450	35
30	9 821265	9 874456	9 946808	10 053192	10 125544	10 178735	30
35	9 821977	9 873896	9 948081	10 051919	10 126104	10 178023	25
40	9 822688	9 873335	9 949353	10 050647	10 126665	10 177312	20
45	9 823397	9 872772	9 950625	10 049375	10 127228	10 176603	15
50	9 824104	9 872208	9 951896	10 048104	10 127792	10 175896	10
55	9 824808	9 871641	9 953167	10 046833	10 128359	10 175192	5
60	9 825511	9 871073	9 954437	10 045563	10 128927	10 174489	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

48 Degrees.

42 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 825511	9 871073	9 954437	10 045563	10 128927	10 174489	60
5	9 826211	9 870504	9 955707	10 044293	10 129496	10 173789	55
10	9 826910	9 869933	9 956977	10 043023	10 130067	10 173090	50
15	9 827606	9 869360	9 958246	10 041754	10 130640	10 172394	45
20	9 828301	9 868785	9 959516	10 040484	10 131215	10 171699	40
25	9 828993	9 868209	9 960784	10 039216	10 131791	10 171007	35
30	9 829683	9 867631	9 962052	10 037948	10 132369	10 170317	30
35	9 830372	9 867051	9 963320	10 036680	10 132949	10 169628	25
40	9 831058	9 866470	9 964588	10 035412	10 133530	10 168942	20
45	9 831742	9 865887	9 965855	10 034145	10 134113	10 168258	15
50	9 832425	9 865302	9 967123	10 032877	10 134698	10 167575	10
55	9 833105	9 864716	9 968389	10 031611	10 135284	10 166895	5
60	9 833783	9 864127	9 969656	10 030344	10 135873	10 166217	0
M	co-sine.	sine.	co-tang.	tangent.	co-sec.	secant.	M

47 Degrees.

43 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 833783	9 864127	9 969656	10 030344	10 135873	10 166217	60
5	9 834460	9 863538	9 970922	10 029078	10 136462	10 165540	55
10	9 835134	9 862946	9 972188	10 027812	10 137054	10 164866	50
15	9 835807	9 862353	9 973454	10 026546	10 137647	10 164193	45
20	9 836477	9 861758	9 974719	10 025281	10 138242	10 163523	40
25	9 837146	9 861161	9 975985	10 024015	10 138839	10 162854	35
30	9 837812	9 860562	9 977250	10 022750	10 139438	10 162188	30
35	9 838477	9 859962	9 978515	10 021485	10 140038	10 161523	25
40	9 839140	9 859360	9 979780	10 020220	10 140640	10 160860	20
45	9 839800	9 858756	9 981044	10 018956	10 141244	10 160200	15
50	9 840459	9 858157	9 982309	10 017691	10 141849	10 159541	10
55	9 841116	9 857543	9 983573	10 016427	10 142457	10 158884	5
60	9 841771	9 856934	9 984837	10 015163	10 143066	10 158229	0
M	Co-sine	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

46 Degrees.

44 Degrees.

M	sine.	co-sine.	tangent.	co-tang.	secant.	co-sec.	M
0	9 841771	9 856934	9 984837	10 015163	10 143066	10 158229	60
5	9 842424	9 856323	9 986101	10 013899	10 143677	10 157576	55
10	9 843076	9 855711	9 987365	10 012635	10 144289	10 156924	50
15	9 843725	9 855096	9 988629	10 011371	10 144904	10 156275	45
20	9 844372	9 854480	9 989893	10 010107	10 145520	10 155628	40
25	9 845018	9 853862	9 991156	10 008844	10 146138	10 154982	35
30	9 845662	9 853242	9 992420	10 007580	10 146758	10 154338	30
35	9 846304	9 852620	9 993683	10 006317	10 147380	10 153696	25
40	9 846944	9 851997	9 994947	10 005053	10 148003	10 153056	20
45	9 847582	9 851372	9 996210	10 003790	10 148623	10 152418	15
50	9 848218	9 850745	9 997473	10 002527	10 149255	10 151782	10
55	9 848852	9 850116	9 998737	10 001263	10 149884	10 151148	5
60	9 849485	9 849485	10 000000	10 000000	10 150515	10 150515	0
M	Co-sine.	Sine.	Co-tang.	Tang.	Co-sec.	Secant.	M

45 Degrees.